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# Evaluation of undrained shear modulus $G_u$ of cohesive soils in a Hollow Cylinder Apparatus

Evaluation du module de cisaillement non drainé  $G_u$  de sols cohésifs dans un appareil à cylindre creux

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**ABSTRACT:** The paper presents test results in a Hollow Cylinder Apparatus to determine the shear modulus  $G_u$  in undrained conditions. Values of the undrained shear modulus  $G_u$  were determined at shear strain 0.1% and 0.5%. Laboratory tests were performed on lightly overconsolidated clay (*Cl*) and sandy silty clay (*sasiCl*) with an overconsolidation ratio *OCR* about 3.5 and 2.7 and a plasticity index  $I_p$  equal to 77.6% and 34.7%, respectively. HCA tests were carried out with anisotropic consolidation and shearing in undrained conditions. The obtained results have allowed to assess the influence of rotation of the principal stress directions on the value of the shear modulus  $G_u$  in undrained conditions.

**RÉSUMÉ:** L'article présente les résultats d'essai dans un appareil à cylindre creux pour déterminer le module de cisaillement  $G_u$  dans des conditions non drainées. Les valeurs du module de cisaillement  $G_u$  ont été déterminées à la contrainte de cisaillement 0.1% et 0.5%. Les essais en laboratoire ont été effectués sur de l'argile légèrement surconsolidée (*Cl*) et de l'argile limoneuse sableuse (*sasiCl*) avec un coefficient de surconsolidation *OCR* d'environ 3.5 et 2.7 et l'indice de plasticité  $I_p$  égal à 77.6% et 34.7%. Les essais HCA ont été réalisés avec la consolidation anisotrope et le cisaillement dans des conditions non drainées. Les résultats obtenus des essais au laboratoire ont permis d'évaluer l'influence de la rotation des directions des contraintes principales sur la valeur du module de cisaillement  $G_u$  dans les conditions non drainées.

**KEYWORDS:** shear modulus, cohesive soil, Hollow Cylinder Apparatus, principal stress directions.

## 1 INTRODUCTION

Prediction of subsoil deformation around retaining structures requires determination of the deformation and strength characteristics appropriate for the applied soil model in numerical analysis. Common deformation parameters in elastoplastic models, such as the shear modulus  $G$  and the bulk modulus  $K$  do not have constant values for a particular soil. The value of these parameters depends mainly on the stress state and history, as well as strain range. In practice, the strain range does not exceed 0.5% for serviceability limit states of diaphragm walls (Atkinson and Sällfors 1991, Burland 1989, Lo Presti et al. 1999). The shear modulus  $G$  characterizes the soil reaction to shear deformation and is commonly used in geotechnical engineering. The shear modulus  $G'$  refers to drained conditions and the shear modulus  $G_u$  corresponds to undrained conditions.

The triaxial test is most commonly used for evaluation of the shear modulus in the laboratory. Large progress in the capability of the triaxial apparatus for the determination of parameters of soil stiffness has been made by using a triaxial cell with internal linking bars, internal measurement of sample deformation and shear wave velocity measurement (Jardine et al. 1984, Jardine 2013, Lipiński and Wdowska 2015). The device that is also used to determine the shear modulus is resonant column, which allows to specify the value in the range of very small strains ( $10^{-4}$ – $10^{-1}$  %). The shear modulus  $G$  can be determined in a Hollow Cylinder Apparatus, which allows to assess the influence of rotation of the principal stress directions (Zdravković and Jardine 2000, Wrzesiński and Lechowicz 2013, Jardine 2015).

This paper presents the results of laboratory tests carried out in a Torsional Shear Hollow Cylinder Apparatus on undisturbed cohesive soils collected from the excavation of selected stations of the II underground line in Warsaw. Laboratory tests have allowed to determine the influence of rotation of the principal

stress directions caused by the construction of diaphragm walls on the values of the undrained shear modulus  $G_u$  of overconsolidated cohesive soils.

## 2 LABORATORY TESTS

The tests were carried out in the Water Centre Laboratory of the Warsaw University of Life Sciences – SGGW using a Torsional Shear Hollow Cylinder Apparatus. Cylindrical specimens had an internal diameter of 60 mm, external diameter of 100 mm and height of about 200 mm. This geometry was selected to minimize the stress non-uniformities across the wall of the hollow cylindrical specimen when different internal and external pressures or torsional shear stresses were applied (Sayao and Vaid 1991).

