

Analysis of pressuremeter testing: evaluating anisotropic behaviour, operational stiffness, and non-linear elasticity in soils and soft rocks

Analyse des essais pressiométriques : évaluation du comportement anisotrope, de la rigidité opérationnelle et de l'élasticité non linéaire dans les sols et les roches tendres

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ABSTRACT

This paper presents ongoing research on pressuremeter testing at the Universities of Saskatchewan and Alberta, Canada. It discusses methods to evaluate the transverse anisotropic behaviour of soils and soft rocks, and the determination of the operational stiffness of structured formations. Additionally, it outlines a method for analyzing the unloading arm of tests in sands using hyperbolic non-linear elastic conditions, which provides a seamless transition from elastic to plastic deformations. The hyperbolic analysis also introduces a new method for estimating the small strain shear modulus from the loading branch of unload-reload cycles.

RESUME

Cet article présente les recherches en cours sur les essais au pressiomètre menées aux universités de la Saskatchewan et de l'Alberta, au Canada. Il discute des méthodes pour évaluer le comportement anisotrope transversal des sols et des roches tendres, ainsi que la détermination de la rigidité opérationnelle des formations structurées. De plus, il décrit une méthode pour analyser la phase de déchargement des essais réalisés dans les sables en utilisant des conditions élastiques non linéaires hyperboliques, ce qui permet une transition fluide entre les déformations élastiques et plastiques. L'analyse hyperbolique introduit également une nouvelle méthode pour estimer le module de cisaillement en petites déformations à partir de la phase de chargement des cycles de déchargement-rechargement.

Keywords: Stiff overconsolidated clays; non-linear elastic analysis; anisotropic behaviour; operational stiffness

1. Introduction

It is an honour to be selected as a keynote lecturer at the 8th International Symposium on Pressuremeters (ISP8). The work carried out at the Universities of Saskatchewan and Alberta over the last 5 years has focused mainly on providing geotechnical engineers with a means to translate high-resolution pressuremeter data into advanced numerical models for complex geotechnical problems.

Initially, my interest in using direct measure pressuremeter measurements arose from the need to classify fissured, heavily overconsolidated soils typical of most of Canada. This work has naturally led to characterisation of very weakly cemented, Cretaceous aged clay shales present in most of Western Canada. These shales are extremely difficult to sample and test, given their high propensity to swell when exposed to non-polar fluids. These materials are essentially stiff, fissured clays that tend to self-destruct when sampled. When tested in a laboratory, recovered samples provide

seemingly usable (albeit conservative) design parameters. Morgenstern and Thomson, (1971) and Elwood (2014) demonstrated that the material stiffnesses measured in the laboratory are, significantly lower than those encountered at the field scale. Similar findings were reported by Dreger et al. (2022). These concerns have led to us pursuing new developments for scaled, operational stiffness solutions that can readily translate to field level designs. It would also appear that usage of the non-linear elastic behaviour can be a strong indicator of the likely mode of failure, either shear or tensile. The data also suggest whether the material will respond as truly elastic or whether non-linear elastic behaviour must be considered when designing.

The final topic discussed is a brief description of a new method for analyzing pressuremeter data to evaluate the initial modulus (G_0) and a new method for interpreting unloading data in sands using both power law and hyperbolic models. At this time, it would appear promising that the hyperbolic interpretation method can be used to develop inputs for hyperbolic, elasto-plastic

constitutive models directly. This finding is somewhat contrary to the discussion provided by Jardine (1992) where the direct comparison of the non-linear elastic behaviour of a pressuremeter test and of high-quality triaxial test data is not permissible for several reasons. Our work has shown that when the unloading interpretation is used as inputs for the Hardening Soil (HS) model, that numerical simulations of the field data are nearly identical suggesting a strong relationship. Work is ongoing to assess whether these very preliminary findings are correct, but on the surface, the methods appear promising.

The last item to discuss is the importance of the dedicated students who have contributed to this paper and have done the bulk of the work. Without these bright young engineers, nothing herein would be possible. To the students, I wish to extend my deepest gratitude and thanks for all your hard work and perseverance over the years. I would also like to thank my collaborators; Cambridge Insitu and ConeTec Investigations have both been supporters through ongoing discussions and test data. I would also like to thank Dr. John Hughes for all his time and encouragement.

