Influence of the block shape on the impact process by an advanced rheological model

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Abstract

We presented an advanced elasto-visco-plastic impact model in which blocks with prismatic shapes are taken into consideration together with their rotation. The model considers a failure mechanism below the impacting block equivalent to that of a function.

After a brief description of the procedure employed to evaluate the equivalent base of the foundation for the new block shapes and the illustration of the new constitutive mathematical equations, a model validation is carried out. For this purpose, comparisons between the experimental data and the numerical simulation results for vertical impacts are shown.

The calibrated model parameters are used in the subsequent parametric analysis, which has been performed to investigate the influence of the blocks shape. To this purpose, the following group of numerical simulations are performed: (i) spherical and prismatic shapes with triangular basis and free rotating, impacting with different angles on an inclined slope; (ii) same as the previous group but with locked rotation; (iii) vertical impacts for different slope angles considering only blocks free to rotate.

The numerical results for a fixed slope gradient demonstrate that the block shape is a major parameter affecting the impact process while rotations have less influence even if their evolution change according to the block shape. Vertical impacts allow to obtain analogous results. Block symmetry is considered the main reason for the small rotational components generated during impacts.

1 INTRODUCTION

Rockfall is an important hazard affecting the highways, railways and habitants centers and therefore it threats human lives (Varnes, 1978; Whalley, 1984; Selby, 1993; Crudens and Varnes, 1996). For this reason, the design of protection structures is essential and to achieve this target is necessary forecasting the block trajectory and its velocity at the impact in a robust and variable way.

This prediction is performed by means of mathematical models implemented in numerical codes, which consider the block trajectory as a composition of four elementary types of motions: free flying, rolling, sliding and impacts with possible bouncing (Descroudes, 1997).

In contrast to the well-known description of the first three above-mentioned types of motions from the physical viewpoint, the bouncing phenomenon, which affects significantly the block trajectory, remains most difficult to predict and limited in understanding (Labiouse and Heindereich, 2009). The basic approach consists in evaluating the block velocity after the impacts (once decomposition in the normal and tangential directions is done) by using the initial velocity components and the restitution coefficients. This approach is followed by many authors (Descroudes and Zimmerman, 1987; Azzoni et al, 1995; Guzzetti et al., 2002) but the coefficient of restitutions cannot be considered as constants depending only on the impacted materials as it was demonstrated by Wu (1985), Spang and Sonser (1995) and Richards (1988).

In particular, some authors found that the block geometry and the block rotation play an important role because in some cases normal restitution coefficients greater than one have been observed (Spadari et al., 2012).

To study impacts and bouncing phenomenon many numerical and analytical methods have been developed: some of these methods are based on the DEM approach. This method provides useful information but it is time consuming and so not always suitable for practical applications. An alternative approach consists in developing rheological models, as the one proposed by di Prisco and Vecchiotti (2006), which describe the physics underling the phenomenon through the combination of elementary models. In this way di Prisco and Vecchiotti (2006) developed the BIMPAM model. This approach was successfully applied to the design charts in di Prisco and Vecchiotti (2010) and the design of shelter structures in Calvetti and di Prisco (2007). That BIMPAM model considered only spherical blocks without rotational degree of freedom and consequently the effects induced by block shape cannot be analyzed. To overcome the problem of the effect of rotation Dattola et al. (2016) extended the model by adding the rotational degree of freedom but keeping the spherical shape for the impacting block.

The aim of this work consists in extending the original BIMPAM model adding the possibility to model the blocks with prismatic shapes and showing their influence on the blocks motion.

The configuration of the impact scheme is shown in Figure 1 where the block trajectory before and after the impact, the incidence angle θ^{in} and the corresponding restituted angle θ^{out} , measured with respect to the normal to the contact plane, are shown. The slope has an inclination ω and the tangents of block trajectory give the direction of incident and restituted velocities.

2 MATHEMATICAL FORMULATION

In this section, a brief description of the mathematical model is carried out considering mainly the difference introduced with respect to the BIMPAM model (di Prisco and Vecchiotti, 2006, Dattola et al., 2016).