The research was performed with anisotropic consolidation and shearing in undrained conditions (*CAU*) on two types of undisturbed cohesive soil – clay (*Cl*) and sandy silty clay (*sasiCl*). Undisturbed samples of cohesive soils were collected from the depth of 22 m and 13 m during the excavation of the Copernicus Science Centre Station of the II underground line in Warsaw. The effective vertical stresses  $\sigma'_v$  were equal to 310 kPa in clay (*Cl*) and 220 kPa in sandy silty clay (*sasiCl*). Based on oedometer tests the overconsolidation ratio *OCR* for tested soil samples were determined. The clay samples had an overconsolidation ratio *OCR* = 3.5 and plasticity index  $I_p$  = 77.6%, whereas the samples of sandy silty clay had *OCR* = 2.7 and  $I_p$  = 34.7%. The index properties of the tested soils are presented in Table 1.

The undrained shear modulus  $G_u$  was determined at angles of the principal stress rotation  $\alpha$  equal to 0°, 30°, 45°, 60° and 90° for clay and equal to 0°, 15°, 30°, 45°, 60°, 75° and 90° for sandy silty clay. HCA tests were performed in six consecutive stages: flushing, saturation, consolidation, change of intermediate principal stress parameter  $b$ , change of angle of the

Table 1. Index properties of the tested soils.

Type of soil	$w_L$ [%]	$w_p$ [%]	$I_p$ [%]	$I_L$ [-]	$I_C$ [-]	Fraction (ISO 14688-2:2004)			
						[%]			
						<i>gr</i>	<i>sa</i>	<i>si</i>	<i>cl</i>
Cl	112.9	35.3	77.6	-0.06	1.06	0	6	37	57
sasiCl	59.0	24.3	34.7	0.13	0.87	0	21	50	29

Explanations:  $w_L$  – liquid limit,  $w_p$  – plastic limit,  $I_p$  – plasticity index,  $I_L$  – liquidity index,  $I_C$  – consistency index, *gr* – gravel, *sa* – sand, *si* – silt, *cl* – clay.

principal stress rotation  $\alpha$  and finally, shearing in undrained conditions. During flushing, air and gases with the largest dimensions were removed from the samples and tubes. Saturation of soil samples was performed using the back pressure method. This stage lasted until the value of Skempton's parameter  $B$  exceeded 0.95. After that, anisotropic consolidation was performed. In the case of clay, the value of  $K_o$  during the consolidation process was equal to 0.97, whereas for sandy silty clay, the  $K_o$  was equal to 0.83. The next step was to change parameter  $b$  from value 0 to 0.5. After that the value of angle  $\alpha$  changed to the determined value in a particular test. Finally, the process of sample shearing was carried out in the stress path involving increase in the deviator stress  $q$  and constant value of the total mean stress  $p$  (Figs. 1 and 3). The values of total mean stress  $p$  during tests were equal to 1002 kPa for clay (*Cl*) and 696 kPa for sandy silty clay (*sasiCl*) while

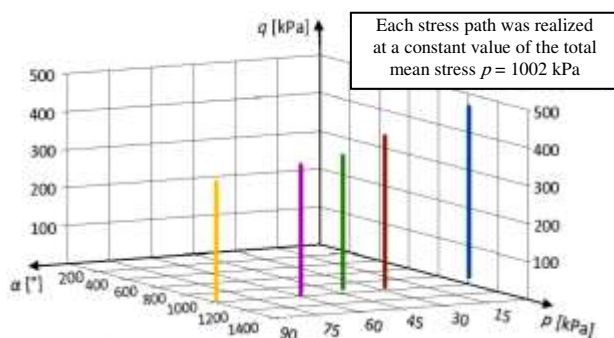


Figure 1. Total stress paths in clay (*Cl*) presented in a  $q$ - $p$ - $\alpha$  space ( $q$  – deviator stress,  $p$  – total mean stress,  $\alpha$  – angle of the principal stress rotation).

