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# Statistical analysis of morphometric variables for basins associated with debris flow events in Colombia: a comparison within types of flows and with non-susceptible basins.

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## Abstract

*The occurrence of debris flows<sup>1</sup> in mountainous basins is a major risk in the Andean region of Colombia. Characterization of those debris-flow-prone basins and comparison with other mountainous areas that are not susceptible to the occurrence of debris flows is a significant step towards susceptibility analyses. Usually, the attention is focused on a particular event or a rather reduced set of events observed at a particular location, whereas a more general, thorough understanding is not typically addressed. Morphometric parameters and rainfall variables were used in this article to describe some of those watersheds in Colombia, the calculation of 25 variables for 151 basins associated with actual debris flows events and 54 non-susceptible basins was meant to describe different aspects of each basin, specifically: geometry, drainage conditions, hypsometry, rainfall and slope. A statistical examination, including an analysis of variance (ANOVA) and a principal component analysis (PCA), is presented and discussed. Results suggest that some variables can be used to distinguish between basins that exhibit different type of flows (i.e. mud-flows, debris flows and hyper-concentrated flows) while others can potentially separate the non-susceptible basins from the susceptible ones, besides, PCA results grouped naturally into categories representing the different aspects mentioned above. However, morphometric variables were found not sufficient to fully differentiate the occurrence of different types of flows nor discriminate between susceptible and non-susceptible basins. This analysis can be used to calculate indicators that account for the debris flow susceptibility, as it can be associated with weights for assessing the relative importance of all the different aspects of the basins' characterization.*

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<sup>1</sup> The term “debris flow” has been used to convey several different meanings throughout time, so it is worth noting that, in this article, the term will be used as described by Hungr (2005, p. 10): on the one hand, as an established keyword that allude the entire phenomenon –including all the different types on flows– that have an initiating process, followed by rapid flow along a steep confined channel, and finishing with deposition on a debris fan; and on the other hand, as a particular type of flow, with specific characteristics in terms of materials, water content, typical velocity and behavior.

## 1 INTRODUCTION

Debris flows are relatively common phenomena worldwide (Coussot & Meunier, 1996), various authors have presented an important collection of events showing that mountainous regions with a wide variety of conditions can exhibit such events (Jakob & Hungr, 2005; Takahashi, 2019; Lay, et al., 2019). In Colombia, several debris flow events have occurred, both in recent years (García-Delgado, et al., 2019; Prada-Sarmiento, et al., 2019; Velásquez, et al., 2018) and in the past (according to the UNGRD (2014) hundreds of events are known to have happened). In addition, many of the debris-flow-prone areas are in the Andean region of the country, where most of the people live, thus generating risk for the families inhabiting those areas. Detailed characterization of the basins where debris flows occur may be used to help further analyses, thereby improving land-use planning and decision making.

Some attempts to characterize susceptible basins have been made. For example, Chen & Yu (2011), used morphometric variables to characterize several basins where debris flows had occurred in the Chenyoulan River region of Taiwan, where they found topographic and moisture indicators to be useful as indicators of potential debris flow hazard in the study area. Another example for the Latin American case is presented by Sepúlveda & Padilla (2008), who not only demonstrated the importance of rainfall variables in the triggering process of debris flows, but also showed that they are insufficient to fully characterise the hazard. García *et al.* (2010) also presented rainfall variables and combined them with some morphometric variables in the debris flow susceptibility assessment for El Cofre Creek in Colombia. However, the focus has been on particular study cases rather than general, broader analyses encompassing susceptible and non-susceptible basins for considerably large regions. This work was partly conceived to address that gap. Rogelis & Werner (2013) posed an interesting idea to approach the problem, although limited to a small subset of watersheds, not including non-susceptible basins and without distinguishing the type of debris flow in their analyses. To attend those problems, we further developed on that idea and applied it to a detailed inventory of events acquired for Colombia, where information about morphometry and the flow type was collected for each event and rainfall variables for each basin were included as well.

