Monte Carlo risk assessment of the bearing capacity of shallow foundations on slopes

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Abstract

Shallow and deep foundations transmit loads from the superstructure to the ground, thus their design determine how reliable the construction is. Normally, when it comes to the projects themselves, the safety analysis for the bearing capacity are based on deterministic methods, which assume fixed values for the calculated parameters. Thus, it is common to observe the negligence of some peculiarities of each civil work, as the deterministic approach overlooks the geological-geotechnical and even geomorphological variability that is present in the soil-foundation system. Following this reasoning, risks exist and need to be defined and evaluated. Specifically, it is worth paying attention to cases of shallow foundations placed throughout slopes, as investigations of their load capacity may impact both the stability of the slope as well as the stability of the foundation itself. In the present paper, methods presented in the literature are applied to estimate the bearing capacity of shallow foundations placed throughout slopes as well as the stability of such slopes. The infinite slope theory has been used. In order to find the statistical distribution of the factor of safety for this coupled analysis, the Monte Carlo probabilistic method was considered. A given statistical distribution was assumed for the values of cohesion and friction factor of the soil mass based on a real dataset presented in the literature. The simulations revealed that the safety factors, given as acceptable under Brazilian standards, do not necessarily have a low probability of failure. Therefore, the importance of correctly assessing the probability of failure of a given geotechnical work is exemplified and discussed. The findings of the present paper can be used as a basis for a better assessment of the interactions between the loads imposed by the work and the real ability of the subsoil to support it, assisting in the decision-making process.

1 INTRODUCTION

The basic role of the foundation is to transmit the building loads to the ground, so that it serves as a foundation for the work and ensures that it remains in place, without ruptures and without suffering instabilities. In addition, it must be ensured that the soil has resistance and stiffness to support all transmitted loads without suffering excessive deformation (settlements) or rupture. The strip footings are elements whose visual inspection is difficult to achieve, which requires that their design, dimensioning and execution are carried out with the utmost care and safety. Commonly, constructions are built on or near slopes, making it necessary to link the studies of the determination of the load capacity to the stability of the massif.

According to the literature, as Falconi et al (1998) and Carvalho and Pinheiro (2009), foundations must have an adequate safety coefficient in order to guarantee the stability of the construction throughout its lifetime. In the present paper, the following points are discussed: Are the safety analyzes based on deterministic methods enough? How do the common cases of shallow foundations near or located on embankments behave?

We must keep in mind that, when it comes to the safety of foundations, there is a great concern in its assessment, as it is directly related to geotechnical variability and uncertainty. The uncertainties are due to various phenomena, construction techniques, different conditions of soil types and landscape morphology found. Besides, one deals with projects and executions carried out by human beings, therefore eventual mistakes may be made. Thus, unfortunately, it is not possible to reduce the probability of accidents to zero.

Since the NBR 6122 standard (ABNT, 2010) - Design and Execution of Foundations, updated in 2019, new concepts for safety analyzes have been incorporated to Brazilian standards. From that moment on, the analysis that was exclusively based on a deterministic safety factor, started to be elaborated also by probabilistic methods, recognizing that it is not possible to deal with “absolute truths” and that risks exist and need to be defined and evaluated.

Following this line of thought, bearing capacity assessment methods for strip footings near or on slopes were studied, being presented in the later topic. In order to check how far are deterministic methods from probabilistic approaches, based on the equations presented in the methodology and the mechanical parameters of local soils in the region of Brasilia, DF, Brazil, a Monte Carlo routine was carried out in order to quantify the probability of failure to which the strip footings are subjected.

2 BEARING CAPACITY METHODS

Foundation structures are often forced to be built on or adjacent to slopes. Investigating the bearing capacity of strip footings on loaded slopes is very important in this case, because they are more susceptible to failure while compared to other types of conventional structures. Generally, for small and medium-sized buildings, shallow foundations are often used. In such situations, the design consists of obtaining the minimum value of the bearing capacity, normally obtained by the generalized shear failure of the foundation or by the general stability of the slope. In the case of non-cohesive soils, the load capacity is always governed by the failure in the foundation, while on cohesive soil the load capacity of the foundation can be dictated by stability (Saran *et al*., 1989).