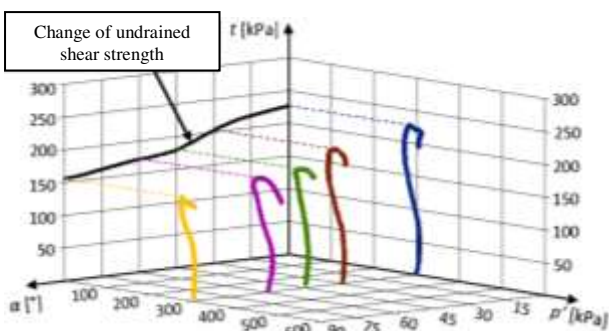


Figure 2. Effective stress paths in clay (*Cl*) presented in a  $t$ - $p'$ - $\alpha$  space ( $t$  – half of deviator stress,  $p'$  – effective mean stress,  $\alpha$  – angle of the principal stress rotation).

the values of initial effective mean stress  $p'$  were equal to 302 kPa for clay (*Cl*) and 196 kPa for sandy silty clay (*sasiCl*). During the entire shearing process of the soil samples, constant values of parameter  $b$  and angle  $\alpha$  were retained. Effective stress paths for clay (*Cl*) and sandy silty clay (*sasiCl*) are presented in Figures 2 and 4. A detailed description of the laboratory tests is presented in the Phd thesis by Wrzesiński (2016).

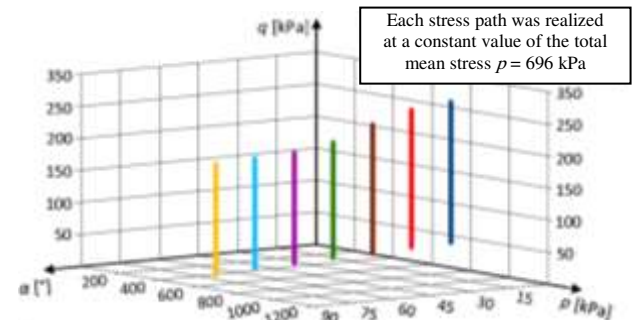


Figure 3. Total stress paths in sandy silty clay (*sasiCl*) presented in a  $q$ - $p$ - $\alpha$  space ( $q$  – deviator stress,  $p$  – total mean stress,  $\alpha$  – angle of the principal stress rotation).

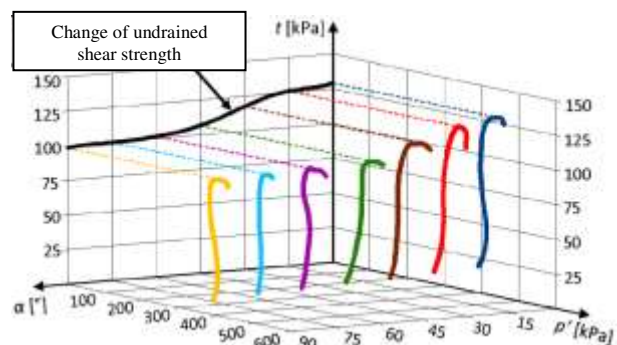


Figure 4. Effective stress paths in sandy silty clay (*sasiCl*) presented in a  $t$ - $p'$ - $\alpha$  space ( $t$  – half of deviator stress,  $p'$  – effective mean stress,  $\alpha$  – angle of the principal stress rotation).

### 3 TEST RESULTS

Studies carried out in a Hollow Cylinder Apparatus enabled the determination of the characteristics, which were used to determine the shear modulus  $G_u$  in undrained conditions at shear strain equal to 0.1% and 0.5%.

Selected characteristics such as deviator stress  $q$  normalized by initial effective mean stress  $p'_0$  for clay (*Cl*) and sandy silty clay (*sasiCl*) are presented in Figures 5 and 6. To better describe the stress state of a specimen under combined axial-torsional loading, a coordinate system with abscissa representing the difference between the effective vertical stress and the effective circumferential stress  $\sigma'_z - \sigma'_\theta$  and the ordinate of shear stress  $\tau_{z\theta}$  were used as shown in Figures 7 and 8. Both axes were normalized by initial effective mean stress. The presented stress paths show that the angles of principal stress rotation were controlled very precisely during the tests. Failure envelopes were obtained for peak points for the following failure criteria: maximum deviator stress and maximum effective principal stress ratio.

