Evaluation of Extrapolation Methods for Estimating Limit Pressure in Stiff Soils of Western Mexico City

Évaluation des méthodes d'extrapolation pour l'estimation de la pression limite dans les sols rigides de l'ouest de la ville de Mexico

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ABSTRACT

The pressuremeter test is a fundamental tool for in situ characterization of the stress–strain behavior of soils. However, in stiff soils of western Mexico City, the measured expansion often does not reach the failure limit, requiring estimation of the limit pressure through mathematical extrapolation of the pressure–volume curve based on available experimental data. This study evaluated the validity of the double hyperbola method by comparing it with three-dimensional simulations in Plaxis 3D, using Mohr-Coulomb and Hardening Soil constitutive models. The extrapolated limit pressure was 2.36 MPa, only 0.85% above the numerical limit pressure (2.34 MPa) obtained by extending the simulation to the limit volume at 19 m depth, confirming its accuracy for this type of soil. Other criteria, such as the inverse volume and maximum pressure methods, showed greater deviations (underestimations of 6.8% and 15.4%, respectively). This research continues with the development of a local database to strengthen limit pressure estimates in deep foundation design and considers complementing extrapolation and modeling with pressuremeter tests using higher capacity probes.

RESUME

L'essai pressiométrique constitue un outil fondamental pour la caractérisation in situ du comportement effort–déformation des sols. Cependant, dans les sols rigides de l'ouest de la ville de Mexico, l'expansion mesurée n'atteint souvent pas la limite de rupture, ce qui rend nécessaire l'estimation de la pression limite par extrapolation mathématique de la courbe pression–volume, à partir des données expérimentales disponibles. Cette étude évalue la validité de la méthode de la double hyperbole en la comparant à des simulations tridimensionnelles dans Plaxis 3D, utilisant les modèles constitutifs de Mohr-Coulomb et Hardening Soil. L'extrapolation a donné une pression limite de 2,36 MPa, soit seulement 0,85 % au-dessus de la pression limite numérique (2,34 MPa) obtenue en étendant la simulation jusqu'au volume limite à une profondeur de 19 m, ce qui valide sa précision pour ce type de sols. D'autres critères, comme les méthodes du volume inverse et de la pression maximale, ont montré des écarts plus importants (sous-estimations de 6,8 % et 15,4 %, respectivement). Cette recherche se poursuit par la constitution d'une base de données locale visant à renforcer les estimations de pression limite dans la conception des fondations profondes, tout en envisageant de compléter l'extrapolation et la modélisation par des essais pressiométriques à l'aide de sondes de plus grande capacité

Keywords: pressuremeter, limit pressure, extrapolation, double hyperbola, numerical modeling, stiff soils

1. Introduction

The pressuremeter test, developed by Ménard in 1957 to measure the radial expansion of soil in a borehole, has become a well-established method for *in situ* mechanical characterization. It allows the determination of parameters such as the pressuremeter modulus (E_m) and the limit pressure (P_L), which are highly useful for the design of deep foundations and for analyzing ground deformations (Baguelin *et al*., 1978; Briaud, 1992). Unlike other in situ tests, the pressuremeter provides a direct stress–strain curve of the soil under radial loading, providing a more realistic description of ground behavior, particularly in complex geotechnical settings (Clarke, 1995).

One of the main challenges in the use of the pressuremeter test is the reliable estimation of the limit pressure P_L, particularly in stiff soils where the deformed volume during testing does not reach twice the initial cavity volume, as originally defined by Ménard. To address this limitation, several extrapolation techniques for the pressure–expansion curve have been developed, including:

- Double hyperbola method: Explicitly outlined in ISO 22476-4, it offers flexibility by fitting both elastic and plastic phases of the curve.
- Initial tangent point method: Widely used in stiff soils for its simplicity (extrapolating the elastic slope until it intersects the pressure axis) and recommended by both ISO and NF P 94-110-1.

- Linear regression in 1/V (reciprocal method): Prescribed in ASTM D4719 and referenced in ISO as the "reciprocal curve method," often used to validate results against the double hyperbola.
- Simple hyperbola and exponential function: Less commonly used; typically reserved for comparative studies rather than standard consulting practice.