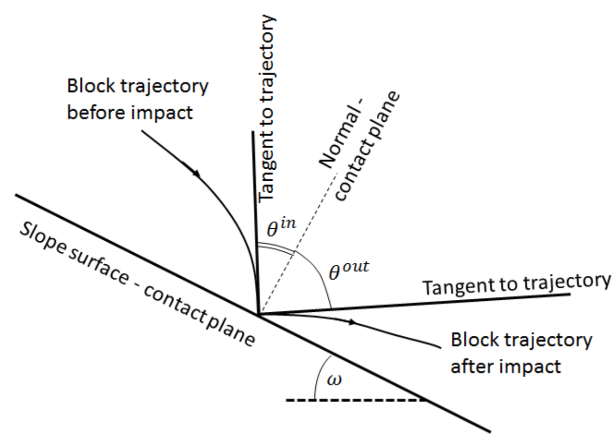

Figure 1. Slope angle ω, block trajectory before and after the impact, normal direction to the contact plane, incident and restitution angles θ^{in} and θ^{out}, respectively.

2.1 Block geometry

According to the original version of the BIMPAM model, the block shape is introduced by considering an equivalent foundation whose size is evaluated as the intersection between the block and a plane representing the ground surface. This method is kept but the mathematical procedure taking into account prismatic blocks shape is updated and is shown hereafter.

The prisms here considered have an out-of-plane size H and polygonal regular base which is inscribed within a circumference of radius R. For a given number of edges n_e and block orientation θ_0, the polygonal vertexes are computed considering a local reference system centered on the block center (2D). After this the maximum value of the polygon vertex ordinates is considered and a vertical translation of the polygon is done to obtain, at the initial impact time, a block tangent to the ground surface and a vertical translation during the simulation equal to the block penetration u_n.

During the impact simulation, the block penetrates into the ground and the computation of the equivalent base is carried out by using the following steps:

A. computation of the intersection points P_1 and P_2 to the left and to the right of the prism center between the polygonal base and the ground surface;
B. computation of the positions of the two extreme points of the polygons, P_l and P_r, that is the vertexes which are on the extreme left and on the extreme right for a given block configuration;
C. comparison of the P_1 and P_l coordinates, if P_l is under P_1 the left point of the equivalent foundation is $P_{fl} = P_l$ otherwise $P_{fl} = P_1$. The same comparison is performed considering P_2 and P_r obtaining the right point of the equivalent foundation P_{fr};
D. the equivalent foundation size, B, is the length of the segment $\overline{P_{fl}P_{fr}}$.

Figure 2 show the four possible configuration for computing the equivalent base.

2.2 The constitutive laws

The BIMPAM model is characterized by four groups of mathematical relationships. First, the dynamic equilibrium equation of the block is introduced to describe the block motion. An elasto-damping model and a visco-plastic model that take into account the soil elastic-property together with the shock wave propagation and the volumetric dissipative mechanism occurring in the region close to the contact point, respectively, are the second and third groups (see Figure 3 for the visco-plastic mechanism).

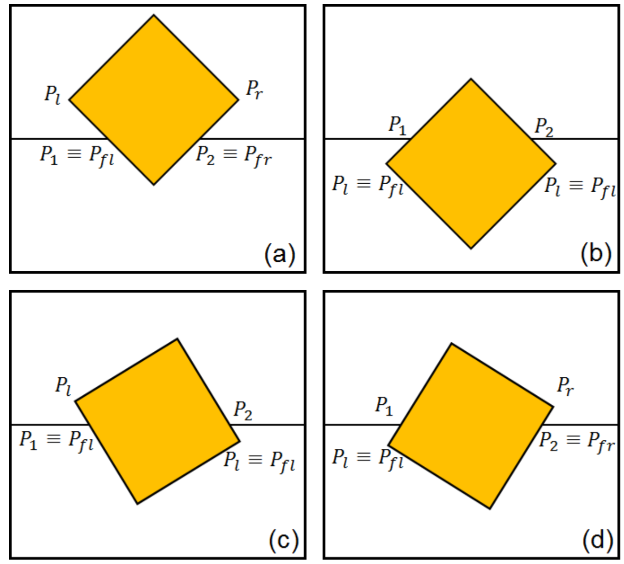

Figure 2. Position of the P_1, P_2, P_l, P_r, P_{fl} and P_{fr}. (a) low level of penetration; (b) high level of penetration; (c) middle level of penetration with a block rotated clockwise; (d) middle level of penetration with a block rotated anticlockwise.

