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Three-dimensional analysis of slope stability

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Abstract

It's necessary to cover, from all applicable areas, the necessary investigations to support and determine with complete veracity the relationship between the application of three-dimensional and two-dimensional methods of slopes stability, not only in terms of soils characterization that make up the slope, but also in the different geometries and geological configurations which the engineer can find in the field. When studying slopes with constant cross sections, the slope failure direction is evident, since it must be parallel to the principal incline in the slope, however, for slopes with complex geometries, in which they are handled by different inclines, stability analysis must guarantee the study on the direction of critical failure, something that in two-dimensional methods is complicated, even impossible to achieve, since the different inclines and its crossing on the slope cannot be modeled in two-dimensional analysis methods.

In this work, a simple and clear methodology is developed to evaluate the safety factor of a slope, depending on the slope geometry, geological configuration and a pseudo-static coefficient. The method is developed for the three-dimensional analysis of slope stability based on pre-existing methods of two-dimensional analysis.

This method is based on the theory of limit equilibrium, adapting Janbu's simplified method, which does not consider shear stresses in the vertical planes of the slices. With this adapted method, the analysis of slope stability in three dimensions is carried out considering the spherical fault surface and defining the critical direction of sliding.

Comparing the method with two-dimensional analysis, the results indicate that the three-dimensional method developed is not strictly less conservative than the two-dimensional comparison method but responds to greater accuracy according to the different geometric and geological configurations of the slopes evaluated.

The most relevant conclusion is that the safety factor resulting from 2D analysis (SF2D) must not necessarily be greater than safety factor resulting from 3D analysis SF3D). This aspect is of vital importance, since engineering is constantly evolving and requires more and more precise methods for solving problems. The contribution to the discussion regarding the SF3D / SF2D relationship is clear and objective. Through a case of study, it was possible to demonstrate that this relationship can even be less to 0.9 and that it depends largely on the slope and especially on the geometry of the strata that is present.

1 INTRODUCTION

Through this work, the development of a method for the three-dimensional analysis of slopes is presented, as well as some practical applications of the method and, finally, an analysis of the results obtained is performed.

The method developed is based on the simplified procedure of Janbu, in which the solution of a system of equations is carried out, whose unknown strengths are the resistant forces in a slope. To solve the system of equations the balances of forces are used in each coordinate axis, and equations of summations of ground thrusts.

First, the wedge of failure is discretized in vertical columnar elements of rectangular section. The height of these elements is determined by the vertical separation between the fault surface and the ground surface. The fault surface has spherical geometry and its center is the C point with coordinates Xc, Yc, Zc.

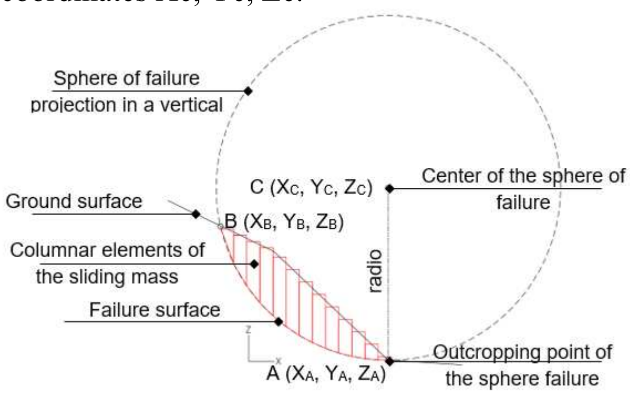

Figure 1. Discretization of the problem - Profile view.

Subsequently, each columnar element is evaluated through a force diagram that allows to identify the acting forces and the resistant forces. The latter represent the unknowns of the system of equations. Additionally, the diagram allows to establish the equilibrium equations in the three coordinate axes “x”, “y” and “z”, as well as the equations of thrust summations in “x” and in “y”. Therefore, these equations, together with the unknowns, form a system of equations that their solutions will result in the safety factors in "x" and "y".