The double hyperbola method has become one of the most frequently used approaches in geotechnical practice and, as mentioned above, is specified in technical standards such as ISO 22476-4 and NF P 94-110-1 due to its ability to reduce bias in the extrapolation of P_L.

The interpretation of pressuremeter tests has evolved with the incorporation of inverse analysis techniques based on numerical models, which iteratively adjust parameters (such as E_m, P_L, p_0) in a finite element model to reproduce the pressure–expansion curve measured in the field. In this context, Hicher and Rangeard (2004) propose the use of inverse analysis techniques to characterize saturated fine soils, emphasizing the importance of using interpretation models capable of capturing the nonlinear response of the ground. Complementarily, Fawaz *et al*. (2014) explore the correlation between the pressuremeter modulus and Young's modulus obtained from numerical simulations, highlighting the importance of validating extrapolation methods using independent tools such as laboratory tests or alternative in situ methods to ensure the reliability of estimated parameters.

In Mexico City, the use of the pressuremeter has expanded into the western zones, where high-stiffness volcanic materials such as tuffs prevail, exhibiting a mechanical response distinct from the soft lacustrine deposits found in the city center. In these areas, the difficulty in reaching the limit volume during testing, combined with limited local experience, creates uncertainty in estimating the limit pressure, which can lead to unreliable design.

In this context, it is pertinent to recall that, according to Ménard's original definition (Baguelin *et al*., 1978; Briaud, 1992), the limit pressure is associated with the radial stress required to expand the soil cavity until reaching a deformed volume equivalent to twice the initial volume. This is expressed as:

$$V_{lim} = V_s + 2V_1 \tag{1}$$

where V_s is the cavity volume at contact, and V_1 is the volume increment corresponding to the contact pressure.

This criterion has been adopted as a theoretical reference for evaluating, through numerical modeling, the simulated limit pressure under controlled conditions and comparing it with extrapolated values obtained from field data.

The objective of this study is to assess the accuracy of the double hyperbola extrapolation method for estimating the limit pressure in stiff materials from western Mexico City. To this end, a real pressuremeter curve and its numerical replica—developed using three-dimensional modeling with Mohr-Coulomb and Hardening Soil constitutive models—are analyzed. The results allow for a comparison between the extrapolated limit pressure and the simulated limit pressure obtained at theoretical failure volume, offering insights based on the percentage difference between extrapolated and simulated PL; deviations in the shape of the stress–strain curve in both elastic and plastic phases; sensitivity of the double hyperbola fit to changes in material stiffness; and the method's ability to replicate the nonlinear soil response under radial loading.

2. Methodology

2.1. Site Description and Pressuremeter Test

The study site is located in the western part of Mexico City, within Zone I (hill zone) according to the city's geotechnical zoning map (Figure 1). This area is characterized by firm volcanic materials which, unlike the soft sediments of the lacustrine valley, exhibit higher strength, lower compressibility, and more stable mechanical behavior under load.

Figure 2 shows the stratigraphic profile of the site, corresponding to a foundation project for a bridge supported on these firm materials. This profile allows identification of the main geotechnical units present. At the top, there is a layer of heterogeneous fill, followed by a pyroclastic-origin tuff composed of low-plasticity silty sands. Beneath this lies a thick layer of andesitic sand, locally known as "blue sand," with high strength and low deformability. This is followed by a second silty-sandy tuff with characteristics similar to the first. Further below is a layer of pumiceous sand with high water content, and finally, a third silty-sandy tuff of high strength, where more than 50 blows per 30 cm were recorded in the SPT.