Finally, the model describes the amount of energy dissipated during the sliding by means of a plastic slider constitutive relationship that constitutes the fourth group.

The dependence of the base of the equivalent foundation on the elasto-damping equations or on the computation of the generalized force variables does not change in this new mathematical formulation. Therefore, the model takes into account the shape dependence directly from the new computation of B.

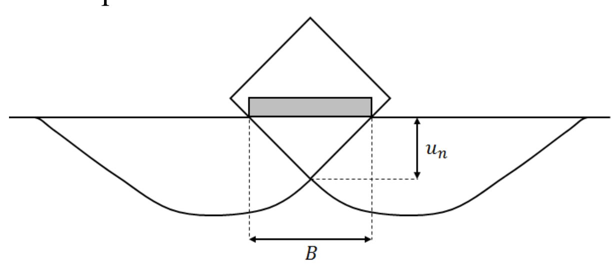

Figure 3. Visco-plastic mechanism occurring at the bottom part of the block. The mechanism is equivalent to failure mechanism generated in a shallow foundation.

When the hardening rule is concerned, some modifications are introduced. The hardening variable ρ_c rate is given by the following hardening rule:

$$\dot{\rho}_c = \left(\frac{B^*}{2R} - \rho_c\right)\frac{R_0}{V_{max}^2}(\alpha_n \dot{q_n} + \alpha_t \dot{q_t} + \alpha_r \dot{q_r}) \qquad (1)$$

where B^* is the maximum size of the equivalent base computed at the original block orientation, V_{max} is the foundation bearing capacity for vertical loads; q_n, q_t, and q_r are the generalized displacement components along the normal, tangential directions and rotation, respectively (Nova and Montrasio,1991). α_n, α_t and α_r are three constitutive parameters, while R_0 is a variable controlling the stiffness of the force-displacement curve along the vertical direction (Nova and Montrasio, 1991; Montrasio and Nova, 1997). In the above equation the upper dots indicates the rate of the corresponding variable.

The computation of the variable R_0 depends on the equivalent foundation size by means of:

$$R_0 = \frac{30V_{max}D_r}{2R}\frac{B}{2R} \quad (2)$$

where D_r is the initial soil density. The solution has been implemented in a code that solve implicitly its mathematical equations.

3 MODEL CALIBRATION

To validate the mathematical model here proposed a comparison between the experimental data and the model results is performed. To this reason, a set of experimental data is taken into account for the calibration of the model parameters.

Some parameters have a direct physical meaning such the block mass, the soil weight for unit of volume, the friction angle so that they are directly assigned. Some of the remaining parameters have a great influence of the numerical results, so that they are calibrated by means of trial and error procedure. Finally, the last one group of parameter are evaluated by refining the differences between the numerical curves and the experimental data.

The comparison is carried out considering the data obtained by Labiouse et al. (1994) in which a non-spherical boulder, is dropped vertically on a horizontal sand layer. These authors measured, by using accelerometers, the block vertical acceleration that was integrated to obtain the block vertical velocity and displacement time history.

Since their block shape is not considered in the model an equivalent spherical block is used, which has the same mass of the block as in Labiouse et al. (1994) and a radius to obtain a circumference tangent to the block shape to guarantee an equivalent foundation size closer as possible to the original shape. The same procedure was adopted in di Prisco and Vecchiotti (2006).

Figure 4 shows the comparison between the experimental data and the model prediction in terms of vertical displacement (a), vertical velocity (b) and vertical acceleration (c). The model prediction are in good agreement with the experimental data since the model is able to capture the initial slope of the curve of the vertical displacement and its maximum value.