Currently, a huge variety of methods are found in the literature to find the bearing capacity of shallow foundations on or adjacent to slopes. In general, the methods are based on concepts of limit equilibrium analysis, sliding line analysis, limit analysis and finite element analysis. Initially, the method for estimating the bearing capacity of strip footings on sloping ground was proposed by Meyerhof (1957) and, later, many researchers contributed in this area, such as Hasen (1970), Vesic (1975), Kusakabe *et al*. (1981), Saran *et al*. (1989), Narita and Yamaguchi (1990), Kumar and Rao (2003), Georgiadis (2009), among others.

The present study is based on the method of bearing capacity by Kumar and Rao (2003), with a null seismic acceleration factor. Equations 1 and 2 referring to the bearing capacity factors, Nc and Nq, were used for different slope inclination (β) and for different soil friction angles (φ). In the case of the $N\gamma$ factor, a fit was made from the graphs present in the authors' article (Figure 1). Failure probability analyzes were also studied, according to the present proposal.

$$N_q = \left(1 + \sin\phi \cos 2\theta_f\right) \times \exp\left(\ln\left(\cos^2\beta/(1 + \sin\phi \cos 2(\theta_{gR} + \beta)) + 2(\theta_{gR} + \theta_f)\tan\phi\right)\right) \quad (1)$$

$$N_c = (1 + \sin\phi) \times \exp(\ln(\cot\phi/(1 - \sin\phi) + (\pi - 2\beta)\tan\phi)) - \cot\phi \quad (2)$$

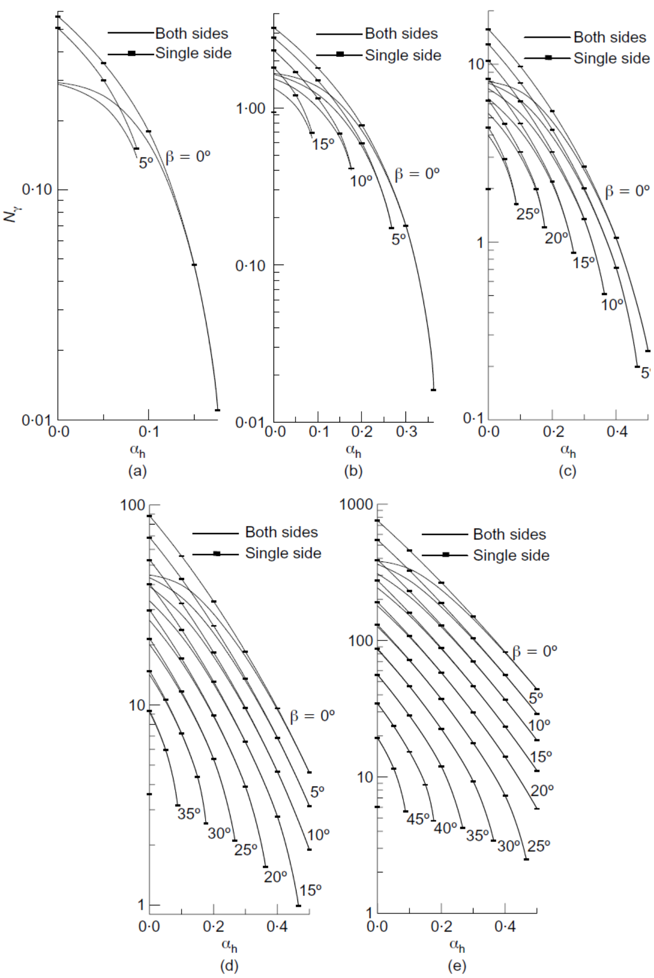

Figure 1. Variation of Nγ with α_h for different values of φ and β: a) φ=10°; b) φ=20°; c) φ=30°; d) φ=40°; e) φ=50° (Kumar and Rao, 2003).

3 MONTE CARLO METHOD

There is no risk assessment procedure applicable for all types of projects. The method's choice will depend on the approach that is most acceptable for the type of problem, the data available, the degree to which there is a dependence on subjective judgment and the criteria that will be used to judge whether or not the risk is acceptable. Thus, it is calculated the probability of success or failure of a structure and its consequence, whose safety is usually expressed in terms of its probability of failure.

