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Review of critical state theory based constitutive models for bio-cemented granular soils

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ABSTRACT

This paper critically examines some of the proposed constitutive models for granular soils within the CST framework and their evolution into a cemented soil model. Understanding the behaviour of biocemented soils has been an active area of research over the past two decades. Progress has been made in developing constitutive frameworks to capture their behaviour. One of the popular approaches to model cemented soil behaviour has been to modify the existing Critical State Theory (CST) frameworks for granular soils. Most of these models are highly complex and require a large number of parameters, making them difficult to implement for practical purposes. Various aspects of CST-based models for cemented soil are explored, including assumptions involved, challenges associated, and limitations of the models along with possible pathways forward.

Keywords: Critical State Theory (CST), biocementation, constitutive models

INTRODUCTION

Liquefaction is one of the most damaging types of failure in geotechnical engineering. The mechanics of liquefaction in saturated clean granular soils (e.g., sand) have been extensively studied through laboratory testing, discrete element methods and constitutive formulations. To enhance liquefaction resistance, various ground improvement techniques have been reported in the literature, e.g., Portland cement or grout treatment (Clough et al., 1981; Ismail et al., 2002), chemical treatment (Maher et al., 1994; Porcino et al., 2015; Saito et al., 2012), geosynthetic applications (Altun et al., 2008; Chew et al., 2000; McDougal & Sollitt, 1984), pre-loading (Karim et al., 2010; Karim et al., 2011), vibrocompaction (Annam & Raju, 2012; Raju & Sondermann, 2015), and other densification methods (Phear & Harris, 2008; Stuedlein et al., 2016).

Portland cement treatment is a popular approach (Biswal et al., 2019; Consoli et al., 2020; Joel & Agbede, 2011) but has a large carbon footprint, as cement production is estimated to contribute ~7% of the total global CO_2 emission (Li et al., 2013). It can also add alkalinity to the ground and surface waters, which can harm aquatic life (Shahin, 2017). Many of the other methods are also highly energy-intensive and can sometimes be toxic to the ecosystem. In recent decades, biotreatment techniques like enzyme-induced carbonate precipitation (EICP), microbially induced carbonate precipitation (MICP), and microbially induced desaturation and precipitation (MIDP) have emerged as potentially sustainable methods for liquefaction mitigation (Ahenkorah et al., 2023; Feng & Montoya, 2016; Hamdan & Kavazanjian Jr, 2016; Karol, 2003). These biogeochemical processes-based techniques precipitate $CaCO_3$ in different crystal forms like calcite and vaterite (see Figure 1). In these methods, the precipitates bridge and coat the soil particles, enhancing shear resistance and stiffness (Montoya et al., 2014). Figure 1 shows SEM images of biocemented granular soils.

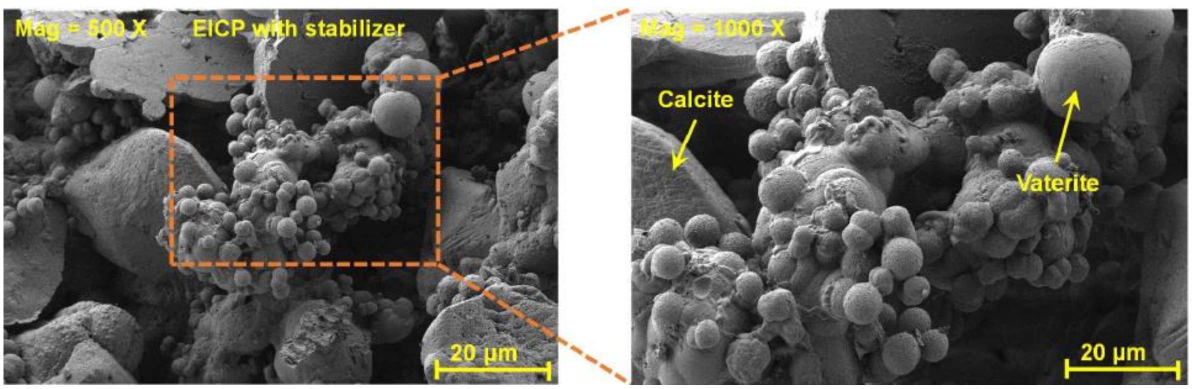

Figure 1. SEM images of CaCO3 spatial distribution (Ahenkorah, 2021)