The method is applied to two cases that correspond to: a theoretical slope, with constant cross-section and geologically shaped by a single kind of material; and a theoretical slope, with a constant cross-section and geologically shaped by three different materials.

The method inherits the same limitations of the Janbu procedure. The following are cited:

- It is assumed that there are no shear forces between the vertical walls of the columnar elements.
- A uniform distribution of stresses is assumed at the base of the columnar elements.
- The material is assumed as isotropic.
- It does not satisfy equilibrium of moments, it only satisfies equilibrium of forces in “x”, “y” and “z”.

2 THREE-DIMENSIONAL ANALYSIS METHOD OF SLOPE STABILITY

The Generalized Procedure of Slices (GPS) was developed in 1957 by Nilmar Oskar Charles Janbu and presented in detail in the Casagrande Volume: Embankment-dam Engineering (1973).

This procedure was developed for 2D slopes in which the sliding mass is divided into vertical slices. The generalized method of Janbu considers the normal and tangent forces on the vertical walls of the slices. To determine the safety factor of the slope, 2 equations of balance of forces are required for each slice, an equation of balance of moments in each slice, and an equation of balance of thrusts throughout the sliding mass.

The method used here is based on the mentioned Janbu GPS method. This method does not calculate the shear on the walls of the slices, for that reason it has been called the simplified method of Janbu.

2.1 Domain discretization

To apply the developed 3D method, the slope is discretized by dividing the sliding mass into a set of elements, called columnar elements by their geometric similarity with a column. The distribution of the columnar elements within the sliding mass can be seen in Figure 2, which shows an isometric view of the slope cut by a vertical plane containing the points A and B.

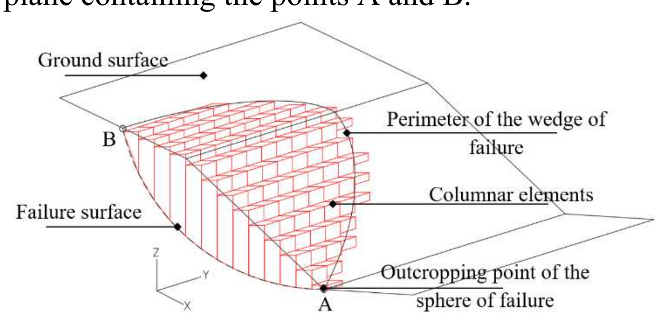

Figure 2. Discretization of the problem - Isometric view.

In addition, this figure details the limitation of the sliding mass from the perimeter of the wedge of failure, as well as the difference in heights that exist between the different columnar elements and that depends on the location of these elements inside the sliding mass. For example, at the edges of the sliding mass, that is, in the areas near the perimeter of the fault wedge, the columnar elements have a very small height, comparatively with the center of the sliding mass, in which the columnar elements has greater height and therefore greater weight in the stability analysis.

2.2 Force diagram

At the base of each columnar element, a curved surface is generated that corresponds to a section of the spherical fault surface with previously defined center and radius. In order to couple the analysis of the balance of forces in the directions of the adopted coordinate system, the base of the columnar element is approximated to a plane, which will have an inclination corresponding to the plane tangent to the central point of the base of the columnar element or point "e".

Both the effective normal force applied at the base of the columnar element, and the shear forces, have an inclined application direction with respect to the coordinate axes, therefore, it is necessary to decompose these reactions into forces that will have "x", "y" and "z" directions.

The forces that interact with each columnar element can be classified into acting forces and resistant forces. The acting forces are the field or body forces, and they are called actions, such is the case of the columnar element's own weight, the water forces or infiltration forces and the forces due to earthquakes. Resistant forces are called reactions and are generated in each element to counteract actions. In this study, the reactions are the forces leading to failure of the material and are directly related to the parameters of soil resistance, such as cohesion and internal friction angle. In accordance with the classification of forces, the force diagram for a columnar element is shown in Figure 3.

For the purposes of this paper, infiltration forces are considered by incorporating a permanent flow network into the model, using Darcy's law and the Dupuit solution for flow through porous media.

