Constitutive and rheological modelling: rock avalanches

Modélisation constitutive et rhéologique: avalanches rocheuse

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ABSTRACT: Rock avalanches present high velocities of propagation, and large volume of materials that generate extremely mobility capable of travelling long distance and spreading over large areas causing high destructive power. The accurate prediction of runout, depth and velocity of rock avalanches is of paramount importance since, in general, preventing it is not possible.

The purpose of this paper is to assess the influence of the rheology on the avalanche properties using a depth integrated, SPH model. The paper compares the performance of different rheological models to reproduce the track, runout and depth of the final deposit for a real event such as Thurwieser rock avalanche. The paper provides information on the proposed model accuracy and limitations.

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RÉSUMÉ: Les avalanches de roches présentent des vitesses de propagation élevées et un grand volume de matériaux qui génèrent une mobilité extrême capable de parcourir de longues distances et de se répandre sur de vastes zones provoquant une forte puissance destructrice. La prédiction exacte du décalage, de la profondeur et de la vitesse des avalanches rocheuses est d'une importance primordiale puisque, en général, la prévention de ce n'est pas possible.

Le but de cet article est d'évaluer l'influence de la rhéologie sur les propriétés d'avalanche à l'aide d'un modèle SPH intégré en profondeur. Le papier compare les performances de différents modèles rhéologiques pour reproduire la piste, l'évanouissement et la profondeur du dépôt final pour un événement réel tel que l'avalanche de roche de Thurwieser. Le document fournit des renseignements sur la précision et les limites du modèle propose.

KEYWORDS: rock avalanche propagation modelling, depth integrated model,

1 INTRODUCTION

Landslides cause a large number of casualties around the world every year. In order to assess the hazard and the vulnerability of the territory, engineers could use advanced simulation tools suitable for foreseeing the landslide propagation path, its velocity and the height of the deposits. These tools aid to understand the mechanisms involved in the processes of both triggering and propagation.

Rock avalanches present high velocities of propagation, and large volume of materials that generate extremely mobility capable of travelling long distance and spreading over large areas (Hungr, 2004). The accurate prediction of runout, depth and velocity of rock avalanches is of paramount importance since, in general, preventing it is not possible.

The approach we propose in this work is based on continuum mechanics, and consists in a depth integrated mathematical model, which is discretized by using the SPH method. Depth integrated models present a reasonable compromise between computational cost and accuracy (Savage and Hutter (1991), Pastor et al (2009).

A fundamental ingredient is the model used to describe the behaviour of the fluidized material. Most of the approaches used so far are based on rheological laws relating effective stress and rate of deformation tensors. On the other hand, triggering is usually described using constitutive models where the increments of the stress and strain tensors are given by a suitable constitutive law.

This makes the whole process difficult to model, because at a certain moment one has to switch from a constitutive law to a rheological model (Cuomo et al. 2012)

A possible alternative has been to use viscoplasticity of Perzyna type, because it can provide suitable laws both for the solid and the fluidized behaviours, as is shown in (Pastor et al. 2010,Pastor et al 2013)

One most interesting conclusion is that from Perzyna's viscoplasticity, it is possible to derive simple rheological laws, which can be used for frictional fluids.

The purpose of this paper is to apply such types of laws to a special case of a rock avalanche: Thurwieser Avalanche (Sossio and Crosta 2007), comparing the results obtained with them and with other more classical approximations based on Voellmy or frictional fluid laws, such as those presented in Pastor et al (2009) and Manzanal et al. (2016).

2 GENERAL FRAMEWORK

2.1 Depth average mathematical model

Depth integrated models are a convenient simplification of 3D models, providing an acceptable compromise between computational cost and accuracy.

Savage and Hutter (1991) proposed their 1D lagrangian model for the case of avalanche dynamics, where a simple Mohr-Coulomb model allowed a description of the granular material behaviour.