The basic geochemical processes involved in $CaCO_3$ precipitation are presented in the following equations.

$$\mathbf{CO(NH_2)_{2\,(s)} + H_2O_{(l)} \xrightarrow{\text{urease enzyme}} NH_{4\,(aq)}^{+} + CO_{3\,(aq)}^{2-}}$$

(1)

$$\mathbf{Ca^{2+}{}_{(aq)} + CO\,{}_{3\,(aq)}^{2-} \xleftrightarrow{\text{precipitation}} CaCO_{3(s)}}$$

(2)

MICP and EICP are very similar biocementation processes, with the primary difference being the source of urease enzyme. MICP uses urease producing microbes to accelerate the process and the EICP uses urease enzymes extracted from plant sources. MIDP (O'Donnell et al., 2017) is a two-stage process for liquefaction mitigation. In Stage 1, the denitrifying bacteria causes N_2 and CO_2 gas emissions leading to desaturation of the soil, causing short-term mitigation. In Stage 2, carbonate precipitation takes place which further enhances liquefaction resistance.

Implementation of bio-cementation techniques in real-life engineering problems needs a thorough understanding of the material behaviour and constitutive relationships that capture their stress-strain response under a wide range of stress and boundary conditions. There have been several constitutive models with different levels of accuracy and sophistication proposed for biocemented soils in the literature. These models range from simple elastic to complex elastoplastic frameworks. Critical State Theory (CST) framework is well known and has been applied to many different soils. Attempts have been made to use the CST framework developed for granular soils to characterize cemented soil behaviour (El Kortbawi, 2022; Nweke, 2017; Zhang et al., 2023). The CST constitutive framework for clean sand is reasonably well-understood (Barnett et al., 2020; Dafalias & Manzari, 2004; Jefferies, 1993; Khayyer et al., 2024; Kolapalli et al., 2022; Wang et al., 1990). However, natural soil often contains some amount of fines. Some past work has investigated CST constitutive formulations of granular soil with non-plastic fines, e.g., silty sand (Barnett et al., 2021; Rahman & Dafalias, 2022; Rahman et al., 2014). Limited work can also be found in the literature that captures the behaviour of biocemented granular materials with different levels of fines content (f_c).

This paper reviews the existing constitutive formulations for biocemented granular soils. Due to expected behavioural similarity, attention has also been paid to models developed based on Portland cement-treated soils. Many of the existing simple correlations and elastoplastic frameworks are also summarized. Various aspects like assumptions involved, challenges associated, and limitations of the models following the cementation process are explored.

BIOCEMENTED SOIL BEHAVIOUR

The behaviour of soil is complex due to its dependency on stress path and stress history. The task becomes even more complicated when the effect of cementation is to be taken into account. It is to be noted that liquefaction is a phenomenon mostly relevant to loose granular soils and the discussion here is limited to treated and untreated granular soils. Various aspects associated with the incorporation of the cementation effect in constitutive formulations are discussed below.