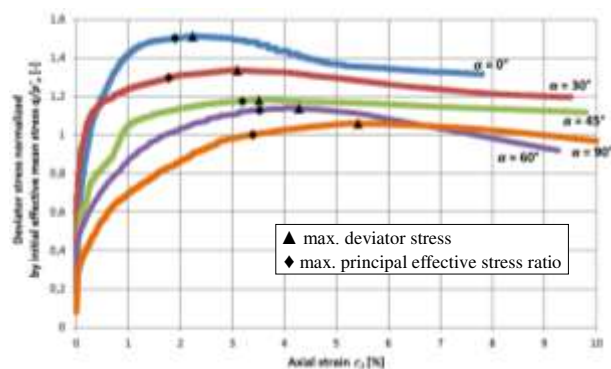


Figure 5. Deviator stress  $q$  normalized by initial effective mean stress  $p'_0$  for clay ( $Cl$ ).

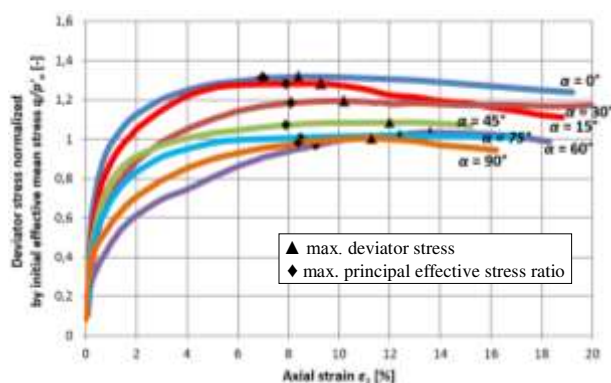


Figure 6. Deviator stress  $q$  normalized by initial effective mean stress  $p'_0$  for sandy silty clay ( $sasiCl$ ).

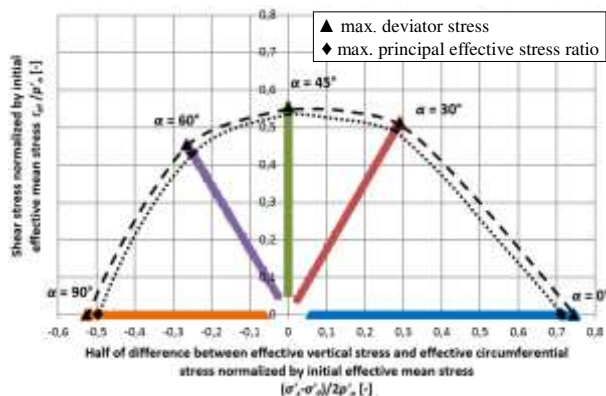
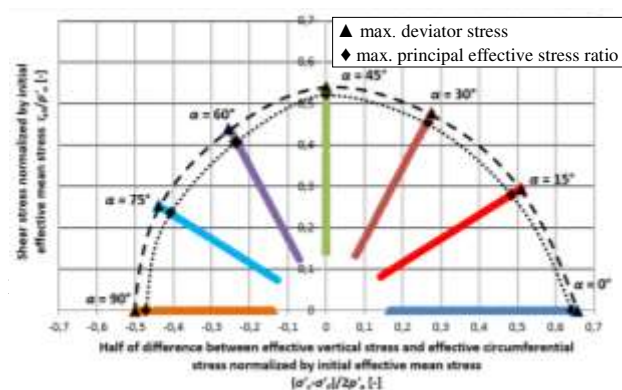


Figure 7. Stress path and failure envelope for clay ( $Cl$ ).



#### 4 EVALUATION OF THE UNDRAINED SHEAR MODULUS $G_U$

Values of the undrained shear modulus  $G_u$  in the performed tests were determined as the ratio of the increase in deviator stress  $q$  to change of the shear  $\gamma_{\theta z}$  according to the equation:

$$G_u = \frac{\Delta q}{\Delta \gamma_{\theta z}} \quad (1)$$

The increase in deviator stress  $\Delta q$  used in equation (2) are equal to values obtained during change of the shear strain  $\gamma_{\theta z}$  by 0.1% or 0.5% compared to the initial value in the particular tests. Shear strains were determined based on the equation (Hight et al. 1983):

$$\gamma_{\theta z} = \frac{2\theta(r_o^3 - r_i^3)}{3H(r_o^2 - r_i^2)} \quad (2)$$

where:

$\theta$  – angle of rotation of the sample [-],  
 $r_o$  – outer diameter of the sample [mm],  
 $r_i$  – inner diameter of the sample [mm],  
 $H$  – sample height [mm].