2. Anisotropic shear stiffness response of Clearwater Formation Clay shale

This research evaluates the operational strength in the highly overconsolidated clays of the Clearwater Formation. It involves three main stages: field test analysis, laboratory test analysis, and computational modelling. To date, stage 1 has been completed, and stage 2 is in progress.

The analysis of the field tests was carried out for high-resolution pressuremeter tests performed with the Cambridge Pressuremeter and additional data from Downhole Seismic Test, campaigns carried out in collaboration between ConeTec and Cambridge Insitu.

The main objective of the field test analysis is to identify the anisotropic response of the in-situ clay shale, in respect to in situ lateral stress (P_0), in situ yield stress (P_Y), undrained strength (s_u) and Secant shear modulus (G_s). In the analysis stage, an automated spreadsheet was created, which takes the raw data from the field test and processes the response considering an average of each pair of axes. In this way, the soil response is obtained in 3 different axes, allowing measurement of anisotropy.

2.1. In-situ Lateral Stress

The in-situ lateral stress was determined by considering the lift-off and non-linear elastic pore water pressure response (PWP method) based on Bolton and Whittle (1999). The Lift-off Method is particularly suitable for the Self-Boring Pressuremeter (SBPM). Since the membrane is in direct contact with the cavity wall, it decreases the distortion of initial measurements, providing a clear determination of the initial radial displacement point, known as in-situ lateral stress. Our research has shown that the in-situ lateral stress calculated with the average of the arms is defined by the axis with primary response, omitting the zones of higher resistance. Clarke (2022) was one of the earliest to

propose the variable response in a 3-arm pressuremeter, creating the caution that an inclination of the probe could contribute a portion of the vertical component of the stress on the arms. In our research, the verticality of the probe was measured during the test and was found to not deviate more than 0.1 degrees in the tests analyzed. Additionally, the processing was performed considering the average of the arms per axis; in this way, the response obtained is limited to the cavity pressure in the three axes.

The PWP Method considers the pore pressure measurement of the SBPM during loading. In this case, the in-situ lateral stress is determined when there is a constant increase in pore pressure. The limitation of this method consists in the fact that the pore pressure is an average value (isotropic) of the primary response of the soil, unlike the arms, the PWP sensors cannot be associated to a specific orientation that provides an anisotropic response of the tested material, instead it is a direct reaction to the lowest principal stress.