Peak strength and stiffness

Observations from past studies suggest that biocementation increases the small strain stiffness, cohesion intercept (c'), and to some extent peak friction angle (φ_{peak}), resulting an increase in peak strength and stiffness (Cui et al., 2017; Lin et al., 2016; Nafisi et al., 2020; Van Paassen, 2009). The stiffness increase could be a challenging task to incorporate into a constitutive model because the amount of calcite precipitation is often not a valid indicator of a level of improvement due to variations in the uniformity of precipitation. Nweke (2017) defined the shear modulus in terms of a cementation parameter that is a function of induced shear strain to account for the increased shear stiffness due to biocementation. To account for the gain in peak strength, the failure criteria is defined in terms of the cementation component. Xiao et al. (2024) proposed expansion of the bounding surface to the left to consider the tensile strength due to biocementation. However, in the Xiao et al. (2024) model the right side of the bounding surface remained unchanged. Gai and Sánchez (2019) expanded the yield surface to account for cementation, introducing a hardening parameter to account for mechanical bond strength which is directly dependent on the mass of the calcite precipitation.

Critical state line (CSL)

Past studies reported that biocementation can change the CSL in both $q - p'$ and $e - p'$ space where deviatoric stress $q = \sigma'_1 - \sigma'_3$, mean effective stress $p' = {\sigma'_1 + 2\sigma'_3}/{3}$, and σ'_1 and $\sigma'_2 = \sigma'_3$ are the principal effective stresses, depending on the degree of cementation. Figure 2a illustrates a downward shift of CSL in $e - p'$ space for biocemented Adelaide Industrial (AI) sand (Ahenkorah, 2021). This shift may be attributed to the increased compressible nature of broken cementation bonds in between soil grains. Depending on the level of cementation, the CSL in $q - p'$ space may also change, as shown in Figure 2b. This change is likely due to cohesive bonds that remain intact in the soil mass, increased surface roughness due to surface coating, increase in particle angularity, and degraded calcite fines contributing to the density (Clough et al., 1981; DeJong et al., 2010; Riveros & Sadrekarimi, 2020). This non-uniqueness of the CSL with the degree of cementation is challenging because it needs to account for all contributing factors. Zhang et al. (2023) accounted for this effect by altering the critical stress ratio (M_c) and critical void ratio (e_c) with initial cement content (C_c) for Portland cement treated soils. El Kortbawi (2022) accounted for this effect by connecting the relative density-based state parameter (ζ_R) with tensile strength due to cementation.

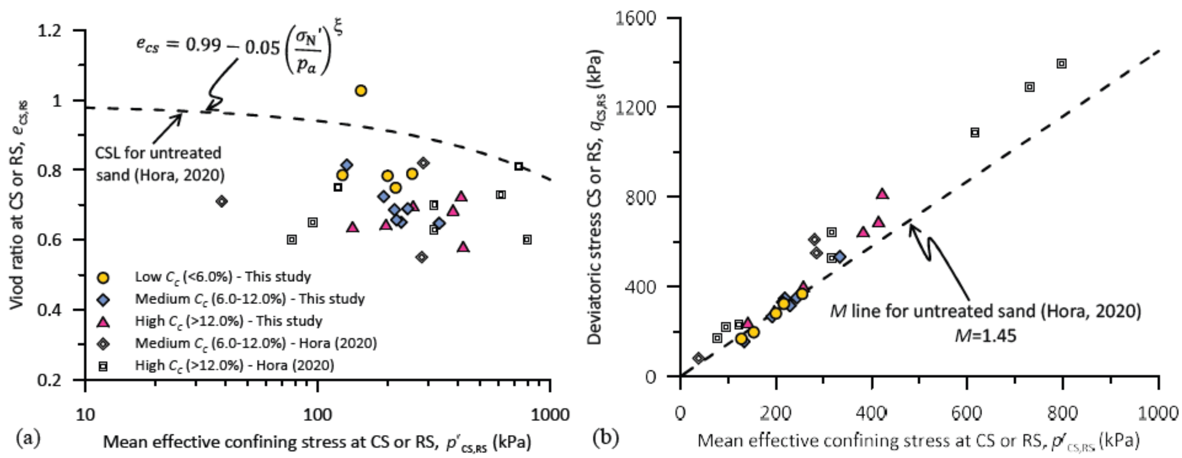

Figure 2. CSL in $e - p'$ and $q - p'$ space for treated and untreated AI 30/60 sand samples (Ahenkorah, 2021)