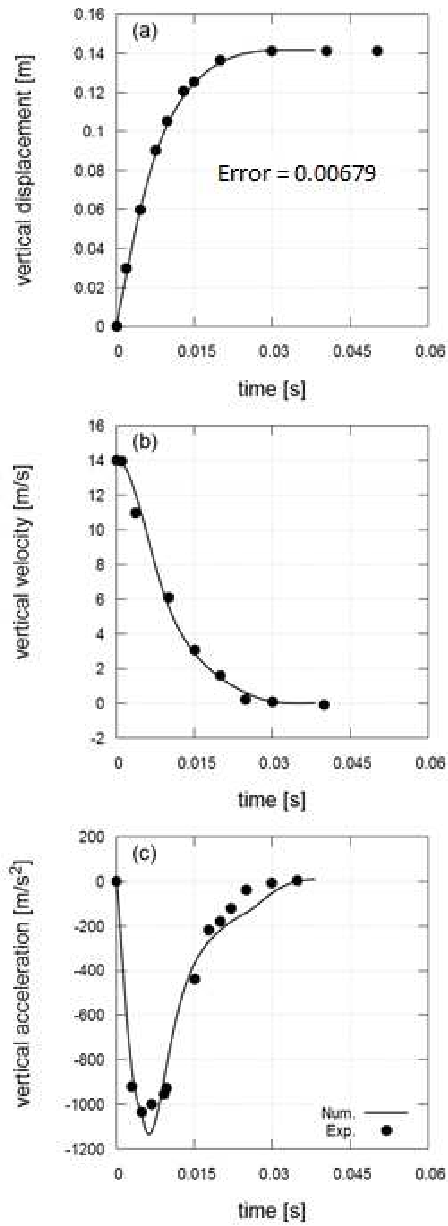

Figure 4 Comparisons between the advanced rheological model results and the experimental data from Labiouse et al. (1994). Evolution with time of (a) vertical displacement; (b) vertical velocity and (c) vertical acceleration.

The vertical velocity trend is well reproduced and, finally, also the trend of the vertical acceleration. Some differences are noted in the vertical acceleration peak that is slightly overestimated and also in the trend of the last part of the curve. Nevertheless, the model reproduces well the peak time.

In the displacement curve, the error, computed by means of the last square method without considering the last two points of experimental data since they are not on the curve, is shown in Figure 4a.

Once the model parameters were calibrated, a parametric analysis was performed to investigate the role of block shape in the main parameters governing the impact process. This is shown in the next section.

4 PARAMETRIC ANALYSIS

Many authors have demonstrated that the block shape has a relevant effect on the impact and on the bouncing because it affects mainly the restitution coefficients and the impact force (Chau et al., 2002; Yan et al., 2018, Shen et al., 2019). Chau et al. (2002) shows also a change in the ratio between translational and rotational kinetic energies between before and after the impact.

After the prismatic blocks implementation in the new model, it is possible to investigate the effects of block shape. To this purpose, the following numerical simulations were carried out using the parameters calibrated against Labiouse et al. (1994) dataset.

Figure 5 shows the numerical results obtained considering a slope inclined at 20° with respect to the horizontal for spherical and prismatic with triangular base blocks. The initial impact angle θ^{in} with the respect to the slope normal ranges from 20° (vertical impacts) to 80° (nearly tangent to the slope).

The block shape affects significantly the block trajectories (Figure 5a): the blocks with triangular base have a greater value of vertical displacements than the spherical boulders. Furthermore, the trajectories of the prismatic blocks generally lay to the left of the trajectory observed for the spherical block.

Figure 5b shows the time evolution of the block rotation for the two class of blocks here considered. For both block shapes, the rotation develops during the impact process and its final value generally increases with the initial impact angle. It is important to note that the triangular block graphs are between the graphs of the spherical blocks according to the initial incidence angle.

Figure 6 shows a comparison between the numerical results obtained considering spherical and prismatic with triangular base blocks in which the rotation is both inhibited and activated.

In both cases no significant differences are observed since the rotation as shown in the previous graphs is not very important.