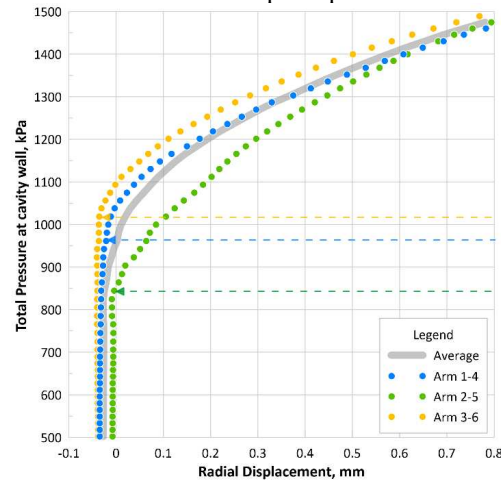


Figure 1. Lift-off Response of Borehole 7030-T1

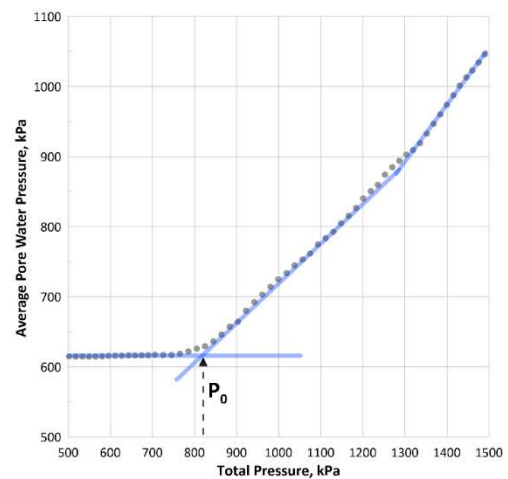


Figure 2. Porewater Pressure (PWP) Response

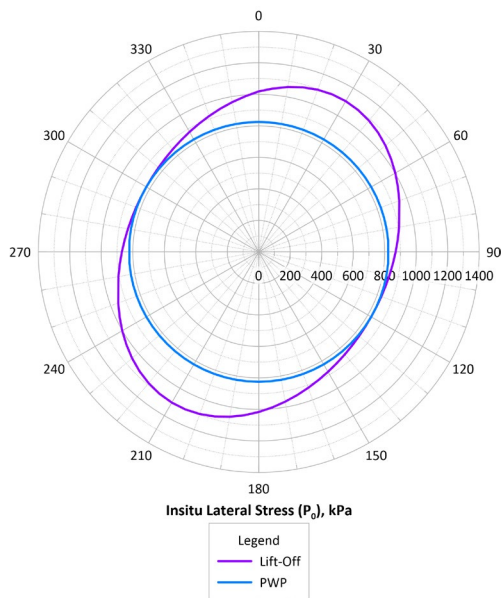


Figure 3. Anisotropic Initial Pressure (P_0) Response

Figures 1 and 2 show the in-situ lateral stress for Borehole 7030 Test 1 by the Lift-off and PWP methods, respectively. Figure 3 shows the in-situ lateral stress distribution for this same test, where it can be seen how the lower principal stress of the lift-off method matches that of the PWP Method. The Lift-off method shows a marked anisotropy with a difference of 200 kPa between the lowest and highest principal stress.

2.2. Initial Yield Stress (P_Y)

The initial yield stress is related to the stress history and shear strength in-situ. In this case, the method of Marsland & Randolph (1978) and non-linear elastic, PWP response (Bolton and Whittle, 1999) were used to determine the onset of yielding. As shown in Figure 2, there is a clear transition from initial loading up to the end of elastic behaviour and the rapid increase in measured excess porewater pressures. Although the anisotropic response is not as marked as that observed in the in-situ lateral stress, the in-situ yield stress shows a slight anisotropy as depicted in Figure 4. In this case, the response of the PWP method is isotropic and is the average of the major and minor principal stresses determined by the Marsland & Randolph method. The analyses show that the stresses exhibit isotropic behavior at the yielding point. Ultimately, the differences in the yield stress are not significant, suggesting that the stress history within the formation is largely transversely isotropic.

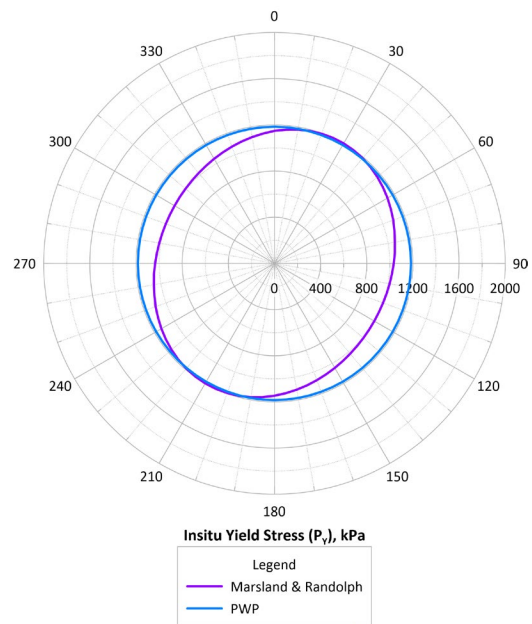


Figure 4. Anisotropic P_Y Response

2.3. Secant Shear Modulus (G_s)

Analyzing anisotropy in stiffness is one of the major contributions of this research. As part of the stiffness analysis, shear stress and strain were calculated. The shear modulus was determined for each measurement point taken during the various unload-reload cycles. This allowed for an accurate representation of the behavior under small deformations.

The shear modulus was calculated by manually defining the initial point of the reload phase. This point was identified according to Bolton & Whittle (1999), using the lowest shear stress from the unloading phase and defining the shear stress that best fits the non-linear trend described by the power law of Bolton & Whittle (1999).

This process was repeated for all three axes of the test, yielding values for the secant shear modulus and shear strain from direct field measurements, illustrated in Figure 5. In this figure, it is evident that the initial shear modulus decreases as yielding approaches, clearly illustrating the degradation of the shear modulus.