Dilatancy behaviour

The dilative tendency of a granular soil was observed to be more pronounced in biocemented soils (He & Chu, 2014; Montoya et al., 2014; Wu et al., 2021). The dilatant behaviour of cemented soil is attributed to the densification of soil by filling the voids as well as the creation of angular large-size aggregates. Zhang et al. (2023) in his constitutive formulations captured this effect by defining a modified dilatancy stress ratio in terms of cementation level and bond strength. Xiao et al. (2024) used energy dissipation-based state-dependent equation for dilatant behaviour to capture the effect of bio-cementation.

Strain softening

Past studies suggest that cementation increases the brittleness of the soils leading to more pronounced strain-softening behaviour (Lin et al., 2016). It is believed that this strain softening is caused by cementation bond degradation rather than the pore pressure development under undrained loading conditions (Lu et al., 2021). For any constitutive framework, it is imperative to model the post-peak strain softening response.

Bond degradation

Degradation of cementation progresses with shearing until soil reaches its critical state (DeJong et al., 2006; Montoya & DeJong, 2015; Weil et al., 2012). At critical state, the soil is expected to behave like a granular soil due to degraded bonds. A constitutive model formulation for biocemented sand needs to be capable of simulating the bond degradation. El Kortbawi (2022) proposed capturing the elastic cemented shear modulus (G_{cem}) and the degradation of other cemented parameters based on shear wave velocity ($V_{s,cem}$). Gai and Sánchez (2019) proposed a damage factor in his model to account for bond degradation. Lu et al. (2021) proposed an evolution rule in the constitutive framework for cemented soils to describe cementation degradation.

EXISTING CONSTITUTIVE MODELS FOR CEMENTED GRANULAR SOILS

A wide range of types of constitutive models exists in the literature starting from simple empirical correlations to complex elastoplastic frameworks. A selected set of such models developed based on artificially cemented granular soil are summarized in Tables 1 and 2 along with targeted material, modelling base, validation data sets, correlations developed, and limitations. Two of the more recent models are of special interest here: the models of El Kortbawi (2022) and the model of Zhang et al. (2023).

El Kortbawi (2022) extended the plane strain plasticity model proposed by Boulanger and Ziotopoulou (2015) to biocemented soil. The overall predictive capability of the model was good; however, the model is only applicable for light to moderate levels of cementation (up to 3%) for clean sand. The constitutive formulation of El Kortbawi (2022) is based on the relative state parameter, $\zeta_R = D_{R,cs} - D_R$, where $D_{R,cs}$ is the relative density at critical state and D_R is the relative density at the current state at given confining pressure (p'). Furthermore, the dependency of the elastic cemented shear modulus (G_{cem}) and degradation of other cemented parameters are based on the concept of the shear wave velocity of the cemented soil ($V_{s,cem}$), which is difficult to monitor. Zhang et al. (2023) proposed a bounding surface CST-based constitutive framework for Portland cement treated sand for both monotonic and cyclic loading. The model captures all aspects of the stabilized sand. An upward shift of the CSL in $e - p'$ space is considered in this model. However, some studies on biocemented samples suggest a downward shift (Ahenkorah, 2021; Riveros & Sadrekarimi, 2020). Additionally, this model requires 26 parameters making it quite complex.

Table 1. Summary of existing simple empirical correlations for artificially cemented granular soil.