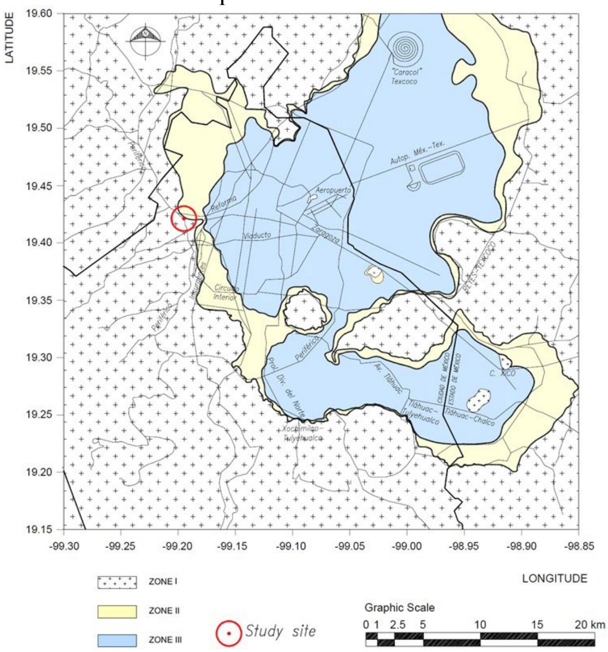

Figure 1. Location of the study site in the geotechnical zoning map of Mexico City

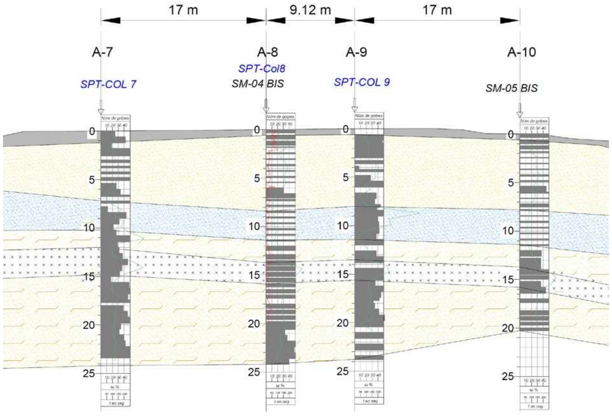

Figure 2. Stratigraphic profile of the study site

The pressuremeter test analyzed in this study was carried out at a depth of 19 m, which corresponds to the foundation level of the bridge elements. At this depth, the soil consists of tuffaceous material with a predominance of silty sand, typical of the local volcanic deposits. This material exhibits high strength and low deformability, preventing the test from reaching the limit volume, and thus hindering direct interpretation of the pressure–volume curve.

The test was performed in accordance with ISO 22476-4 (2012), using a Ménard-type probe with controlled radial expansion and following the recommended loading rates. The pressure–volume curve obtained displayed a well-defined pseudoelastic segment, followed by a change in slope indicating the onset of plastic behavior, without reaching the theoretical limit volume. Therefore, the application of extrapolation methods, particularly the double hyperbola method, is justified to estimate the limit pressure of the stratum.

Table 1 summarizes the mechanical properties assigned to each geotechnical unit in the model, and Figure 3 shows the pressure–volume curve obtained in the test.

Table 1. Mechanical properties assigned to each geotechnical unit in the model

Strata	γ (kN/m³)	c (kN/m²)	ϕ (°)	E (kN/m²)	ν	K_0
UG1: Fill	14	1	37	15,000	0.3	0.6
UG2: Tuff 1	14	1	42	20,000	0.3	0.6
UG3: Blue sand	15	1	45	25,000	0.3	0.6
UG4: Tuff 2	14	1	36	35,000	0.3	0.6
UG5: Pumiceous sand	12	1	45	20,000	0.3	0.6
UG6: Tuff 3	16	1	25	44,900	0.3	0.6

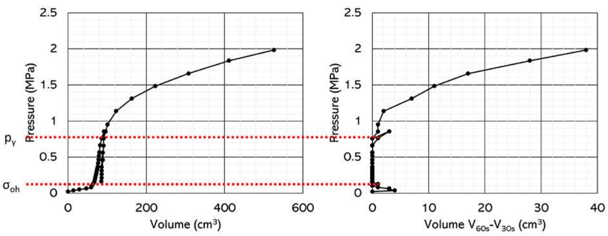

Figure 3. Pressure–volume curve obtained from the pressuremeter test

2.2. Numerical Modeling

To validate the limit pressure estimated by extrapolation and simulate the field loading conditions of the pressuremeter test, a three-dimensional numerical model was developed using the commercial finite element software Plaxis 3D.