To calculate the infiltration force in the "y" direction, it is assumed that the water table follows a parabolic geometry with a given focus and a vertex that contains a point on the upper line of the flow network generated by the Dupuit solution in the "x" direction.

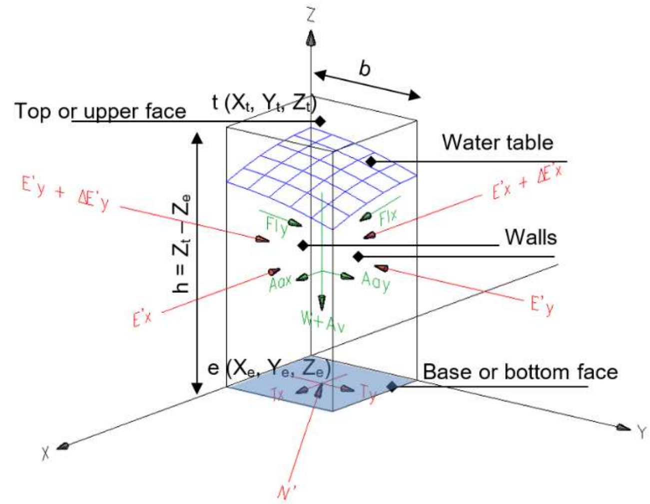

Figure 3. Diagram of forces in each columnar element.

In Figure 3 the actions are presented in green, where

- W: total weight of the columnar element
- Fix, FIy: infiltration force in "x" and "y"
- Av: pseudo-static acceleration in "z"
- Aax, Aay: pseudo-static acceleration in "x" and "y".

While the reactions are presented in red, where

- N': effective normal force at the base
- Tx, Ty: shear force at base in "x" and "y"
- Ex, Ey: thrust force in "x" and "y"
- ΔEx, ΔEy: Change in thrust forces in "x" and "y".

2.3 Safety factor

The safety factor for the Janbu limit equilibrium method according to the force method is:

$$SF = \frac{T_{available}}{T_{required}} = \frac{c'\Delta A + N'\tan(\emptyset')}{T} \quad (1)$$

With:

- SF: Safety factor
- c': Effective cohesion
- ΔA: Base area of the columnar element, shown in figure 3 as the shaded area.
- Ø': Internal friction angle

This safety factor is used in the developed method, except that this one has two safety factors, one in "x" and another in "y" direction.

The area of the base of the columnar element must be calculated considering that the plane of the base is inclined because the base of any element, share its surface with a little part of the sphere of failure. Since the point (X_C, Y_C, Z_C) is the center of the fault sphere, the area of the base of the columnar element is:

$$\Delta A = \left|\frac{b^2 r}{Z_e - Z_c}\right| \qquad (2)$$

And by replacing (2) in (1) the safety factor is obtained based on the geometry of the base of the columnar element and the normal force and shear force acting on it:

$$SF = \frac{c'\left|\frac{b^2 r}{Z_e - Z_c}\right| + N' \tan(\emptyset')}{T} \qquad (3)$$

2.4 System equations

The actions indicated in green in Figure 3 are known forces or variables, while the reactions, shown in red, are unknown forces or variables of the problem to be solved. The normal force at the base of each columnar element is an unknown variable, as well as the thrust in “x” and the thrust in “y”, so, in total there are 3 unknowns variables for each columnar element. There are “n” columnar elements, so the number of unknown variables amounts to 3n.

The shear forces at the base of each element, is expressed as the shear force available on the safety factor, that is:

$$T = \frac{c'\left|\frac{b^2 r}{Z_e - Z_c}\right| + N' tan(\emptyset')}{SF} \qquad (4)$$

With cohesion, the area of the base of the element and the angle of internal friction being known, the shear force is a function of two unknown variables: the normal force and the safety factor. The normal force has already been considered within the 3n unknown variables of the system of equations, so the safety factor of the sliding mass remains to be added.