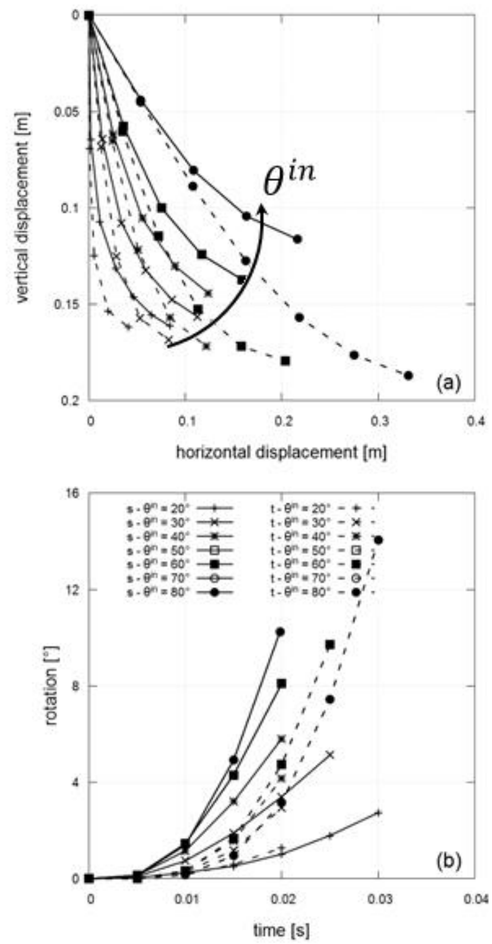

Figure 5 Numerical simulation results obtained considering a slope inclined of 20° and spherical and triangular blocks impacting with different inclinations. (a) Blocks trajectories during sinking in the layer; (b) rotation versus time. The prefixes "s-" and "t-" stand for sphere and triangular prisms, respectively.

In order to evaluate the effect induced by the slope inclination, a set of numerical simulations for the blocks falling vertically on a slope with different inclination was carried out ($\theta^{in} = \omega$). The slope inclination ranges from 2° to 30°. The numerical simulation results for both spherical and prismatic with triangular base blocks are shown in Figure 6.

The boulder shape affects significantly the block trajectory since the blocks with triangular base have a greater value of the maximum penetration depth than the spherical blocks and in general, their trajectories lay to the left of the spherical block trajectories Figure 6(a). When the rotations are concerned, the results demonstrate that the spherical blocks have greater values than triangular blocks. Furthermore, the graphs associated with prismatic blocks with triangular base lay on the left of the corresponding curves for of the spherical boulders, and therefore, a sort of anticipation of the rotations is observed.

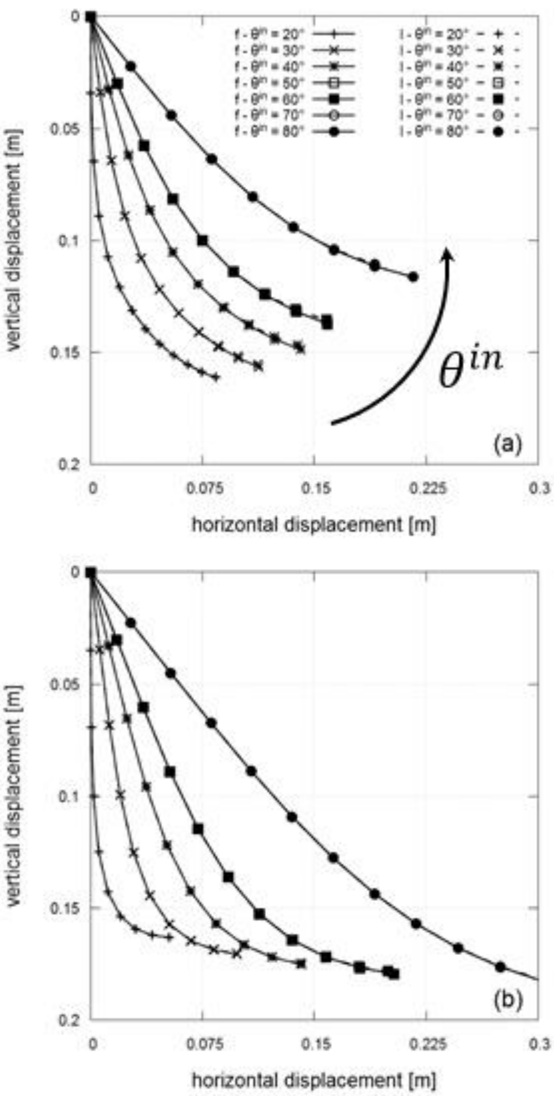

Figure 6 Block trajectories for blocks with free and locked rotating: (a) spherical blocks, (b) prismatic with triangular base blocks.