This paper evaluates the information obtained in the inventory, thinking towards potential criteria for differentiating behaviours and susceptibility assessment. Each of the numerical variables' variability was visually inspected with boxplots, discriminating between types of flow and treating non-susceptible basins as a different group, then, an analysis of variance was conducted for each variable using two different datasets (i.e. one excluding the non-susceptible basins, and the other using all the available basins), finally, a principal component analysis was performed using different groups of data, related with the inclusion of non-susceptible basins and a data transformation (they are described in more detail in section 2.2). Those analyses were centred on differentiating the basins according to the type of flow and the possible occurrence of debris flow events. The following sections present an insight on our dataset and basic background on the statistical procedures used, after which the results are presented and discussed and, finally, conclusions, along with some concluding remarks, are raised for discussion.

## 2 DATA AND METHODOLOGY

All the procedures and analyses are based on an inventory of basins that fulfil two conditions: first, they have to either be associated to an actual debris flow event or to a non-susceptible basin, according to the criteria described below; and second, they must have sufficient information to compute all the variables and morphometric indices used in the statistical analysis.

Another consideration for the basins associated with a debris flow event is that information should be enough to confidently classify the flow type into hyper-concentrated flows, mudflows, or debris flows (understood here as a particular type of flow), and to accurately locate the apex of the dejection cone formed by the deposit.

The selection of non-susceptible basins was primarily based on a geomorphological criterion, avoiding basins with depositional landforms, avalanche boulder tongues, debris levees and other characteristic landforms associated with debris flows. The absence of active mass movements, landslide scars and gully erosion were considered to be favourable against the occurrence of debris flows, and, therefore, necessary for non-susceptible basins. Additionally, basins located in volcanic environments were completely avoided when selecting the non-susceptible ones, as volcanic activity is known to be a triggering factor for lahars.

The information in the inventory was acquired using secondary sources, being the Colombian Geological Survey (SGC) reports of the events, the ALOS-PALSAR Digital Elevation Model and the CHIRPS rainfall data the most important ones. Based on the inputs mentioned above, and using a script developed for the GRASS GIS software, 25 numerical variables were calculated for each basin, the variables computed are listed in Table 1. The characterised basins have catchment areas between 0.1 km<sup>2</sup> and 2000 km<sup>2</sup> showing great variety regarding land use and climatic conditions.

## 2.1 The study area

The study area comprises a significant part of the Colombian territory, covering the mountainous Andean region (See Figure 1.). The three branches of the Colombian Andes shape the landscape, creating topographic elevations that range between 0 and more than 5000 meters above the sea level. The westernmost branch of the Andes mountain chain is known as the Cordillera Occidental and, as described by Zapata-Villada et al. (2017), it is mainly formed by basalt and plutonic rocks, the oceanic origin of those rocks has been known for several decades and the region is strongly influenced by igneous activity. The second branch is known as Cordillera Central and it is also primarily formed by igneous rocks, although metamorphism and sedimentary formations are significant in certain regions (Restrepo, 1986). The last branch is the Cordillera Oriental, in which sedimentary rocks constitute the most common type of rocks, especially sandstones constituting various tertiary sedimentary units, the eastern range is an inversion orogen that coincides with a Lower Cretaceous rift and, in the east, thrusting has uplifted basement rocks to higher elevations than on the western flank (Mora, et al., 2008).

Depending on the place and the orography, the mean annual precipitation varies from 500 to more than 5000 mm along the Andean Region, as a general pattern, the area exhibit a bimodal regime with two rainy seasons between April and May as well as between September and November. Rainfall influences the weathering conditions of rocks, facilitating the production of potentially erodible deposits, therefore increasing the availability of material that can trigger a debris flow event. The landscape is dominated by recent transformations due to the extensive livestock production and display some crops concentrated in the valley bottom (Etter & van Wyngaarden, 2000).