The probability of rupture is linked to the theory of reliability, which represents the link between the level of safety and the procedure adopted. This probability is quantified through analyses with a probabilistic emphasis in conjunction with professional experience.

The method requires that the variables of the mathematical system are described in terms of probability density functions (PDF). Thus, for each variable, a possible result is generated obeying its probability distribution. Deterministic analyses are performed for the set of generated values and the safety factor is determined. This process must be repeated as many times as necessary to determine the probability distribution of the safety factor. The probability of failure is given by the percentage of safety factors less than the unit. It is observed in Figure 2 an example of obtaining the failure probability, being represented by the area under the normal distribution curve with FS ≤ 1.

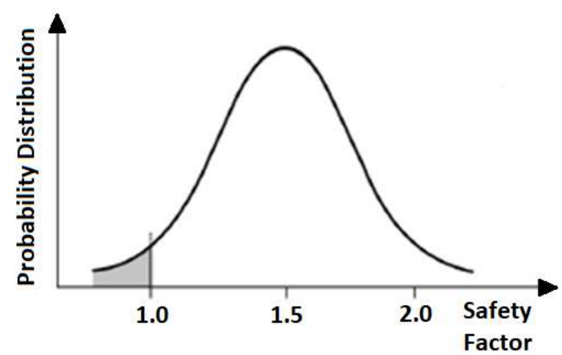

Figure 2. Probability Distribution of the Factor of Safety.

Several types of distributions are easily found in the literature. As an example, we can mention the normal, log-normal, exponential, uniform, triangular, gamma, geometric, hyperbolic distribution, Poisson, Dagum, among others. Examples of the literature that addressed the use of the Monte Carlo method for risk analysis in geotechnical infrastructures can be seen in Ozelim *et al*. (2014), Albuquerque *et al*. (2015 and 2016) and Mascarenhas *et al*. (2017).

The next step is to generate a sample of the distribution of the safety factor, from which any statistical parameters may be obtained. For example, the confidence intervals of measurements can be evaluated. For the study, a certain statistical distribution was assumed for the values of cohesion and friction angle of the soil mass based on a real data set presented in the literature.

4 LITERATURE DATA

The data used in the present study is taken from Albuquerque et al. (2016). In their paper they find saturated geomechanical parameters of Brazilian soils present in the superficial layers (0 to 10 meters) of Brasilia's stratigraphic profile, a massif typically composed of tropical soil, especially by clays with high void ratio (ranging from 1 to 3) and high contents of iron and aluminium oxi-hidroxides (which comes from the weathering process). Another interesting point to note is that this soil is mostly composed of clay and its cohesion tends to zero when saturated. This is a common characteristic of the local soil, due to its collapsibility and granular behaviour (clay particles tend to join with each other leading to grains of the size of silts and thin sands).

The probability distributions related to the parameters are as shown in Table.

Table 1 – Soil parameters best fit distributions (adapted from Albuquerque et.al. 2016)

Parameter	Distribution	Parameters
Specific Weight	Inverse-Gamma	$\alpha = 24.72$ $\gamma = 3.47$ $\beta = 43.00$ $\mu = 0.0$
Friction angle		$\alpha = 59.63$ $\beta = 1731.99$
Cohesion	Dagum	$p = 1.44 \times 10^{-6}$ $a = 240{,}960$ $b = 29.0$

5 RESULT ANALYSIS AND DISCUSSION

The Monte Carlo method (100,000 statistical realizations for each case) was used to determine the probability of failure of a strip footing in an infinite slope. Several inclinations were studied in order to give a complete insight of the influence of this parameter in the probability of rupture, P(FoS < 1). The other parameters were a load of 50 kN, base width and depth of 1 m each for the footing and a height of 3 m for the infinite slope layer, all of them considered deterministic. No influence of the ground water lever was considered, and the hypothesis is of an undrained loading. The results are presented in Table 2 and Figure 3.

As it can be seen from Table 2 and Figure 3, for the distributions considered, the soil shows a high probability of rupture in case of inclinations greater than 20˚.