Depth averaged models are obtained by integrating along depth the balance of mass and momentum equations. Details of the general framework can be found in Pastor et al. (2009).

2.2 SPH approach

Smoothed particle hydrodynamics (SPH) is a meshless method introduced independently by Lucy (1977) and Ging old and Monaghan (1977). It has been applied to a large variety of problems. For avalanches propagation we can menetion the recent work of Rodriguez-Paz and Bonet, 20 05; McDougall and Hungr, 2004; Pastor et al 2009, 2013.

Fast landslides is treated as fluidized masses of either soil or rock blocks to simulate their propagation. SPH is a numerical technique able to describe these phenomena.

Smoothed particle hydrodynamics is based on the appro ximation of given properties and its spatial derivatives by integral approximation defined in term of the smoothed function or kernel function. An interpolation process calcu- lates the relevant properties on each "particle" over neigh- bouring"particles". Therefore, SPH is based on introdu- ci ng a set of nodes $\{x_k\}$ with K=1..N and the nodal variabl es on landslide problem are: height of the landslide atno de I, depth averaged, 2D velocity, surface force vector at the bottom and pore pressure at the basal surface.

If the 2D area associated with node I is Ω_I, we will introduce for convenience: (i) a fictitious volume m_I mov ing with this node: $m_I= \Omega_I h_I$ and (ii) an averaged pressu- re term $p_I = 0.5b_3h_I^2$. m_I has any physical meaning, as when node I moves, the material contained in a column of base Ω_I has entered it or will leave it as the column mo- ves with an averaged velocity which is not the same for all particles in it. Details of the formulation can be found in Pastor et al 2009a, Manzanal et al 2016.

3 CONSTITUTIVE AND RHEOLOGICAL MODELS

3.1 Introductory

Rock avalanches, at large scale, behave as fluidized granular materials. At small scales we can observe phenomena such as inverse grading and crushing of rock blocks, which results into a change in granulometry and in dilatance properties.

The behaviour of this granular fluid can be modelled using either discrete element methods or continuum based rheological models. The former presents the advantage of reproducing in a natural manner crushing and inverse grading phenomena, but the computational cost of modelling a real avalanche is still difficult to afford.

On the other hand, the latter reduces the cost, but suitable models have to be used to describe the constitutive/rheological behaviour of the granular fluid.

And it is because of this fluid like behaviour that rheological models have been used traditionally to model rock avalanches, as in other types of avalanches.

While mudflows, lahars, and some cases of debris flows have been approximated using models such as Bingham or Herschel-Bulkley (Coussot 2005), rock avalanches present a dominant frictional behaviour requiring a different approach.

3.2 Simple infinite landslide

Simple shear infinite landslide models are on of the simplest, which can be used to describe the behaviour of a landslide. The main assumptions are the following (see Fig. 1): (i) flow is steady, (ii) all variables are independent on the position along the landslide, which is assumed to have an infinite length. We will use here x as the abscissa along the infinite plane and z the axis perpendicular to x within the plane.

As the acceleration is zero, the shear stress varies linearly along depth:

$$_{(z)} = \ _b\left(1 \ \frac{z}{h}\right) \tag{3}$$

where $\tau_b = \rho\gamma h \sin\theta$ is the basal shear stress, ρ is the density of the mixture and g the acceleration of gravity. The shear stress depends on the rheological law used. A general approach can be defined as:

$$= s + \left(\frac{\partial v}{\partial z}\right)^m \tag{4}$$

where s is the basal shear strength, μ is the viscosity, m is a model parameter and v is depth average velocity. From here, it is possible to derive simple cases as: (i) newtonian fluids with $s = 0$ and $m = 1$; (ii) Bingham fluids with $s = \tau_y$ and , $m = 1$ where τ_y is the cohesive strength of the fluid; (iii) $s = 0$ and $m = 2$ (Bagnold, 1954) (iv) Visco-Frictional $s = \sigma_n \tan\phi$ where σ_n is the effective stress normal to the basal plane, and ϕ the friction angle, m =2 (Chen et al. 1988, Pastor et al 2009) and for the velocity profile given in Manzanal et al, (2016) is obtained:

$$_b = \ gh\cos \ \tan \ + \frac{25}{4}\frac{1}{h^2}\ _F\bar{v}^2 \tag{5}$$

It is interesting to note the similarity with Voellmy's law (Voellmy 1955):

$$_b = \ gh\cos \ \tan \ + \frac{g\bar{v}^2}{} \tag{6}$$

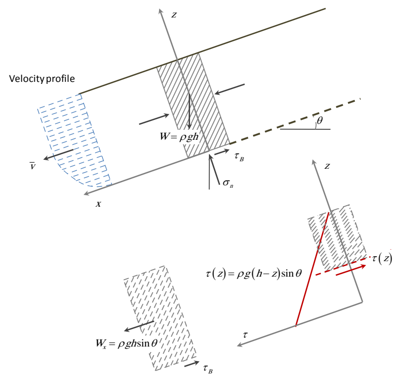


Figure 1. The simple shear infinite landslide model

3.3 Perzyna based rheological models for frictional fluids

There exists an interesting similitude between simple shear rheological laws of the type of equation 4 (see eq. 7) and Perzyna elasto-viscoplastic models in 1D (Perzyna, 1963), where the rate of viscoplastic shear strain is given by equation 8.

$$\left(\frac{\partial v}{\partial z}\right) = \frac{1}{^{1/m}}\left(\ s\right)^{1/m} \tag{7}$$

we can derive a simple shear model which can be used for frictional fluids. By neglecting the elastic components, the shear strain rate is given by:

$$\left(\frac{\partial v}{\partial z}\right)^{vp} = \ _F\left(\frac{\ s}{s}\right)^N \tag{8}$$

The basal shear stress is

$$_b = s_b\left(1+\left(2\ _{PZ}\bar{v}/h\right)^{1/N}\right) \tag{9}$$

where we have introduced $\mu_F = 1/\gamma_F$, which is the inverse of the fluidity γ_F.

As an alternative, it is possible to define

$$\frac{v}{z} = \ _F\frac{\left(\ s\right)^N}{p_{atm}^{n1}s^{n2}} \tag{10}$$

where $N = n1 + n2$ and p_{atm} is the atmospheric pressure or an alternative reference pressure. The velocity profiles depend now on $n1$, being given by $v = v_{max}(1-(1-z/h)^{n1+1}$, where v_{max} is the velocity at the top (z = h). It is important to note that the curvature of the velocity profile depends on $n1$.
Regarding the basal shear stress, it is given by:

$$_b = s_b\left(1+\left(2\ \ _{PZ}\overline{v}/h\right)^{1/N} s_b^{\frac{n2\ \ N}{N}}\right) \qquad (11)$$

where v is the depth averaged velocity. The viscoplastic constitutive model presented derives from the continuum mechanics rather than empirical results as previous rheological laws. This approach allows reproducing the transition between initiation and propagation of the failure in a consistent manner and represents a generalization of the previous rheological law.

So far, we have considered three rheological laws, which can be applied to model frictional fluids: (i) Voellmy law (eq. 6); (ii) The viscous frictional law (eq. 5) and (iii) The Perzyna based law (eq. 12). These laws have been implemented in a SPH depth integrated code, and they will be applied to simulate real rock avalanches and to assess the influence of the rheology on the avalanche properties.

4 VALIDATIONS

4.1 Case study description

Thurwieser rock avalanche took place in the Central Italian Alps on 18th September 2004. The location was the south slope of Punta Thurwieser, and it propagated through Zebrú valley. Its propagation path extended from 3500 m to 2300 m of altitude, with a travel distance of 2.9 Km. Sossio and Crosta 2007 have provided the information concerning this avalanche, including a detailed digital terrain model, the triggering area and the volume of rock involved, estimated to 2.4 million cubic meters. Figure 2 provides a general view of the avalanche and its location.