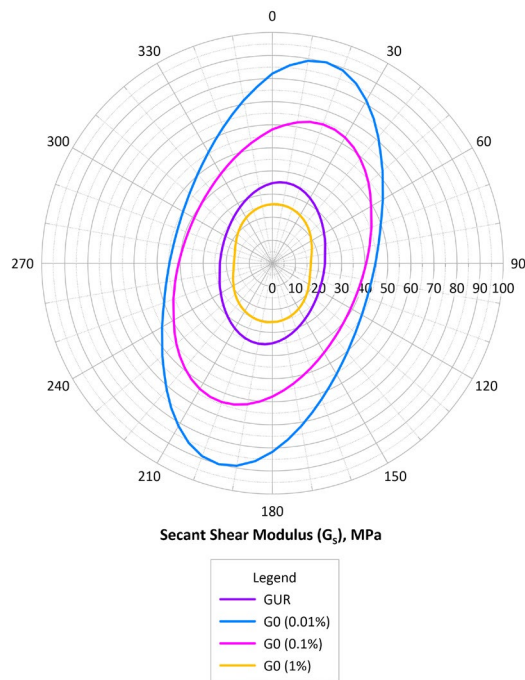


Figure 5. Anisotropic G_s Response

3. Operational Stiffness

Over the life of a project, the usefulness of a Factor of Safety diminishes after initial design as it does little to indicate adequate performance during construction and operation. Serviceability often governs a structures performance during this period, with measured deformations being a key performance indicator. Performance-based design (PbD) links deformations to performance objectives defined at design outset and therefore provides a means to evaluate design acceptability at all project phases. The use of numerical models to estimate rock mass deformations is now common practice, owing to advancements in model capabilities and computing power. Their significance in PbD is that they permit comparison of model predictions with measured deformations and the performance objectives defined. This informs practitioners of the current level of risk for a structure.

Accurate deformation predictions support the PbD framework and model reliably depends on how well the user-defined stiffnesses, and constitutive relations represent the rock mass in-situ. A linear elastic constitutive framework is often assumed to simplify modelling. Although this may yield a sufficient approximation in some cases, most soils and rocks exhibit more complex deformation behaviour. At the small strains typically experienced during service by well-engineered structures (0.01 to 0.1%), stiffness is initially high and reduces non-linearly with strain. This behaviour is prevalent in both soils and rocks and is influenced by the degree of bonding or cementation, void ratio, effective stresses, stress path, history, and anisotropy, among others. Capturing this non-linear behaviour during testing is essential as neglecting it underestimates stiffness, producing a design high in both cost and uncertainty.

In practice, stiffness is often derived from laboratory tests on samples at strains higher than the structure will experience. Considering stiffness degradation with strain, this yields a reduced stiffness. If the small strain behaviour is measured reliably in a laboratory, samples tested have often been damaged due to sampling disturbance and deterioration (Lim et al., 2019). This damage is significant in weak argillaceous rocks, like shales, impacting sample integrity and removing the effects of the insitu state and history. In rocks, indirect empirical relations using rock mass classifications are typically used to estimate rock mass stiffness. These provide unreasonably high values for shales if intact sample stiffness is not considered as they are biased toward stronger rocks. If considered, stiffness is underestimated due to sample disturbance.

Insitu testing is often needed to determine shale stiffness, but conventional tests can be resource intensive and demonstrating that the volume of rock tested adequately represents the greater rock mass is a challenge. Most in-situ tools lack the strain control and resolution necessary to capture stiffness and degradation in rock at small strains, where it matters most. Conventional methods to characterize stiffness are insufficient for shales. As shales comprise nearly half of the emerged rock mass globally and account for a substantial portion of the Western Canada Sedimentary Basin, the implications of these issues are far-reaching in geotechnical design, and a novel approach is needed to produce representative models.

The central objective of this research is to use a practicable approach to evaluate the stiffness characteristics of weak shale at operational strains for geotechnical structures. This work includes analysis of an extensive testing program comprising high-resolution pressuremeter and large-scale lateral load tests shown in Figure 6, in-situ geophysical logging, and laboratory testing of the Shaftesbury Shales. These shales support major structures at a large hydroelectric project under construction in British Columbia, and understanding their behaviour is of great engineering significance. This research will demonstrate how stiffness varies non-linearly as a function of strain, testing scale, and in-situ conditions.

Preliminary analysis of the direct strain pressuremeter data indicates that strains as low as 0.005% can be measured in the shale and that small strains (0.1 to 1%) are typically experienced at pressures up to 20 MPa presented in Figure 7.