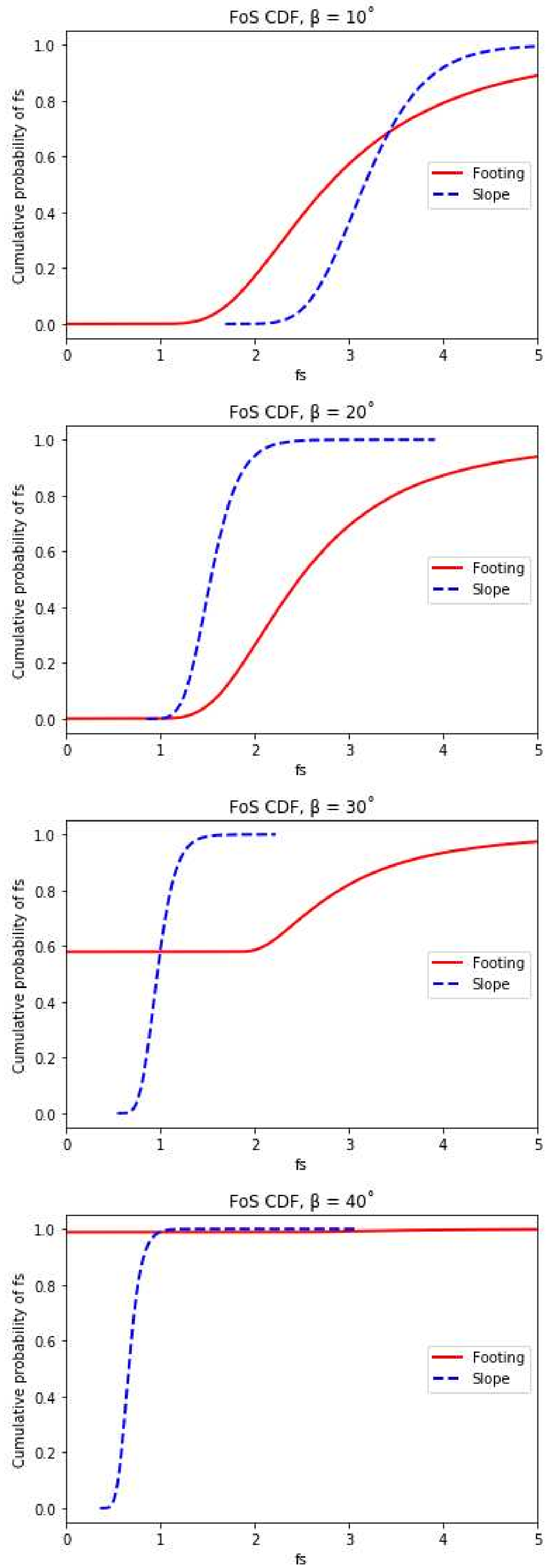

Figure 3. Cumulative Distribution Function of the factor of safety for several values of β.

Table 2 – Probability of rupture for the slope and the footing as a function of the inclination of the slope.

Inclination (β)	P(FoS < 1) Footing	P(FoS < 1) slope
10°	0	0
20°	0.1%	0.1%
30°	58.1%	58.1%
40°	98.8%	98.9%

One should note that the range of the FoS of the slope is smaller than the FoS of the footing. This implies, in a greater probability of failure on the footing than on the slope. However, this can be a characteristic of the local soil and must not be extrapolated for other regions.

Besides, if a deterministic approach was taken, for a load of 50 kN, base width of 1 m and a depth of 0.5 m for the footing, in a slope with inclination of 15 degrees, the safety factor using the mean values of Specific Weight, Friction angle and Cohesion would be of 1.61. The Monte Carlo simulation for this same case provides a probability of failure of 1.23%. This probability of failure can be considered high, depending on the impact (number of lives, remediation costs, etc) of the consequences if failure occurs. On the other hand, the deterministic factor of safety would provide some comfort to the designer, which may be misleading.

Another thing that can be seen is that the probability of failure of the slope and the footing for large inclinations are always the same. This is due to the collapsivity of the soil, making a cohesive soil behave as a sand when saturated. This can be seen in the CDFs above, in which the initial part of the curves for the footing is a straight line, accounting for all the situations where c=0 and $\beta>\phi$. For low inclinations, on the other hand, footing failure becomes dominant, as seen on Figure 3.

The influence of the other parameters (load, base width and depth) was also accounted for. Figure 4, shows that, as the load increases, the probability of failure rapidly increases. Therefore, the depth, the base width or both must be increased in order to achieve desirable safety (Figures 5 and 6).