This avalanche presents several modelling difficulties, such as crossing of terrains of different materials, such as the Zebrú glacier. There, the basal friction is very small, and erosion of ice and snow is possible. This entrained material can melt due to the heat generated by basal friction, providing extra water, and probably originating basal pore pressures. GEOFLOW-SPH can adopt different shear resistance laws along the topography in order to simulate different materials. For the glacial deposit, the adopted basal shear resistance was zero between the altitude 2900 and 3050 m.a.m.s.l.

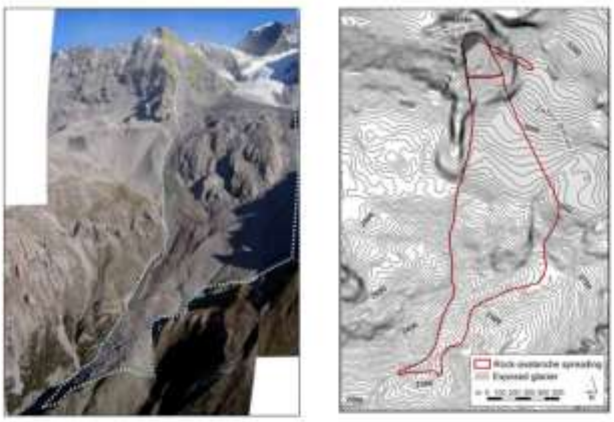


Figure 2. General view of Thurwieser rock avalanche

4.2 Parameter calibrations

In the present work, Thurwieser rock avalanche has been studied with four rheological models (i) the simple frictional rheology, (ii) the Voellmy rheology given by equation (6) (Voellmy, 1955), (iii) cohesive-frictional viscous rheology given by equation (5) (Chen 1988, Pastor et al. 2009b) and viscoplastic- Perzyna rheology given by equation (11). The constitutive parameters have been obtained from back analysis. Since the material involved is mostly dry fragmented rock, the simulations have been carried out with zero pore pressure ratio. Concerning erosion, we have used the law proposed by Hungr (1995).
The rheological parameters for the different rheologies summarized has been:
(i) pure frictional rheological model: $tan\ \phi = 0.39$. (ii) viscous frictional model: $tan\ \phi = 0.39$ and viscous coefficient $\gamma_{CF} = 0.002 kPa.s\text{-}1$ (iii) Voellmy model: $tan\phi = 0.35$,Voellmy coefficient $= 1000\, m/s$ and (iv) viscoplastic - Perzyna model: $s_b = 0.19$, $\mu_{PZ} = 0.001$, $N = 0.5$, $n2 = 0.5$.
These parameters were obtained by trial en error, as the only values reported concerned times of propagation and runout. For the different rheologies used, we adopted a erosion coefficient $0.0030 m^{-1}$.

The longitudinal profiles of Thurwieser avalanche for different rheological models are presented in Figure 3a. Similarly, the comparison of cross section is shown in Figure 3b.

Figure 4 provides the comparison of computed results and field measurements. The results are shown as deposit depth isolines. They are compared with the topographic isolines and the contour of spreading of the rock avalanche. It can be observed that all the rheologies reproduce the run out shape at middle altitude and stay slightly behind for the maximum distal point. However, viscoplastic - Perzyna rheology gives a better approximation of the run out of the spreading and maximum distal point (Figure 4 d).

Concerning deposit height distribution, it can be seen that it fluctuates slightly between different rheologies. Frictional and viscous frictional rheologies concentrate the maximum depth (29 - 31m) at middle altitude of spreading, between 2600 and 2500 m.a.m.l.s. However, reported field measurements (Sossio and Crosta 2007) indicate that the maximum depth of the final deposit focuses between 2450 and 2400 m.a.m.l.s. similar as the numerical simulation of the depth integrated SPH code with viscoplastic - Perzyna models (Figure 4 d). For the Voellmy rheology deposit height distribution varies between 2500 and 2350 and the maximum mobilized mass depth is similar to the averaging values reported (25-28m).