Sl No	Reference	Targeted material	Targeted features	Developed correlation	Testing database
1	Riveros and Sadrekarimi (2020)	Biocemented sand	Shear stiffness increase	• $G_{maxN} = 1161({}^{\sigma'_{vc}}/_{P_a})^{0.38}$ The power is reduced from 0.5 to account for the reduced effect of normal stress on treated samples	Direct simple shear
2	Liu et al. (2019)	Biocemented calcareous sand	Increase in unconfined compressive strength (UCS), splitting tensile (ST) strength, and Young's modulus	• $q_u(kPa) = 41.1\exp\ (0.807R_c)$ • $E_{50u}(kPa) = 3888\ \exp\ (0.807R_c)$ • $E_{50t} = 1926\ \exp\ (0.807R_c)$ • $q_t = 6.4\ \exp\ (0.807R_c)$ Where $R_c = \frac{V_c}{V}$, V_c is the volume of cementation solution ,and V is is the volume of the sample	UCS tests, ST tests, drained triaxial testing
3	van Paassen et al. (2010)	Biocemented sand	Increase in UCS, young modulus and shear modulus	• $E_{50}(GPa) = 0.06e^{0.1502Cc}$ • $E_{ur}(GPa) = 0.25e^{0.1445Cc}$ • $G_o = \frac{E_{ur}}{2(1+\nu)}$ Where C_c is the cementation content (%), E_{ur} is the loading-unloading Young's modulus, ν is the Poisson's ratio.	UCS tests and geophysical measurements
4	Cheng et al. (2020)	Bio-bricks	Increase in UCS	• $UCS\ (kPa) = 54614C_c - 1558.5$ The correlation considers 50% degree of saturation.	UCS tests
5	Ismail et al. (2002)	Portland cemented calcareous soil	Increase in UCS	• $q_{us}(MPa) = 0.09e^{0.204C_p}$ Where C_p is the Portland cement content (%).	UCS tests
6	Bernardi et al. (2014)	MICP-treated, lime, and cement-treated bio-bricks	Increase in strength and young modulus as a function of p-wave velocity tests and calcite content	• $q_u\ (kPa) = -5265 + e^{(0.0232Cc+8.49)}$ • $E_{50}\ (kPa) = -6527 + e^{(0.2052Cc+8.1)}$ Where C_c=calcite content (%)	UCS tests and compression (p-wave velocity) tests

Table 2. Summary of existing constitutive frameworks for artificially cemented granular soil.

Sl No	Reference	Baseline formulation	Targeted material	Targeted features	Model validation	Validation base	Model performance
1	Gajo et al. (2019)	(Gajo et al., 2015)	carbonate cemented sandstone or microbially cemented silica sand	Gain in strength and stiffness with cement augmentation	Yes	Drained monotonic triaxial compression tests on microbially treated Ottawa sand	The model better captures the trend of brittle failure and increased strength of treated soil, however, it overpredicts the initial volumetric strain response. This overprediction is associated with some simplifications like associated flow rule adoption, neglecting elastic anisotropy, increase in bulk stiffness due to soil densification, and elastic stiffness increase due to level of stress.
2	Nweke (2017)	Nor-Sand model (Jefferies, 1993)	Biocemented sand	gain in strength and stiffness, dilation, and degradation of cementation with shearing	Yes	drained monotonic triaxial compression	The model can capture the monotonic response, simulating the increase in dilatancy and strain softening with bond degradation, accounting for up to ~6% cement content. Beyond this limit, the predictions are unreliable because the void space reduction is assumed to be neglected in light cementation which is not the case for high levels of cementation.
3	Gai and Sánchez (2019)	-	Biocemented sand	Bond degradation during shearing, critical yield surface enhancement to account for cementation and sub-loading concept for	Yes	Drained triaxial monotonic tests	Overall, the model captures the stress-strain, volumetric strain response, and effective stress paths very well under varying loading paths, confining pressures, and a wide range of cementation contents. Furthermore, during shearing the bond degradation is well simulated by the model.
4	El Kortbawi (2022)	PM4Sand (Boulanger & Ziotopoulou, 2015)	Biocemented sand	Increased shear stiffness and peak strength due to cementation and degradation of these improvements due to damage accumulation	Yes	Monotonic and cyclic triaxial testing	Overall, the qualitative predictive capability of the model is reasonably good; however, the pore water pressure generation during cyclic loading is overpredicted. The model is applicable for light to moderate levels of cementation (~ 3%).