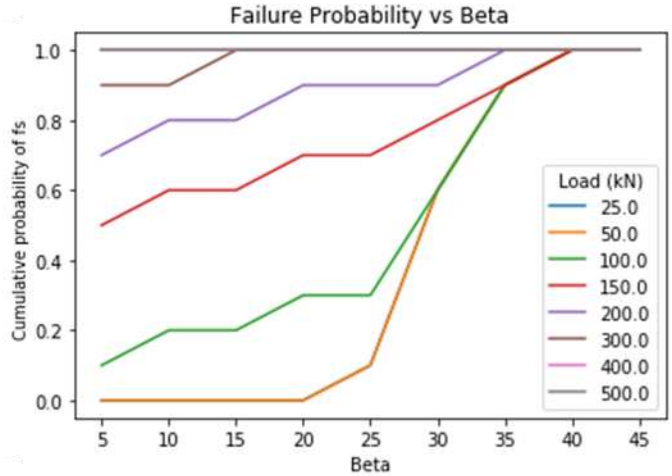

Figure 4. Cumulative Distribution Function of the factor of safety for several loads.

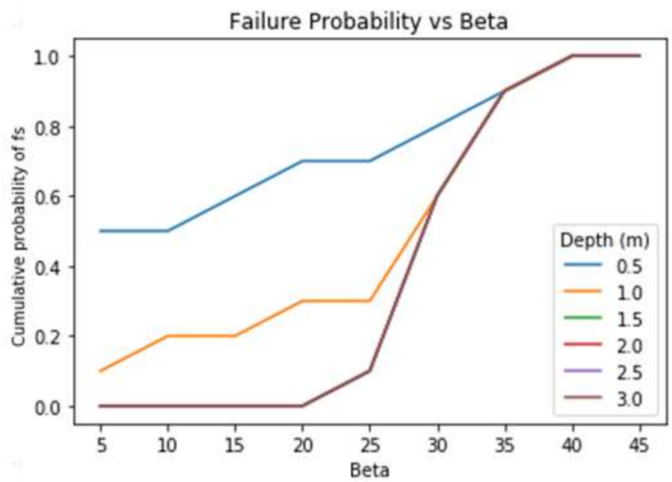

Figure 5. Cumulative Distribution Function of the factor of safety for several depths.

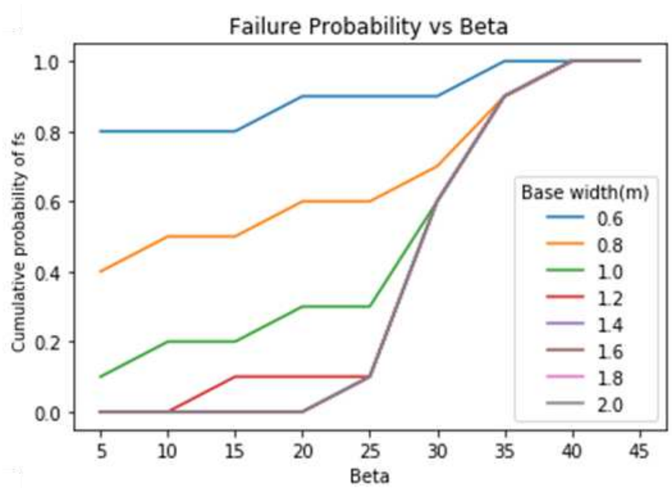

Figure 6. Cumulative Distribution Function of the factor of safety for several base widths.

6 CONCLUSIONS

The main goal of this paper was to account, in a similar approach to Ozelim et al. (2014) and Albuquerque & Campos (2015, 2016), for the importance of the probabilistic approach in the determination of the FoS of geotechnical works.

Based on the data used in this paper, the results show that one should not disregard the effect of the variability of soil parameters when designing footings on slopes, as the properties of materials often demand models that have asymmetric behavior and the probability of failure is highly dependant of those parameters. Furthermore, it should be stated that a misguided use of statistical distributions can result in erroneous assessment of the risks.

The authors recommendation is that designers should build their databases of the local soil parameters from academical and professional studies, making it possible to find the best distributions to model the soil parameters. Thus, proper risk assessment could be achieved, enhancing the reliability of foundation designs.

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