Figure 6. Site C lateral Osterberg load frame

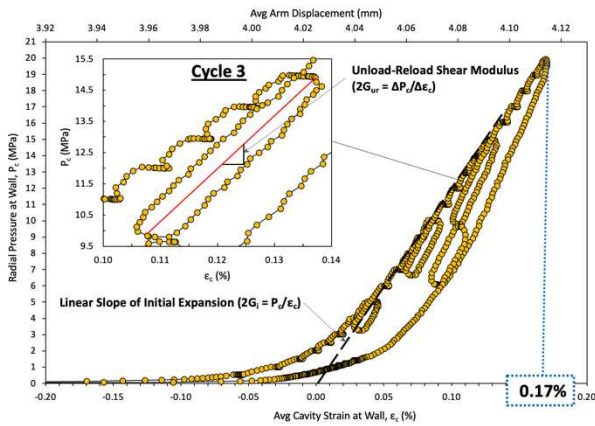


Figure 7. Typical pressuremeter test in R2 rock

In general, the amount of strain experienced at maximum pressure increases near the surface of the shale, with measured shear stiffnesses derived from unload-reload cycles increasing with depth depicted in Figure 8. However, the current analysis uses the stiffness of a line bisecting the unload-reload cycle (Fig 7 inset) and therefore is the minimum value at each mean stress cycles were conducted. Non-linearity in the unload-reload data has been identified and demonstrates that stiffness degradation occurs in this shale. The data presented illustrates that the direct strain pressuremeter can measure non-linear behaviour as shown in the inset of Figure 9. This behaviour is more pronounced in the shallower tests, indicating a zone of relaxation where shale porosity, water content, and frequency of discontinuities are higher (Figure 8). This increase in non-linearity toward the surface may also be indicative of a transition from tensile to shear failure mode. The influence of tensile cracking in the shale is yet to be determined during this work, as well as the impact of pressuremeter system, creep, compliance, and non-linearity.

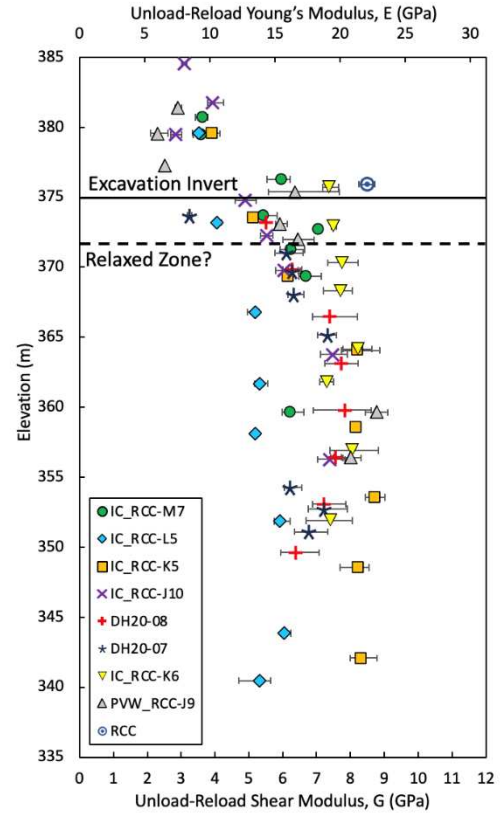


Figure 8. Shaftsbury unload-reload modulus with depth

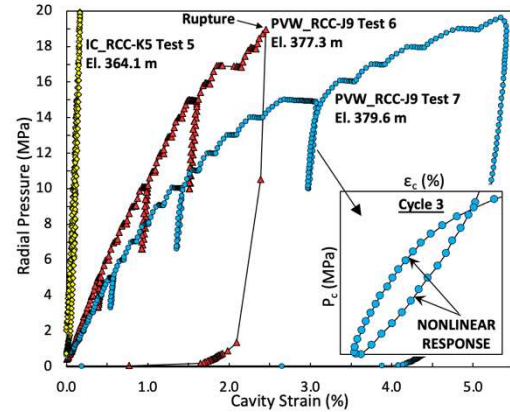


Figure 9. Linear and non-linear behaviour of Shaftsbury

With a height of 4 m and a diameter of 2.6 m, the large-scale split-lateral load tests are the largest scale in-situ tests conducted at the site, geophysical testing excluded. As the shale formation is predominantly massive, the deformation behaviour observed, and stiffness characteristics back calculated from the load tests are considered to be a reference for full rock mass behaviour at small strains. These results will be used as the basis of comparison for the pressuremeter findings to evaluate the small strain, non-linear stiffness characteristics derived using the pressuremeter. Interestingly, preliminary analysis shows that measured stiffness increases with testing scale, contrary to the conventional understanding of scale-dependent behaviour in rock (i.e., stiffness should decrease due to the influence of more discontinuities at larger scales) as shown in Figure 10. This may be due to the lack of discontinuities and deformation required at larger scales to begin mobilizing frictional resistance, but this is yet to

be confirmed. This phenomenon may demonstrate that the pressuremeter tests a volume adequately large enough to represent the rock mass.