Although each columnar element may have a safety factor, for the purposes of the stability analysis of this investigation it is considered that there are two safety factors that apply to all columnar elements, one for the “x” direction and the other for the direction "y".

A summary of the unknown variables is shown in Table 1, where a total number of unknown variables is equal to 3n + 2.

Having 3n+2 unknown variables, 3n+2 equations are required to solve the problem. These equations correspond to the balance of forces on each coordinate axis for each columnar element, which gives 3n equations. The other 2 equations correspond to the sum of thrusts in the “x” and “y” direction for the whole set of columnar elements. A summary of these is shown in table 2.

Table 1. Unknown variables of the system of equations

Unknown variable	Number
Normal force at the base (N)	n
Thrust in “x” direction (ΔEx)	n
Thrust in “x” direction (ΔEy)	n
Safety factor in “x” (SFx)	1
Safety factor in “y” (SFy)	1
Total	3n+2

Table 2. Unknown variables of the system of equations

Equation	Number
Sum of forces in “x” (∑Fx=0)	n
Sum of forces in “y” (∑Fy=0)	n
Sum of forces in “z” (∑Fz=0)	n
Sum of thrust in “x” (∑ΔEx=0)	1
Sum of thrust in “y” (∑ΔEy=0)	1
Total	3n+2

The normal force at the base is found by balancing forces in "z" (∑Fz=0) and corresponds to:

$$N' = \frac{W(1 + Av) - fiz - c'\Delta A(fx + fy)}{\frac{Z_c - Z_e}{r} + tan\,\emptyset'\,(fx + fy)} \qquad (5)$$

The dimensionless factors “fx” and “fy” are a function of the geometry of the fault sphere and the location of the columnar element in the sliding mass. The infiltration force factor “fiz” is a function of the geometry of the water table, of the columnar element and of the gradient “i” in both directions “x” and “y”.

$$fx = \frac{1}{SFx} \cdot \frac{(X_c - X_e)}{\sqrt{(Z_c - Z_e)^2 + (X_c - X_e)^2}} \qquad (6)$$

$$fy = \frac{1}{SFy} \cdot \frac{(Y_c - Y_e)}{\sqrt{(Z_c - Z_e)^2 + (Y_c - Y_e)^2}} \qquad (7)$$

$$fiz = \frac{\gamma_w b^2 (h_x i_x + h_y i_y)(Z_B - Z_A)}{\sqrt{(X_B - X_A)^2 + (Z_B - Z_A)^2}} \qquad (8)$$

The thrust in “x” is found by balancing forces in "x" (∑Fx=0) and corresponds to:

$$\Delta E_x' = N' \frac{X_c - X_e}{r} - fzx + fix + W \cdot Aa_x \qquad (9)$$

The shear force transformation factor “fzx” is:

$$fzx = \frac{(c'\Delta A + N'\,tan\,\emptyset')(Z_c - Z_e)}{SF_x \cdot \sqrt{(Z_c - Z_e)^2 + (X_c - X_e)^2}} \qquad (10)$$

$$fix = \frac{\gamma_w b^2 h_x i_x \cdot (X_B - X_A)}{\sqrt{(X_B - X_A)^2 + (Z_B - Z_A)^2}} \quad (11)$$

The infiltration force factor “fix” is a function of the geometry of the gradient “i” in the “x” direction.

The thrust in “y” is found by balancing forces in "y" (∑Fy=0) and corresponds to:

$$\Delta E'_y = N' \frac{Y_c - Y_e}{r} - fzy + fiy + W \cdot Aa_y \quad (12)$$

The shear force transformation factor “fzy” is:

$$fzy = \frac{(c'\Delta A + N' \tan \emptyset')(Z_c - Z_e)}{SF_y \cdot \sqrt{(Z_c - Z_e)^2 + (Y_c - Y_e)^2}} \quad (13)$$

$$fiy = \frac{\gamma_w b^2 h_y i_y \cdot (Y_B - Y_A)}{\sqrt{(Y_B - Y_A)^2 + (Z_B - Z_A)^2}} \quad (14)$$

The infiltration force factor “fiy” is a function of the geometry of the gradient “i” in “y” direction.