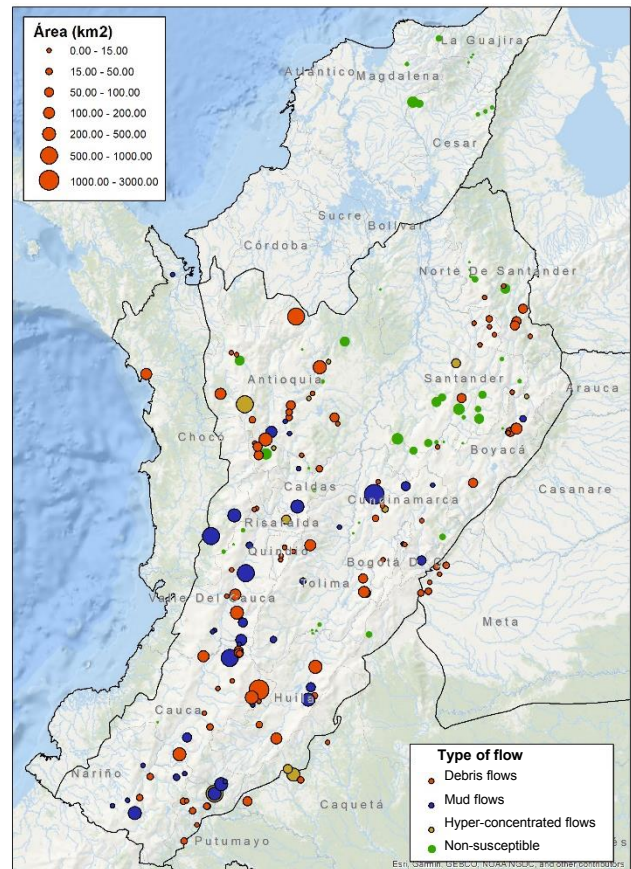


Figure 1. Study area, showing all the basins included in the inventory categorised by the type of event recorded (or as a non-susceptible basin). Note that the great majority of the basins are located in the Andean Region.

As was also described by Etter & van Wyngaarden (2000), a few outstanding aspects of the study area are the existence of very large human settlements and the high degree of geographical variability, i.e. the large altitudinal variations, the climatic anomalies present in the inter-Andean valleys (as a result of the complex topography), and the contrasts in soil mosaics due to the geologic heterogeneity.

## 2.2 Statistical analysis

The statistical tools used in this article are intended to capture the differences and variability of the morphometric conditions present among the debris-flow-prone basins and to help the understanding on how those differences relate to the type of flow. First, the boxplots were used to make some initial observations, after which the empirical cumulative distributions for each variable were constructed separately for the basins with debris flow events and the catchments that were considered as non-susceptible, the cumulative distributions are useful to evaluate the probability changes for a certain value of a variable, helping to identify ranges that, for some variables, occur more frequently in the debris-flow-prone basins.

Table 1. Variables calculated in the inventory and their prospective relevance to the debris flow related analyses.