The obtained values of the undrained shear modulus  $G_u$  for the soil at applied angles of principal stress rotation  $\alpha$  are presented in Tables 2 and 3 and also shown in Figures 9 and 10.

Table 2. Values of the undrained shear modulus  $G_u$  at shear strain  $\gamma_{\theta z} = 0.1\%$  and  $\gamma_{\theta z} = 0.5\%$  for clay ( $Cl$ )

$\alpha$ [°]	0	30	45	60	90
$G_{u0.1\%}$ [MPa]	43.1	42.5	38.6	36.5	33.2
$G_{u0.5\%}$ [MPa]	42.4	42.0	37.1	36.1	32.7

Table 3. Values of the undrained shear modulus  $G_u$  at shear strain  $\gamma_{\theta z} = 0.1\%$  and  $\gamma_{\theta z} = 0.5\%$  for sandy silty clay ( $sasiCl$ )

$\alpha$ [°]	0	15	30	45	60	75	90
$G_{u0.1\%}$ [MPa]	35.6	33.1	32.2	30.3	26.2	25.9	25.8
$G_{u0.5\%}$ [MPa]	34.8	32.1	29.8	28.6	26.0	25.4	24.7

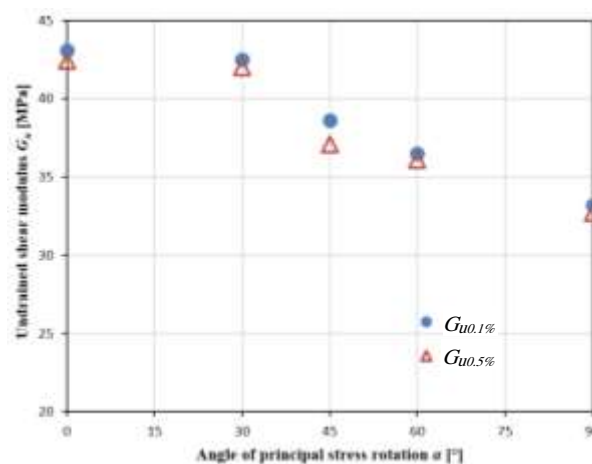


Figure 9. Values of the undrained shear modulus  $G_{u0.1\%}$  at shear strain  $\gamma_{\theta z} = 0.1\%$  and  $G_{u0.5\%}$  at  $\gamma_{\theta z} = 0.5\%$  depending on the angle of principal stress rotation  $\alpha$  for clay ( $Cl$ ).

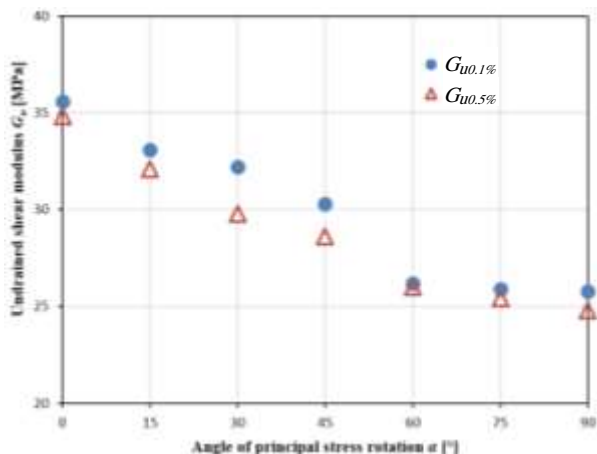


Figure 10. Values of the undrained shear modulus  $G_{u0.1\%}$  at shear strain  $\gamma_{ax} = 0.1\%$  and  $G_{u0.5\%}$  at  $\gamma_{ax} = 0.5\%$  depending on the angle of principal stress rotation  $\alpha$  for sandy silty clay (*sasiCl*).