From Figure 2 it is observed that the columnar elements that are located on the border at which the fault surface and the ground surface are joined, have a practically zero or negligible height, so it is considered that the initials and endings thrusts in the unstable mass are known. In the case of slope stability analysis, it is considered that the total sum of the thrusts of the sliding mass is equal to zero (∑ΔEx=0 and ∑ΔEy=0). In this way, the safety factor in "x" must be:

$$SF_x = \frac{\sum fzx}{\sum \left(N' \frac{X_c - X_e}{r}\right) + \sum fix + \sum (W \cdot Aa_x)} \quad (15)$$

In the same way, the safety factor in “y” must be:

$$SF_y = \frac{\sum fzy}{\sum \left(N' \frac{Y_c - Y_e}{r}\right) + \sum fiy + \sum (W \cdot Aa_y)} \quad (16)$$

Note that the safety factor in “x” and “y” is given as a function of the normal force at the base of the element, and according to equation (5), it is accepted that the normal force at the base of the element is given as a function of the safety factor in "x", but also depending on the safety factor in "y", and therefore, an iterative process is necessary to solve the system of equations.

To solve the system of equations the Jacobi method is used, considering that the system is linear and square, the number of columnar elements can be very high, convergence can be guaranteed (except when the slope geometry is symmetric and the factor of security is infinite) and that the expected number of iterations is low.

3 APLICATIONS

3.1 Case 1: theoretical slope, simple model

Based on the previous developments, the stability study of a theoretical slope, with the geometry showed in Figure 4, is carried out using the EST3D.UN software developed in this investigation.

The slope is initially considered to be in a dry condition and therefore the infiltration forces due to water are zero. Likewise, it is considered a static condition, that is Aa = 0.0. The results of applying the general procedure for the calculation of safety factors are shown below:

- Safety Factor in “x” (SFx): 1.8674
- Safety Factor in “y” (SFy): infinity

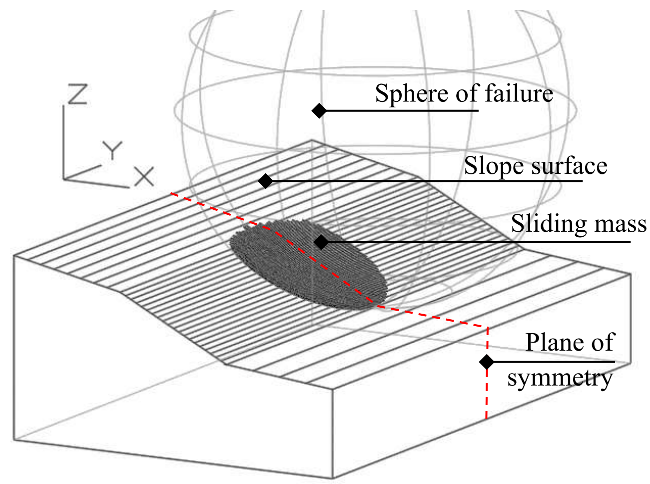

Figure 4. Slope Geometry Analyzed - Isometric View

Due to the symmetry of the slope, the safety factor analyzed in the "y" direction indicates absolute stability because each normal force corresponding to each columnar element has a normal force identical in magnitude, but of opposite direction that cancels it and therefore the sum of all normal forces in the "y" direction is equal to zero. Therefore, and in accordance with equation (16) the safety factor in this direction is infinite.

Following the analysis of the slope under study, a water table is incorporated that generates infiltration forces both in the x direction and in the y direction.