Sl No	Reference	Baseline formulation	Targeted material	Targeted features	Model validation	Validation base	Model performance
5	Xiao et al. (2021)	-	Biocemented calcareous sand	Cohesive bonds due to CaCO3 and stress-induced anisotropy, effect of particle configuration on cyclic response	Yes	Undrained monotonic and cyclic triaxial testing on biocemented sand	The model prediction of stress-strain loop and excess pore water generation during cyclic loading is reasonably well.
6	Xiao et al. (2024)	-	Biocemented calcareous sand	Strain softening, dilation, and bond degradation of biotreated sand	Yes	Drained monotonic triaxial compression tests	The proposed model captures the strain hardening and softening, contracting, and dilatant behaviour as well as state parameter evolution in triaxial tests. However, the stress ratio is underestimated at lower confining pressure because of the unique critical stress ratio value.
7	Fauriel and Laloui (2012)	-	Biocemented	Permeability, porosity, and stiffness change due to calcite precipitation	Yes	Drained triaxial compression tests on biocemented sands	The model can reproduce all mechanisms of interest including physical, biological, and chemical processes occurring during the reactive grout injection in a saturated deformable porous medium.
8	Lu et al. (2021)	Cyclic mobility model proposed by (Zhang et al., 2007)	Biocemented sands and cemented clays	Degradation of cementation, tensile strength via state variable to characterize the degree of cementation	Yes	Monotonic and cyclic triaxial testing, isotropic and uniaxial tests	The proposed model well describes the influence of cementation and stress-induced anisotropy on the mechanical response under both monotonic and cyclic loading conditions
9	Zhang et al. (2023)	SANISAND model (Dafalias & Manzari, 2004)	Portland cement-treated model	The full response of cemented soil	Yes	monotonic triaxial compression tests, undrained cyclic direct simple shear (DSS) tests	The constitutive formulation simulates cemented soil response very well capturing all characteristic features like peak strength and stiffness, dilatancy, bond degradation, and shift of CSL.

CAPTURING THE EFFECT OF CEMENTATION IN MODELLING

As stated previously, some behavioural similarities are expected between the biocemented and parent soil, especially at large shearing strain. To evaluate the shortcomings of approach based upon parent soil behaviour and highlight development needs, a constitutive framework presented by Dafalias and Manzari (2004) was adopted here to simulate the monotonic drained shear behaviour of untreated and biocemented Adelaide industrial (AI) sand. The model selection was rather arbitrary, however observations made here are applicable to similar elastoplastic models. The model is first calibrated for untreated AI sand and then validated against treated soil. The model parameters are summarized in Table 3. Further details on the behaviour of uncemented soil can be found in Hora (2021).

Figure 3 presents the observed and predicted consolidated isotopically drained (CID) response of untreated AI sand after consolidation to 200 kPa. The trend of the deviatoric stress-strain response, volumetric behaviour, and effective stress path (ESP) is captured by the constitutive model with reasonable accuracy. For the same AI sand, the cemented shear response after consolidation at 45 kPa is presented in Figure 4. The cemented soil exhibits larger dilation throughout shearing and has a higher normalized peak strength as compared to the untreated soil. The strain softening is more pronounced in the cemented case. Adopting guidelines proposed by Zhang et al. (2023), the elastic shear modulus (G) and plastic modulus (K_p) are defined in terms of cementation content and bond strength, keeping all other parameters unchanged. The modelling constants defining bond strength and degradation with shearing are $h_p = 80, a^p = 1872 \ and \ b^p = 1$. The model in which cementation effect is considered is renamed as "cemented parameters". Fairly good agreement can be observed between experimental and predicted data trends. However, the peak strength is underestimated by the model, which suggests the need for changes to the model to capture the cementation effect. Similarly, the predicted volumetric response after ~10% axial deformation deviates from the observed behavior. Table 3 summarizes model constants.