This behavior can be explained by means of the following considerations. First, the prismatic blocks due to their shapes, which is characterized by a vertex, penetrate more easily than the spherical boulders as the numerical results show. This implies that their rotation is inhibited by the surrounding soil as it is illustrated by the corresponding graphs.

At the beginning of the numerical simulation, the triangular blocks have a greater value of the tangential arm and, consequently, the angular velocity is greater and therefore the anticipation is explained.

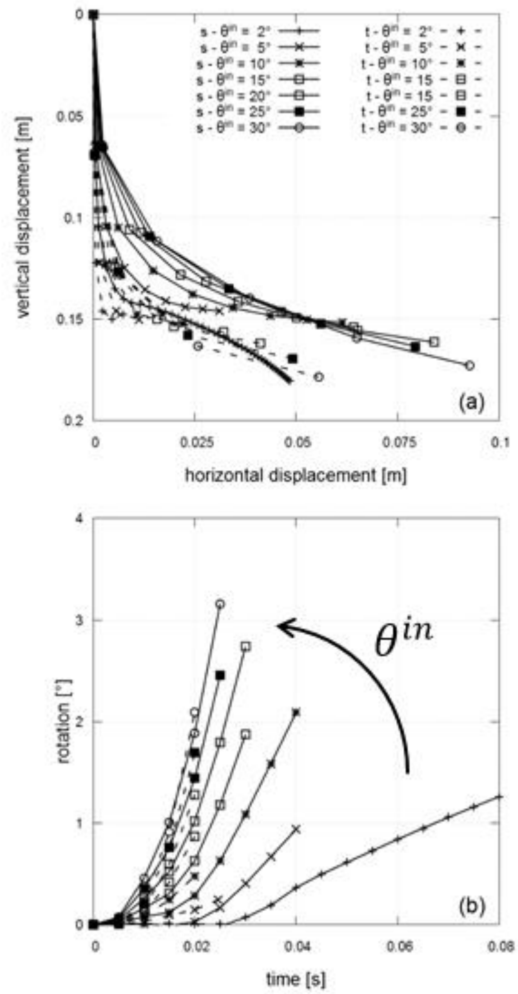

Figure 7 Vertical impacts for both spherical ("s-") and prismatic blocks with triangular base ("t-"): (a) blocks trajectories, (b) time evolution of blocks rotation.

5 CONCLUSIONS

The aim of this paper consists in evaluating the effects of block geometry on the impact of a block on a layer with different properties.

To this purpose, a new model modification is presented starting from the BIMPAM model proposed by di Prisco and Vecchiotti (2006); di Prisco and Vecchiotti (2010) and Dattola et al. (2016). This new model includes the possibility to simulate impacts of prismatic blocks with a regular polygonal base in presence of a block rotational component. To this scope, a parametric analysis was performed. This has been carried out to evaluate the differences in the impact process when the spherical block is replaced a prismatic block with triangular base.

Since by the soil layer is not a horizontal plane, in the numerical simulation a prefixed inclination (20°) is considered and different impact angles are taken into account. The results demonstrate a significant difference when a comparison in terms of block trajectory, maximum penetration depth and block rotation between the blocks with spherical and prismatic shape is performed. Furthermore, the block rotation does not play an in important role as the comparison of the numerical results between blocks with free and locked rotation are considered both considering spherical and prismatic blocks.

This demonstrates that the above differences are only due the block geometry and the change of geometry affects the rotation process but this does not contribute significantly to the block trajectory.

It is the opinion of the authors that this low influence of the block shape to the rotation is due to the symmetry of the adapted block shapes both for the spherical and prismatic blocks.

Finally, in order to investigate the role of the slope inclination, when two type of shapes are considered, a second set of numerical simulations has been performed. This consists in simulating vertical impacts with different values of the slope inclination ranging from 2° to 30° chosen to keep the slope under equilibrium condition.

The results demonstrate that the block geometry influences the trajectory and the evolution of the block rotation. Blocks with prismatic shape exhibit a larger value of the maximum penetration depth.

6 ACKNOWLEDGMENTS

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