Variable	Relevance
Area ( $A$ )	It is correlated with discharge and nearly proportional to sediment storage in the catchment. Wide basins collect a large amount of water, which can dilute the sediment concentration, thereby reducing the probability of debris flow forming in a first-order basin reaching the alluvial fan. It is also correlated with other morphometric parameters (Crosta & Frattini, 2004; Baker, 1976; De Scally & Owens, 2004).
Perimeter ( $P$ )	It is the base for several basin shape indices.
Mainstream length ( $L_s$ )	Stream length has been used to discriminate between different processes in watersheds (Chen, Wang, & Wang, 2010).
Elongation ratio ( $R_e$ )	In elongated basins floods travel less rapidly, have less erosion, less transport potential and have less suspended load. An elongated shape favours floods reduction (and also certain debris flow types reduction) because tributaries flow into the mainstream at greater intervals of time and Space (Zavoianu, 1985).
Gravelius compactness coefficient ( $K$ )	It is related to basin shape in a similar way as elongation ratio (Zavoianu, 1985). Although our results suggest that it is not as meaningful as elongation ratio.
Asymmetry factor ( $AF$ )	A weak relationship between asymmetry factor and erosion of the central zones watersheds has been shown (Baioni, 2007). Higher rates of erosion can imply higher possibility of rainfall triggered hyper-concentrated flows.
Hypsometric Integral ( $HI$ )	Is an indicator of the erosional stage, it is related to several geometric and hydrological properties, such as flood plain area and potential surface storage. Has been used to establish empirical correlations between the hypsometric parameters of a watershed and its observed time to peak. Can be used to differentiate between processes in the watershed (Harlin, 1978).
Hypsometric skewness ( $HS$ )	Indicates the amount of headward erosion attained by streams (Harlin, 1978).
Hypsometric kurtosis ( $Hk$ )	Large values denote erosion in both upper and lower reaches of a basin (Harlin, 1978).
Hypsometric density integral ( $HdI$ )	It is a basic input for several hypsometric density values calculations.
Hypsometric density skewness ( $HdS$ )	Indicates where slope changes are concentrated and whether accelerated forms of erosion, are more probable in the basin's upper reaches. When density skewness equals 0, equal amount of change is occurring (or has occurred) in the upper and lower reaches of the basin (Rogelis & Werner, 2013. After Harlin, 1978).
Hypsometric Density kurtosis ( $Hdk$ )	The density kurtosis is associated to the slope of mid-basin areas (Harlin, 1978).
Averaged annual precipitation ( $P_{ann}$ )	Basic hydrologic variable. It is meant to represent the average rainfall conditions for the basins.
Maximum precipitation in five days ( $P_{5d}$ )	It is associated to the rainfall intensity regime of the region, 5-day period was selected because shorter periods of time cannot be accurately inferred using satellite products.
Mainstream slope ( $S_s$ )	It can be used to discriminate between different types of processes in the watershed (Welsh, 2007).
Mean watershed slope ( $S_w$ )	Used to discriminate between debris-flow-dominated and clear-water-flood-dominated watersheds (Al-Rawas, 2010).
Valley floor width to height ratio ( $V_f$ )	Related to the valley "V" shape and neotectonics. It showed to be useful to discriminate between types of flows.
Melton Index ( $I_M$ )	Frequently used to discriminate between different hydro-geomorphologic processes. (Welsh, 2007)
Torrentiality coefficient ( $C_T$ )	It can be correlated to the magnitude of an event (Rickenmann, 2016).
Time of concentration - Kirpich formula ( $t_c$ )	The time of concentration is correlated to the mainstream length, so it can potentially be used to discriminate between different processes in watersheds (Chen, Wang, & Wang, 2010). Kirpich formula was selected as it is commonly used in hydrologic analyses.
Horton's Stream order ( $\omega_H$ )	Related to the size and the mainstream length of the basin.
Drainage density ( $DrD$ )	It has correlation with base flow, peak flood discharge, flood potential and debris flow sediment volumes (Baker, 1976; Méndez, et al., 2019).
Bifurcation ratio ( $R_B$ )	Used to characterize the drainage conditions within a basin.
Area ratio ( $R_A$ )	Used to characterize the drainage conditions within a basin.
Length ratio ( $R_L$ )	Used to characterize the drainage conditions within a basin.

Analysis of variance (ANOVA) test was applied with the type of flow as the differentiating group category. For a certain variable, the ANOVA test gives an indication of whether different groups of observations (samples) belong to the same population or, in other words, if that variable can be used to distinguish the different groups. Two different tests were conducted: one with the aim of discriminating only between types of flow and the other used with the objective of differentiating the basins that have flows from the basins that do not.