Finally, the analysis is performed simulating a pseudo-static condition. The design seismic movements were defined as a function of the effective peak acceleration for a ten percent probability of being exceeded within fifty years. The factors Aa_X and Aa_Y come from the expression:

$$Aa = Ap * Fa * CI \qquad (17)$$

Where:

- Aa: Maximum horizontal acceleration of the terrain, expressed as a fraction of the acceleration of gravity
- Ap: Coefficient representing the effective peak horizontal acceleration for design
- Fa: Amplification factor due to site effects
- CI: Importance coefficient of possible landslide

The Ap values depend on the region in which the slope is located within the country and are specific to the "Reglamento Colombiano de Construcción Sismo Resistente" (Colombian Construction Regulations Earthquake Resistant) in chapter A.2. The slope localization zone was considered a region with low seismic threat, effective peak acceleration Ap = 0.10, amplification factor of 1.20 and importance coefficient CI = 1.25, which results in a maximum horizontal acceleration Aa = 0.15. The maximum vertical acceleration Av was taken as 1/3 of the maximum horizontal acceleration. In summary, there is one acceleration value for each direction of the coordinate axes "x", "y" and "z" of $Aa_X = 0.15g$, $Aa_Y = 0.15g$, $Av = 0.05g$ respectively where g is the acceleration of the gravity.

The results of applying the method for calculating safety factors are shown below:

Table 3. Summary of results – case 1

Condition	2D	3D (x)	3D (y)
Dry, static	1.633	1.867	∞
Table surface, static pseudo-static	1.156	1.260	∞
Table surface, pseudo-static in "x"	0.828	0.975	∞
Table surface, pseudo-static in "x" and "y"	0.828	0.975	4.682

The earthquake force applied in the "y" direction generates a variation in the balance of forces in this direction and, therefore, the symmetry that governed the previous analyzes in "y" is lost. In the absence of this geometry, the safety factor takes values closer to 1.0, although obviously the slope is far from instability in this direction, it is observed that the earthquake forces act as a destabilization factor even when the symmetric geometry of the slope seems to indicate that theoretically there is no possibility of slope instability in this direction of analysis.

This is important insofar as stability analyzes are commonly studied in 2D conditions and a single direction of movement is chosen, completely ignoring that, under certain geometric characteristics, it could result in lower stability factors in other directions of the same slope analyzed.

In the "x" direction, a reduction in the safety factor is evident to the extent that acting forces such as infiltration forces and earthquake forces are considered.

The 2D analyzes using the Janbu method are performed keeping the same characteristics as the slope studied in three dimensions, that is: the geometry, the geomechanical parameters and the projected fault surface in 2D.

The analysis is carried out including a projected phreatic surface of the 3D analysis and a pseudo-static condition, using the same pseudo-statics coefficients used in the three-dimensional analysis, resulting in a safety factor of 0.828.

As evidenced, the safety factors for the studied geometry and for the evaluated geomechanical parameters are related to conventional 2D analyzes and decrease, as is logical, as the acting forces increase.

3.2 Case 2: theoretical slope, complex model

First, the stability analysis will be carried out in a two-dimensional condition using the Janbu method and using the software Slide developed by Rockscience. This software allows to find the critical fault surface of the slope in 2D by finding the safety factor for about 5000 potential fault surfaces.

Once the critical fault surface is identified in 2D, the three-dimensional stability analysis is performed with the center and radius of the fault sphere obtained from the 2D analysis.

A 2D condition is one in which there are no variations either in the slope geometry or in its geological conformation in the direction perpendicular to the cross section. Therefore, 2D methods do not contemplate the three-dimensional nature of the slopes, their geometric shapes and the morphology of the soil strata that make them up, it is impossible for example to perform an adequate stability analysis with 2D models for slopes formed by strata with an anticline or syncline fold in which there are geological variations that cannot be represented in a single 2D section of the slope. Whereas, a 3D stability analysis can easily contemplate these types of slopes and give an adequate response to the real conditions of the problem evaluated.