Based on the obtained results it can be concluded that the value of the angle of principal stress rotation  $\alpha$  has significant influence on the value of the undrained shear modulus  $G_u$ . In the case of clay (*Cl*) and sandy silty clay (*sasiCl*), the values of the undrained shear modulus  $G_u$  both at shear strain of 0.1% and 0.5% decreases nonlinearly with an increasing angle  $\alpha$ . When comparing the undrained shear modulus  $G_u$  at angle  $\alpha$  equal to 90° and at angle  $\alpha$  equal to 0°, the value of  $G_u$  is almost 25% lower for clay (*Cl*) and almost 30% lower for sandy silty clay (*sasiCl*). The highest decrease in the value of the undrained shear modulus  $G_u$  is observed for angle  $\alpha$  in the range of 30°–60°.

## 5 CONCLUSIONS

The method of determining the undrained shear modulus  $G_u$  was presented on the example of tests carried out on lightly overconsolidated clay (*Cl*) and sandy silty clay (*sasiCl*) with an overconsolidation ratio *OCR* about 3.5 and 2.7 and a plasticity index  $I_p$  equal to 77.6% and 34.7%.

The performed tests have shown that the value of the undrained shear modulus  $G_u$  at shear strain equal to 0.1% and 0.5% decreases with the increasing values of the angle of principal stress rotation  $\alpha$ .

In order to determine the effect of the principal stress rotation on the undrained shear modulus  $G_u$  in other soil types tests should be performed in a Hollow Cylinder Apparatus on soil samples characterized by different values of the following parameters: the overconsolidation ratio *OCR*, the plasticity index  $I_p$  and the coefficient of lateral earth pressure  $K_0$ .

## 6 REFERENCES

- Atkinson J.H., Sällfors G. 1991. Experimental determination of soil properties. Proc. 10th ECSMFE, Firenze, 3, 915–956.
- Burland J.B. 1989: Ninth Lauritis Bjerrum Memorial Lecture: Small is beautiful: the stiffness of soils at small strains. Canadian Geotechnical Journal 26(4), 449–516.
- Hight D.W., Gens A., Symes M.J. 1983. The development of a new hollow cylinder apparatus for investigating the effects of principal stress rotation in soils. *Geotechnique* 33(4), 335–383.
- Jardine R.J. 2013. Advanced laboratory testing in research and practice. Bishop lecture. Proc. 18th ICSMGE, Paris, 1, 35–54.
- Jardine R.J. 2015: Investigating the anisotropic shear strength and stiffness behavior of stiff geologically aged clays. Keynote Lecture. 24th EYGEC. Durham, UK.
- Jardine R.J., Symes M.J., Burland J.B. 1984. The measurement of soil stiffness in the triaxial apparatus. *Geotechnique* 34 (3), 323–340.

- Lipiński M. J., Wdowska M. K. 2015. Capability and limitations in laboratory determination of stiffness parameters of soils. *Ann. Warsaw Univ. Life Sci. – SGGW, Land Reclam.* 47 (2), 139–151.
- Lo Presti D.C.F., Pallara O., Cavallaro A., Jamiolkowski M. 1999: Anisotropy of small strain stiffness of undisturbed and reconstituted clays. Prefailure Deformation Characteristic of Geomaterial. Balkema, Rotterdam. 3–9.
- Sayao A.S.F., Vaid Y.P. 1991. Deformations due to principal stress rotation. In Proc. 12th ICSMFE. Rio de Janeiro. 1, 107–110.
- Wrzesiński G., Lechowicz Z. 2013. Influence of the rotation of principal stress directions on undrained shear strength. *Annals of Warsaw University of Life Sciences - SGGW. Land Reclamation* 45(2), 183–192.
- Wrzesiński G. 2016. *Stability analysis of an embankment with influence of the principal stress rotation on the shear strength of subsoil*. PhD Thesis. Warsaw University of Life Sciences – SGGW. [in Polish].
- Zdravković L., Jardine R.J. 2000: Undrained anisotropy of  $K_0$ -consolidated silt. *Canadian Geotechnical Journal*. Vol. 37(1), 178–200.
- ISO 14688-2:2004. Geotechnical investigation and testing. Identification and classification of soil. Part 2: Principles for a classification.