The computational domain consisted of a soil column measuring 4 m × 4 m in plan and 25 m in depth, with a central borehole of 6 cm diameter (representing the pressuremeter cavity). The active length of the probe was 20 cm, located between elevations −18.9 m and −19.1 m. A refined mesh was applied around the borehole, and boundary conditions were established to restrict displacements. The numerical model captured all stress states, from borehole formation to the application of radial pressures. Figure 4 illustrates the characteristics of the numerical model.

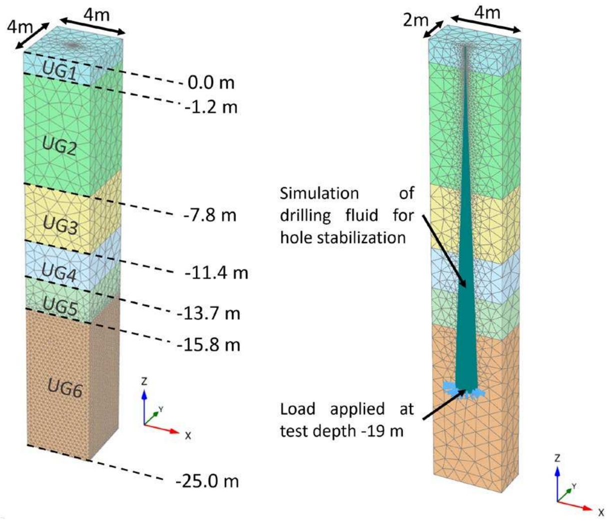

Figure 4. Numerical model configuration in PLAXIS 3D

To simulate the behavior of the soil in the stratum of interest (UG6: Tuff 3), two constitutive models were used, whose initial properties are presented in Table 2:

- **Mohr-Coulomb (MC):** A perfectly elastic–plastic model with constant parameters. For calibration, cohesion (c) and internal friction angle (ϕ) were adjusted, while the elastic modulus (E) was kept fixed using the value obtained directly from the pressuremeter test.
- **Hardening Soil (HS):** An advanced model incorporating strain-dependent stiffness. Initially, the secant loading modulus (E_{50}), oedometer modulus (E_{oed}), and unloading–reloading modulus (E_{ur}) were defined along with strength parameters. A second calibration adjusted stiffness and reference pressure to better match the experimental curve.

The numerical simulation followed this sequence:

- Generation of initial stress state under self-weight.
- Excavation of the pressuremeter cavity (6 cm diameter).
- Application of radial pressures in increments of 0.25 MPa up to 2.5 MPa.
- Monitoring volumetric deformations along the cavity wall.

For the Mohr-Coulomb model, the elastic modulus was fixed at $E = 44{,}900$ kPa, and c and ϕ were calibrated. The adjustment aimed to replicate the shape of the pressure–volume curve measured in the field, accounting for the initial contact closure between the probe and borehole wall. From the horizontal pressure point onward, both the initial slope and plastic behavior were accurately modeled (Figure 5).

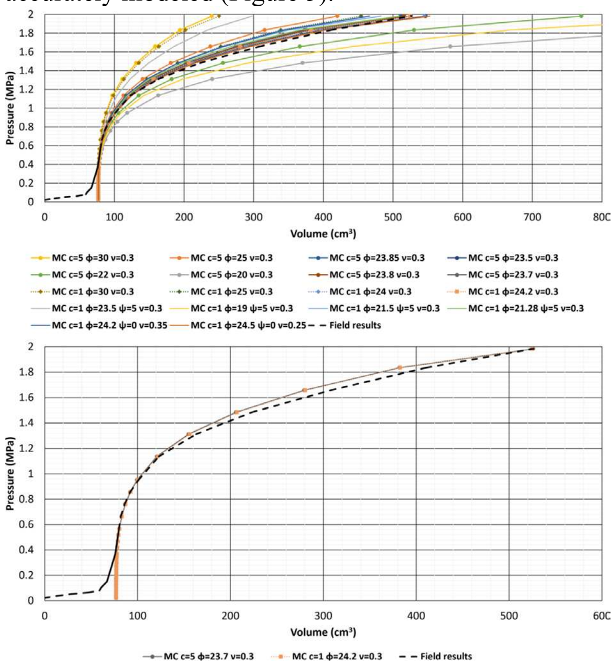

Figure 5. Calibration of the numerical model using Mohr-Coulomb

Table 2. Initial parameters for the variation of the target stratum

Strata	Constitutive model	γ (kN/m³)	c (kN/m²)	ϕ (°)	E_{50} (kN/m²)	E_{oed} (kN/m²)	E_{ur} (kN/m²)
UG6: Tuff 3	**MC**	16	1	25	44,900	-	-
UG6: Tuff 3	**HS**	16	1	25	44,900	44,900	130,100