In this way, the analysis of a slope with three-dimensional geological configuration is performed and modeled in both 2D and 3D to make a quantitative comparison of the results. Figure 5 shows the slope evaluated:

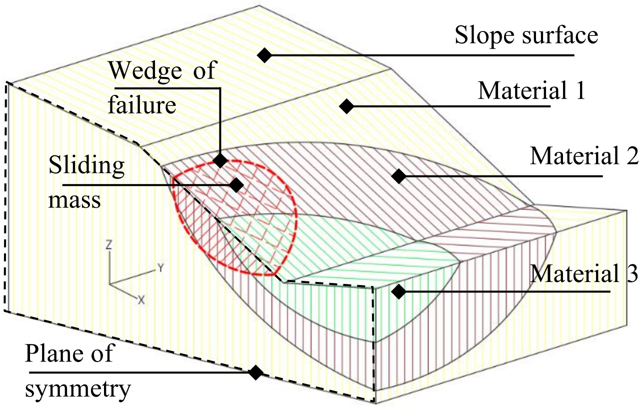

Figure 5. Slope Geometry Analyzed - Isometric View

For this case, the following properties of each of the soil strata are considered:

Table 4. Summary properties of soil strata

Material	Cohesion (kN/m^2)	Friction angle (º)	Specific weight (kN/m^3)
1	25	32	18.0
2	10	25	17.5
3	1	28	17.0

The soil properties selected for the slope geology simulate a condition in which the strata inherit the structures of an ancient rock massif that has been weathered over time. This condition is very common and can be found in many slopes of Colombian geography.

The result obtained by the 2D analysis with a pseudo-static coefficient Aa = 0.15 and the previously established geometry, indicates that the slope is stable with an SF = 1.336.

Next, the stability analysis of the same slope was carried out, with the same fault surface and the same coefficient Aa but considering the three-dimensionality of the slope geology. The method results in a critical safety factor in the "x" direction of 1.242. In the "y" direction, due to the slope symmetry and its geology in this direction, the safety factor is high, with a value of 4.597.

Table 5. Summary of results – case 2

Condition	2D	3D (x)	3D (y)
Dry, pseudo-static in "x"	1.336	1.242	∞
Dry, pseudo-static in "x" and "y"	1.336	1.242	4.597

When comparing the results of the 3D method fully developed in this investigation, against the 2D method of conventional use, the 3D analysis has a critical safety factor of 1.242, while the 2D analysis yields a critical safety factor, for the same fault surface of 1.336, so it is clear that the SF 3D / SF 2D ratio must not necessarily be greater than unity, there are slopes with geometric shapes and geological distribution as discussed above in which the aforementioned relationship may be 0.92 demonstrating that 3D methods are not more "risky" or less conservative, but rather are much more precise.

4 CONCLUSIONS

Through this research work, it was possible to design and program a method for the three-dimensional analysis of slope stability, which is based on the theory of limit equilibrium, specifically based on Janbu's two-dimensional methodology. A spherical fault surface was considered within the method. The effect of water was considered through the evaluation of infiltration forces and a pseudo-static coefficient was included in the model to study the stability of the slopes simulating an earthquake condition.

It is considered that the contribution to the discussion regarding the SF 3D / SF 2D relationship is given in a clear and objective way, finding that this relationship should not necessarily be greater than 1.0. Through a case of analysis, it was possible to demonstrate that this relationship can even be equal to 0.92.

It has been shown that the assumption that 3D security factors are greater than 2D is not, by any means, an incontrovertible truth. As indicated and illustrated, that depends on the direction of the actions and the geomorphology of the slope. If other fault surfaces are considered, different from the spherical ones, the conclusions would be more evident. In rock mechanics, for example, a wedge stability analysis would never be changed, when kinematically required, by a planar analysis.

The limitations in 2D analyzes reduce the accuracy of the results, which may result in oversized works in some cases or in other worse, under-sized works with the serious consequences that could be caused.

The direction of the acting forces, the geomorphological conformation of the slope and the presence of weak areas within the slope will finally define the potential fault surface and the direction of greater mobility of the fault wedge. Therefore, the spherical fault surface that has been worked on in this investigation is only a first approximation.

In order to strengthen the investigation of 3D methods, it is suggested to develop three-dimensional methods based on formulations of the mechanics of granular media, whether it is soils, or the mechanics of block systems in rock mechanics.

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