Table 3. Model parameters.

Parameter	Symbol	Value
Elasticity	G_o	80
	ν	0.3
Critical state	M	1.45
	e_Γ	0.99
	λ_c	0.05
	ζ	0.64
Dilatancy	d_o	1.2
	m	2.4
Hardening/plastic modulus	h_o	13
	c_h	0.85
	n	3

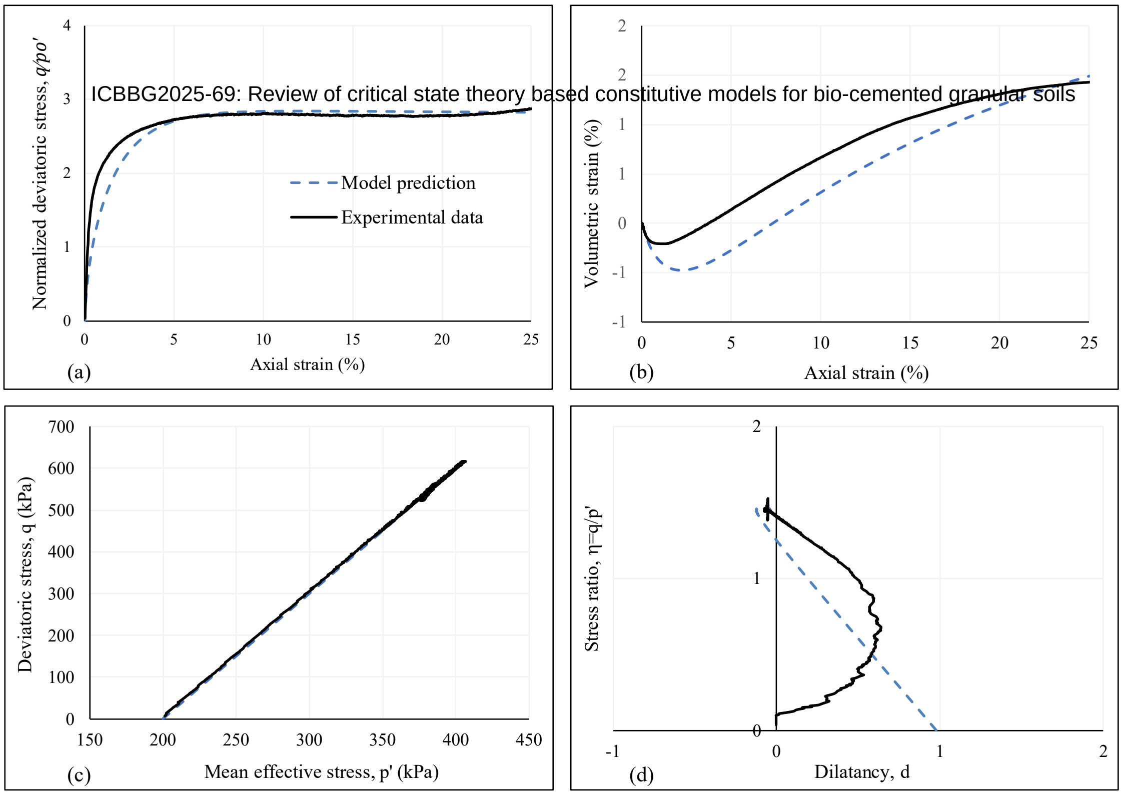

Figure 3. Comparative analysis of experimental behaviour and model prediction for untreated AI sand ($e_o = 0.827, p'_o = 200\ kPa$) (a) normalized stress-strain response (b) volumetric strain curve (c) effective stress path (ESP) (d) dilatancy-stress ratio.