In the case of the Hardening Soil model, the initial strategy maintained the moduli as obtained from the test. However, this approach failed to accurately reproduce the curve in the intermediate zone between the end of the pseudoelastic segment and the start of plasticity (Figure 6) suggesting that the stiffness inferred from the test might overestimate the actual behavior of the stratum, resulting in a numerically stiffer response.

To improve the fit, a second calibration was performed by reducing E_{50}, E_{oed}, and E_{ur} values and adjusting the strength parameters. With this new configuration, the simulated curve showed improved agreement with the experimental curve, especially in the pseudoelastic–plastic transition zone (Figure 7).

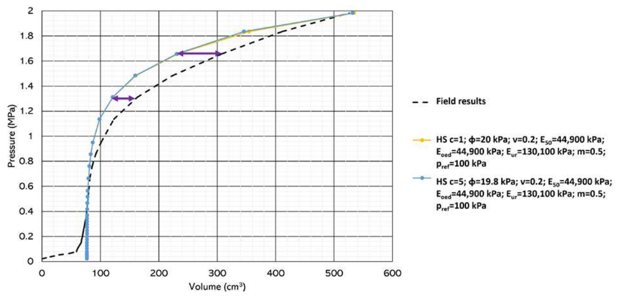

Figure 6. Results using constant moduli in the Hardening Soil model

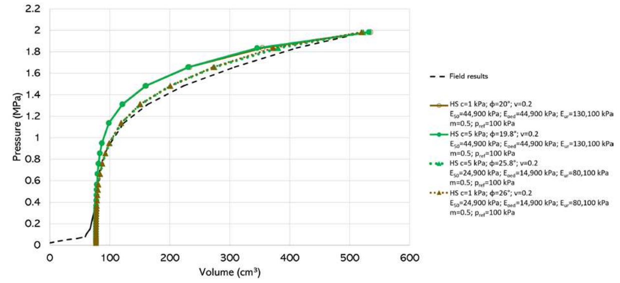

Figure 7. Improved fit after reducing stiffness in the Hardening Soil model

Once the model was calibrated to closely replicate the field curve, the simulation continued with increasing pressure until the theoretical limit volume (V_{lim}) was reached as defined by Ménard. The pressure at that point was defined as the numerical limit pressure, which serves as a reference value to evaluate the accuracy of analytical extrapolation methods. Figure 8 shows the extension of the simulated curve up to that volume.

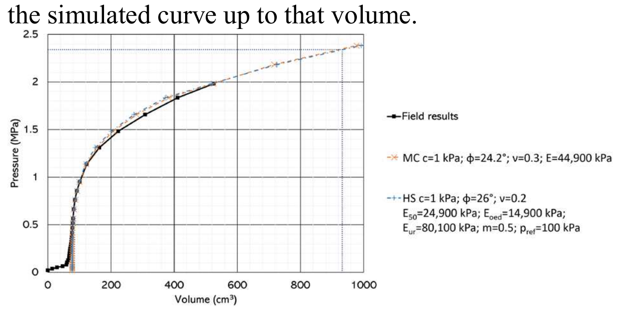

Figure 8. Extension of the simulated curve up to the limit volume

2.3. Limit Pressure Validation Criterion

To obtain a reliable reference value for limit pressure (P_L), the criterion previously described in the introduction was adopted based on the radial stress required to double the initial cavity volume during expansion. This criterion, originally proposed by Ménard, was implemented in the numerical model by progressively applying radial pressures until the defined volume was reached.