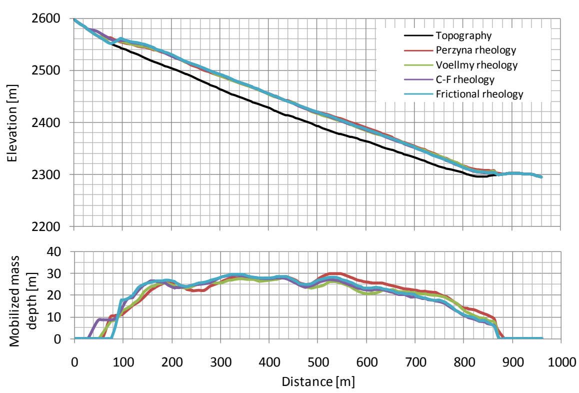
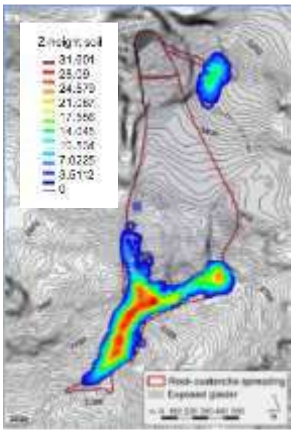
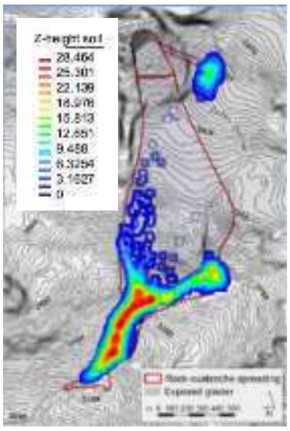


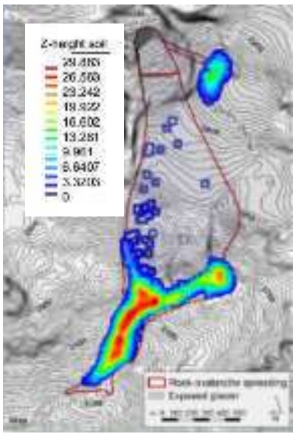
Figure 3 Longitudinal profiles of Thurwieser avalanche for different rheological models



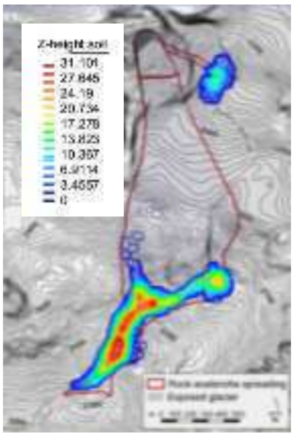
a) Frictional rheology



b) Voellmy rheology



c) Viscous Frictional rheology



d) Viscoplastic-Perzyna rheology

Figure 4. Thurwieser avalanche after 90s. for different rheological models: Computed results (colour isolines and deposit height) versus field measurements (black isolines and red line for the spreading).

5 CONCLUSIONS

The model presented combines a depth-integrated model with basal friction laws obtained from different rheological models to reproduce the propagation of rock avalanches with GEOFLOW-SPH code (Pastor et al 2009). The real case of Thurwieser rock avalanches is analysed. The validation presents accurate prediction of runout, depth and velocity of Thurwieser rock avalanches.

In the authors' opinion, the visco-plastic based models provide a consistent bridge with continuum models and their results agree well with the benchmark presented. This approach allows to reproduce the transition between initiation and propagation of the failure in a consistent manner and represents a generalization of the classical rheological law (Voellmy and frictional).

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