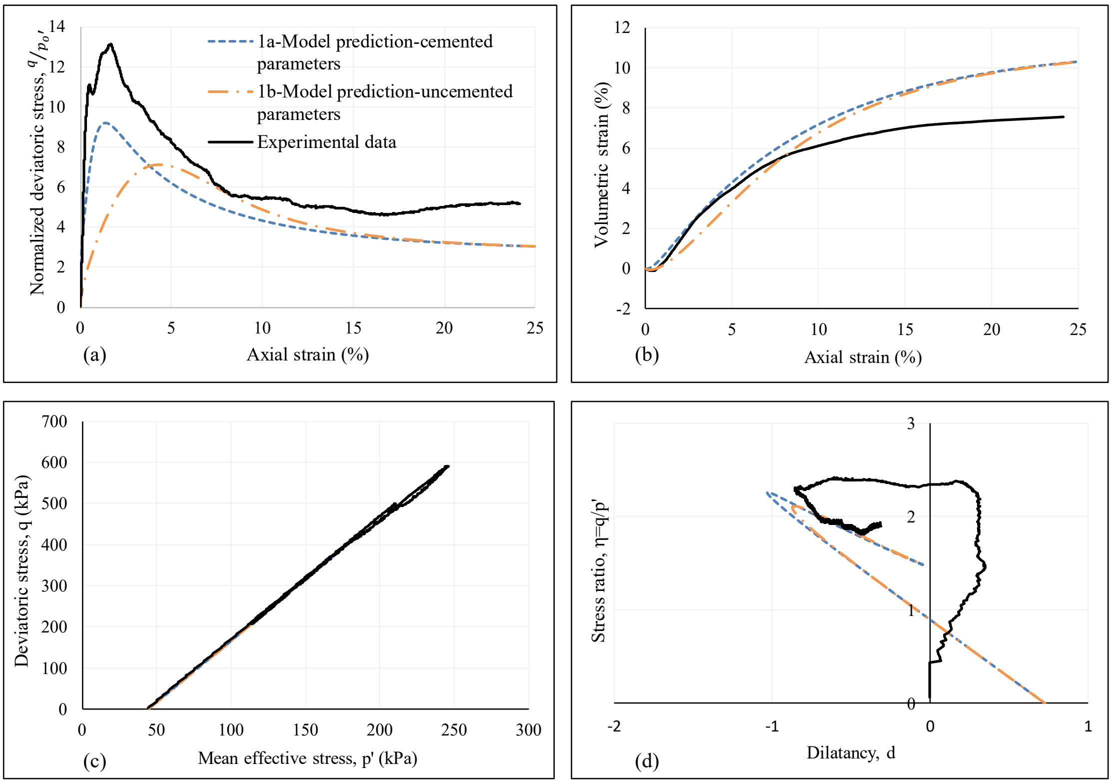

Figure 4. Comparative analysis of experimental behaviour and model prediction for biocemented AI sand ($e_o = 0.751, p'_o = 45\ kPa, Cc = 13\%$) (a) normalized stress-strain response (b) volumetric strain curve (c) effective stress path (ESP) (d) dilatancy-stress ratio.

CONCLUSION

The following conclusions can be drawn from the discussions in this paper:.

- Use of granular soil constitutive models to capture biocemented soil behaviour has deficiencies in capturing peak strength, dilative tendency, and the shift of CSL.
- Studies suggest that cementation increases peak strength, dilatancy and shift CSL in both $q - p'$ and $e - p'$ space. Hence, a more elaborate constitutive framework is needed that capture all these characteristic features.
- The shift of the CSL in $e - p'$ due to biocementation is still unclear, as some studies reported an upward shift and others suggest a downward shift.
- Modifications of existing clean sand constitutive formulations to capture the behaviour of cemented soil can be highly complex, needing a large number of parameters, making them difficult to implement.
- Very limited research is available on the shear behaviour of biocemented sand with fines to evaluate the combined effect of fines content and cementation level on critical state behaviour.
- No constitutive framework can be found in the literature that captures the behaviour of biocemented granular soils with different fine contents.

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