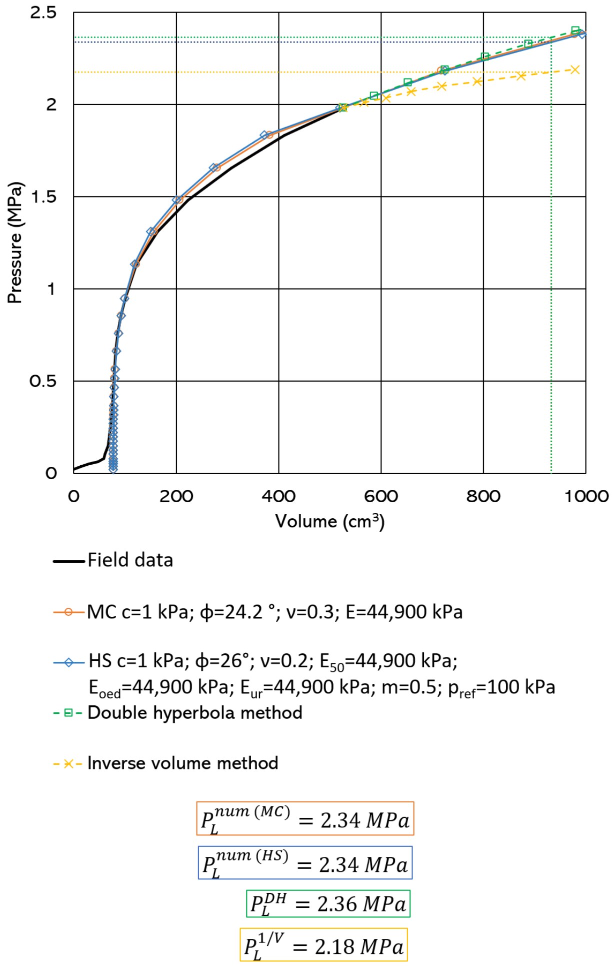

Figure 9. Comparison between limit pressure obtained from numerical modeling and extrapolation using the double hyperbola method

The pressure obtained at that point was recorded as P_L^{num} and it is considered the most representative value of the soil's load-bearing capacity in this simulation, as it is directly tied to the material's deformational behavior.

In parallel, the limit pressure was also estimated from the pressure–volume curve obtained in the field, using the double hyperbola extrapolation method in accordance with NF P 94-110-1 (2000). The method uses two hyperbolic segments (elastic and plastic), intersecting at the extrapolated limit pressure.

The comparison between (P_L^{num}) and the extrapolated value allows for evaluating the accuracy of the method within the context of this study, where the pressuremeter test did not reach the failure state and the limit pressure had to be indirectly estimated.

By extending the simulation until the limit volume (V_{lim}), was reached, a numerical limit pressure of 2.34 MPa (P_L^{num}) was obtained. This value is adopted as the internal reference for the calibrated model—that is, it serves as a benchmark within the scope of this simulation, acknowledging that it is also an adjustment rather than a direct measurement of failure in the field.

For a more thorough validation, pressuremeter tests using higher capacity equipment could be considered, as they may allow direct attainment of the V_{lim} criterion in these stiff soils. Similarly, the double hyperbola extrapolation (P_L^{DH}) is also an adjustment, and both approaches should therefore be regarded as complementary methods for estimating limit pressure.

Table 3. Comparison of limit pressure obtained by modeling and extrapolation

Estimation Method	P_L [MPa]	Relative difference (%)
Numerical modeling (MC/HS)	2.34	—
Double hyperbola (extrapolation)	2.36	0.85
Inverse volume method	2.18	−6.8
Maximum pressure reached in test	1.98	-15.4

Additionally, the inverse volume method and the criterion based on the maximum pressure recorded during the test were applied.

The inverse volume method yielded a limit pressure of 2.18 MPa (P_L^{inv}), corresponding to an underestimation of 6.8% relative to (P_L^{num}). While this degree of deviation may be acceptable during preliminary design stages, its accuracy is highly dependent on the shape of the pressure–volume curve and the quality of in situ data.

In comparison, using the maximum recorded pressure (1.98 MPa) as an estimate of limit pressure would imply an underestimation of 15.4%, which could result in overly conservative safety factors and unnecessary design costs.

These findings highlight the importance of using validated extrapolation methods (e.g., the double hyperbola as per NF P 94-110-1) and complementing them with numerical modeling at representative strata of the site.