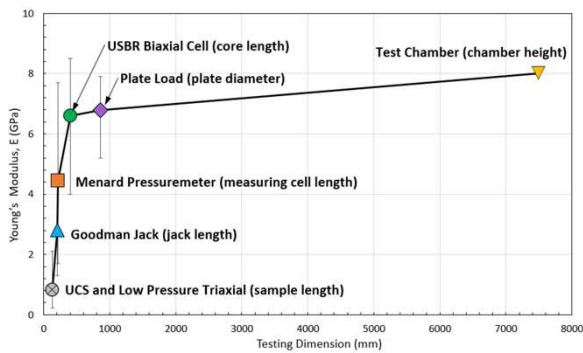


Figure 10. Scaled modulus from Shaftesbury Shale

Historical laboratory and in-situ testing data have been collected in the Shaftesbury Shales since the 1970's. This data will be combined with additional laboratory testing on well-preserved samples recovered from the lower shale facies to establish a correlation between the shale properties and in-situ stiffnesses measured. The degree of sample disturbance will be identified during testing and the effects demonstrated through a comparison between stiffness measured in the laboratory versus in-situ.

The intent of this study is to provide practitioners with key information on properties, conditions, and aspects of testing methods that affect shale stiffness at small strains. It will provide useful what practitioners can expect regarding shale deformation behaviour to aid in the development of a directed investigation and testing program which will ultimately inform deformation models supporting a PbD methodology.

4. Cavity Contraction of Sands

To date, there has not been a widely accepted model for non-linear elastic interpretation of the pressuremeter test in sands. The lack of a solution is likely due to a number of reasons, including an appropriate flow rule, definition of yielding how to best define the stress path.

Fundamentally, sands undergo intrinsic changes in their material properties during cavity expansion due to their drained behaviour. The elastic modulus of soils has been shown to decay non-linearly with increasing strain, as demonstrated by Mair (1993), Bolton and Whittle (1999), and Clayton (2011). Conversely, it increases non-linearly with confining stress during cavity expansion, as noted by Wroth (1984), Robertson and Hughes (1986), and Bellotti (1989). The magnitude and rate of volumetric strain in sands tend to decrease with increasing confining stress (Rowe, 1962; Bolton 1986; Maeda and Miura, 1999), while the rate of dilation varies with increasing shear strain. For a dense sand, the dilation rate was assumed by Hughes et al. (1977) to remain constant throughout yielding at a given confining stress. This behaviour leads to non-uniform volumetric changes and varying stiffness across the plastic radius.

Shear strength in sands is influenced by initial confining stress, relative density, and sand fabric. In dense sands, the high stiffness results in a heightened sensitivity to disturbances during installation, where even

slight strains can significantly alter the stress state at the cavity wall. This disturbance affects the strain origin, reducing the peak mobilized friction angle and peak mobilized dilation, as observed in Hughes et al. (1977) analysis (Robertson and Hughes, 1986).

Carter et al. (1986) derived a small-strain analytical solution for cohesive-frictional materials, assuming a linear elastic-perfectly plastic behaviour that accounts for elastic strains within the plastic radius, which, were previously neglected in Hughes et al. (1977) analysis. However, this solution fails to capture the nonlinear response of sands due to its linear elastic assumptions and is applicable only to SBPM tests within the small-strain regime. Similarly, Houlsby (1986) provided an analytical framework for cylindrical cavity contraction in sands considering linear elasticity. This solution provides shear strength at yield defined as ϕ_{cv} and it does not capture the observed nonlinear behaviour. On the surface, it would appear that Houlsby's solution is well suited for definition of the ϕ_{cv} as opposed to calculation of ϕ_p .

Houlsby (1986) mentions that the stiffness in sands is influenced by both the stress state and the stress range in which it is measured. This phenomenon can be attributed to the fact that unload/reload cycles do not experience stress reversal as by their very nature, do not invoke yielding in reverse plasticity, whereas the unloading arm does. Following stress reversal, sands exhibit reduced strength and more compliant deformation characteristics, depending on prior deformation history (Tatsuoka and Ishihara, 1974). Furthermore, sands yield at lower deviatoric stresses after stress reversal than predicted by the Mohr-Coulomb stress envelope, a phenomenon known as the Bauschinger effect (Schofield and Wroth, 1968; Roy and Campenella, 1997; Jefferies, 1997; Davis and Selvadurai, 2002).