Following that characterization, a Principal Component Analysis was conducted. PCA can be useful to define the weight of each factor entering the computation of an indicator, as showed by Rogelis & Werner (2013), it can also be valuable to the understanding, reduction and interpretation of the gathered data, especially considering the number of variables included in the inventory. For example, recent exercises have used PCA to reduce and create new relevant variables that are independent of each other, generating inputs for susceptibility analyses (Wang, Sun, Li, & Meng, 2020), similarly, other authors showed that PCA can be used to generate linearly independent inputs in neural network techniques to generate susceptibility maps (Qing, et al., 2016). In this analysis various cases were considered and a transformation was applied to the variables in some of those cases as follows: the first two PCA (with and without non-susceptible basins) used variables without any transformations, the second group consisting of two PCA applied the transformation to all the variables analysed, and the third PCA only transformed the variables when the skewness of the empirical cumulative distribution of the transformed variable was less than the corresponding skewness for the original variable. The transformation applied in the cases described above is shown, for a particular value  $v$  that belongs to the variable  $V$ , in equation (1). The transformation was meant to reduce skewness so as to make the variables more suitable for PCA, which is known to work the best when data is nearly normally distributed and has little skewness (Shlens, 2014, pp. 9-10). Given that many indicators rely on other variables in the inventory, only the measured variables were used in PCA.

$$T(v) = \ln(v - \min(V) + 1) \quad (1)$$

The inventory comprising numerous basins with information regarding type of flow and a thorough description of each basin in terms of the morphometric variables, constitute one of the few exercises in the Country with those characteristics.

### 3 RESULTS

After the described analyses it was found that variables like area and perimeter can be useful to differentiate the type of flow, which might be explained by the fact that large basins (with a large amount of water) dilute the flow, thereby changing its sediment concentration. Likewise, mainstream length can be explained by the fact that flows with less internal friction dissipating energy can travel further for the same conditions of roughness and slope. The elongation ratio ( $R_e$ ) is the variable that reduce its p-value the most when non-susceptible basins are included (~100%), showing a strong trend of more debris flow occurrence in nearly circular watersheds. Interestingly, Melton index or Melton ratio ( $I_M$ ) does not reach the significance level when only susceptible basins are considered (although it is not so far from 0.05 value, that typically defines the significance threshold) but appears to be significant to discriminate susceptibility. Table 2 show the results of ANOVA test for selected variables, p-values below 0.05 were highlighted in bold text, as they may indicate that the variable can be useful to distinguish behaviours between the groups (types of flow).

Table 2. Levels of significance (p-values) for selected variables according to the analysis of variance (ANOVA) test.

Var.	Only basins with associated events		Including non-susceptible basins	
	F value	p-value	F value	p-value
<i>A</i>	4.032	<b>0.0197</b>	3.2376	<b>0.0232</b>
<i>P</i>	5.520	<b>0.0049</b>	4.8609	<b>0.0028</b>
<i>R<sub>L</sub></i>	0.814	0.4453	0.5958	0.6184
<i>P<sub>5d</sub></i>	0.873	0.4197	2.6012	0.0532
<i>P<sub>ann.</sub></i>	1.618	0.2017	1.8458	0.1401
<i>L<sub>s</sub></i>	4.871	<b>0.0090</b>	3.2684	<b>0.0223</b>
<i>S<sub>s</sub></i>	1.828	0.1643	2.4555	0.0643
<i>S<sub>w</sub></i>	0.669	0.514	2.8297	<b>0.0396</b>
<i>R<sub>e</sub></i>	3.440	<b>0.0347</b>	7.9693	<b>4.8E-5</b>
<i>K</i>	0.283	0.7539	0.9053	0.4395
<i>I<sub>M</sub></i>	2.563	0.0805	2.6907	<b>0.0474</b>
<i>C<sub>T</sub></i>	0.132	0.8768	1.0439	0.3742
<i>AF</i>	1.329	0.2678	1.2497	0.2929
<i>t<sub>c</sub></i>	3.263	<b>0.0410</b>	2.4624	0.0637
<i>V<sub>f</sub></i>	3.980	<b>0.0207</b>	5.220	<b>0.0017</b>

As stated before, principal component analysis can be used to reduce the number of dimensions in the problem. With that in mind, the percentage of the explained variance for the first four components in each PCA exercise is presented in the Table 3, the cases summarized are: the basins with events, without any transformation (EB); all the basins, without any transformations (AB); the basins with events, with all the variables transformed (EBTa); all the basins, with all the variables transformed (ABTa); the basins with events, with the variables selectively transformed (EBTs); and all the basins, with the variables selectively transformed (ABTs).