The second phase of this study is currently underway and involves the validation of multiple pressuremeter tests through numerical modeling. The objective is to develop a robust database to improve the reliability of extrapolation methods for estimating limit pressure in stiff soils of western Mexico City.

To enhance this database, it is proposed to include additional tests using higher-capacity probes, allowing for a greater number of data points and the possibility of directly reaching the limit volume (V_{lim}) in high-stiffness soils.

Moreover, both calibrated constitutive models (Mohr-Coulomb and Hardening Soil) successfully reproduced the field data. However, in the case of the Hardening Soil model, it was necessary to reduce the elastic modulus initially derived from the pressuremeter test, suggesting that its nonlinear behavior warrants further analysis.

Continued calibration and validation of a larger number of tests including those performed with high-capacity probes will help accurately assess the representativeness of the target stratum under different constitutive frameworks and refine the criteria for limit pressure extrapolation.

3. Conclusions

This study evaluated the validity of the double hyperbola extrapolation method for estimating the limit pressure in stiff soils of western Mexico City, where the standard pressuremeter test does not reach the failure state due to soil characteristics and equipment limitations.

The comparison between the extrapolated value (P_L^{DH} = 2.36 MPa) and that obtained from calibrated numerical modeling (P_L^{num} = 2.34 MPa), revealed a difference of only 0.85%, supporting the method's applicability to this type of materialto capture data up to the theoretical limit volume and enhance the validation of analytical methods. Additional tests using higher-capacity probes could confirm whether this level of agreement remains valid when the limit volume criterion is reached in the field.

Other estimation methods showed wider deviations: the inverse volume method underestimated by 6.8%, and the maximum pressure recorded during the test yielded a 15.4% underestimation potentially leading to overly conservative safety factors and inflated design costs.

Numerical modeling using both the Mohr-Coulomb and Hardening Soil models satisfactorily reproduced the field pressure–volume curve. However, in the case of the Hardening Soil model, it was necessary to reduce the elastic modulus initially inferred from the test, suggesting the need for more detailed calibration. Additional tests with higher-capacity equipment will help assess whether these adjustments result from scale effects or specific characteristics of the stratum.

It is concluded that double hyperbola extrapolation, when applied to high-quality curves with a well-developed plastic branch, can be a valid tool for the design of deep foundations in stiff soils. Nonetheless, it is recommended that this technique be complemented with numerical modeling at other representative locations and with pressuremeter tests using high-capacity probes, to reach the theoretical limit volume and strengthen method validation.

Although such equipment is not widely available in Mexico, the cost difference between standard and high-capacity pressuremeter tests typically ranges from 10% to 20%. This suggests that, when greater reliability is required for the estimation of limit pressure in stiff soils, the use of high-capacity probes may be both feasible and technically justified.

As future work, this methodology is expected to be expanded to a greater number of pressuremeter tests including those performed with larger diameter probes in order to develop a robust database that standardizes the use of extrapolation methods in estimating limit pressure in the stiff soils of western Mexico City and increases reliability in geotechnical design.

References

Baguelin, F., Jezequel, J., & Shields, D. H. (1978). The Pressuremeter and Foundation Engineering. Trans Tech Publications.

Briaud, J.L. (1992). The Pressuremeter. A.A. Balkema, Rotterdam.

Clarke, B.G. (1995). Pressuremeters in Geotechnical Design. Blackie Academic & Professional.

Combarieu, O. (1995). Détermination de la déformabilité des sols et des roches par l'essai pressiométrique. Revue Française de Géotechnique, (70), 3–24.

Fawaz, A., Hagechehade, F., & Farah, E. (2014). A study of the pressuremeter modulus and its comparison to the elastic modulus of soil. SCEA Journal, 3(7), 7–15.

Hicher, P.-Y., & Rangeard, D. (2004). Interprétation des essais pressiométriques II. Détermination des caractéristiques hydromécaniques des sols fins saturés par analyse inverse. Revue Européenne de Génie Civil, 8(7–8), 819–849.

ISO 22476-4. (2012). Geotechnical investigation and testing — Field testing — Part 4: Ménard pressuremeter test. International Organization for Standardization.

NF P 94-110-1 (2000). Essai pressiométrique Ménard. Association Française de Normalisation