Table 3. Proportion of the explained variance for each dataset in which the Principal Component Analysis was applied.

Case	PC1	PC2	PC3	PC4
<b>EB</b>	25.6%	16.0%	13.8%	10.2%
<b>AB</b>	25.4%	15.1%	13.2%	10.6%
<b>EBTa</b>	29.8%	18.0%	15.7%	9.6%
<b>ABTa</b>	29.5%	16.8%	15.3%	10.3%
<b>EBTs</b>	29.8%	18.0%	15.7%	9.6%
<b>ABTs</b>	30.5%	18.2%	15.0%	10.3%

The principal component analysis revealed a natural grouping between the variables in each principal component for all the cases presented before. As an example, the most important variables in the first five principal components are presented in the Table 4 for the case where all the basins are included and transformations are applied when the skewness of the transformed variable is less than the skewness of the original variable. It should be noted that the first variables within every principal component grouped quite well, allowing the use of the first variables in the component to describe a certain aspect of the basin.

If desired, results shown in Table 4 could be used to generate variables representing each of the categories associated with the principal components, and the use those to compute an indicator according to the percentage of the explained variance of each component, such as the indicator that Rogelis & Werner (2013) produced in their approach. For the cases where all the variables were transformed or some of them were selectively transformed, geometry variables appear to be more important to explain the variance in the dataset, followed by hypsometry and drainage condition. In the cases of no transformations, PC1 was associated with hypsometry, PC2 with drainage condition and PC3 with geometry.

Table 4. Principal components and their corresponding variables, the relative importance of each variable inside the principal component is given in the loading column.

Variable	Loading
<b>PC1 – Geometry</b>	
<b>% of the variance explained = 30.5%</b>	
Area ( <i>A</i> )	-0.3434
Mainstream length ( <i>L<sub>s</sub></i> )	-0.3403
Perimeter ( <i>P</i> )	-0.3278
<b>PC2 – Hypsometry</b>	
<b>% of the variance explained = 18.2%</b>	
Hypsometric integral ( <i>HI</i> )	0.3725
Hypsometric kurtosis ( <i>Hk</i> )	-0.3635
Hypsometric skewness ( <i>Hs</i> )	-0.3154
<b>PC3 - Drainage conditions</b>	
<b>% of the variance explained = 15.0%</b>	
Length ratio ( <i>R<sub>L</sub></i> )	-0.5525
Area ratio ( <i>R<sub>A</sub></i> )	-0.5351
Bifurcation ratio ( <i>R<sub>B</sub></i> )	-0.5186
<b>PC4 – Rainfall</b>	
<b>% of the variance explained = 10.3%</b>	
Annual precipitation ( <i>P<sub>ann</sub></i> )	0.6513
Maximum precipitation in five days ( <i>P<sub>5d</sub></i> )	0.6310
<b>PC5 – Slope</b>	
<b>% of the variance explained = 7.0%</b>	
Mean watershed slope ( <i>S<sub>w</sub></i> )	-0.7702
Mainstream slope ( <i>S<sub>s</sub></i> )	-0.4186

Another of the potentially useful features of PCA is the ability to find aggregations of variables. In many cases, when this can be done it is possible to plot the events in the principal components' space to observe how are located the events in this rotated and stretched vector space. The Figure 2 shows, for the case of all variables selectively transformed, the variables represented as vectors in a plot of the first two principal components, the groups noted before (Table 4) are also visible in the plot, where many variables point in similar directions in the plot. The basins on the other hand, are represented as dots with the type of flow indicated using different colours, they do not exhibit an obvious pattern that allow us to differentiate or aggregate them in this representation, the ellipses are supposed to help identifying the groups, but in none of the six different cases presented in Table 3 the aggregations were possible in this principal components spaces.





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