Bolton and Whittle (1999) demonstrated that the non-linear stiffness degradation with strain in clays can be effectively described using a power-law formulation, where non-linear behaviour is the same owing to the undrained behaviour of clays. Whittle and Liu (2013) further developed a methodology for analysing the strain and stress dependency of the elastic shear modulus during the loading phase of Thanet sand, employing a strain-dependent stiffness model based on a power-law formulation. This approach normalizes the non-linear stiffness trends to a given stress state—typically the mean effective stress—without requiring prior knowledge of the in-situ stress state.

Progressively converging nonlinear trends reveal volumetric changes and indicate the near-field soil's approach to a critical state (Schofield and Wroth, 1968; Atkinson and Bransby, 1978; Muir Wood, 1996). At the critical state, sand deforms at a constant volume, and subsequent stiffness trends overlap, mimicking an undrained response. This provides valuable insight into the sand's state relative to the critical condition.

Whittle adapted Carter et al.'s (1986) framework by incorporating elastic strains using the method proposed in Whittle and Liu (2013). This refined approach successfully captures nonlinear elastic behaviour without requiring small-strain stiffness measurements. However, the derived parameters may not be directly applicable in

advanced numerical models such as Hardening Soil (HS) or Hardening Soil Small-Strain (HSS) formulations.

The actual stress path is illustrated as dashed line C-D-E' in Figure 11, after Hughes and Robertson (1985). The stress path is assumed to follow constant mean effective stress throughout the elastic zone until the stress-ratio gets equal to stress-ratio at yield C-D-E. After yielding, the cavity wall follows a constant stress-ratio as defined by line E-G.

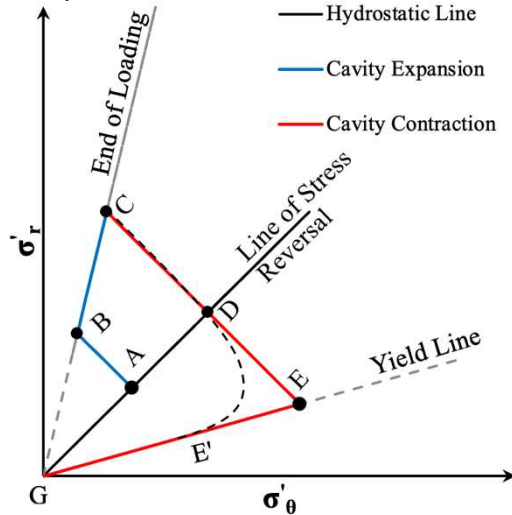


Figure 11. Unloading stress path (after Robertson and Hughes, 1986)

The initial condition occurs at end of cavity expansion, or the end of loading (Point C). At this stage, the cavity wall is expanded from the initial radius to maximum, with a corresponding maximum radial stress and cavity strain. Upon unloading, the cavity pressure and cavity strain are both reduced. Because yielding has not yet occurred, the hoop stress at any given radius must be in equilibrium with radial stress after neglecting overburden pressure to maintain stress continuity. Once the borehole pressure reaches the hydrostatic line, there is a stress reversal that takes place, and it is somewhere around this stage that yielding occurs (Point D). Because there is no non-linear elastic constitutive model that currently exists, it is not possible to actually reproduce this point numerically to accurately determine the true point of yielding. Again, it is well understood that the Bauschinger effect will result in yielding prior to the $-\phi'$ line. For this reason, it has been assumed that yielding will follow previous conventions, and that yielding will occur when the stress path intercepts the $-\phi'$ boundary (Point E).

To capture the non-linear behaviour of soil in elastic zone, two non-linear elastic formulations, power law (Bolton and Whittle, 1999) and hyperbolic law (Kondner and Zelasko 1963; Duncan and Chang 1970; Pye 2011), are considered and compared for a self-bored pressuremeter test data in Canvey Sand. The pressure expansion curve is presented in Figure 12. Both the models are proven to demonstrate the elastic behaviour of sand in the strain range of pressuremeter test with sufficient accuracy. During cavity contraction, the soil element at cavity wall experiences a stress-reversal and is found to possess lower stiffness post stress-reversal.

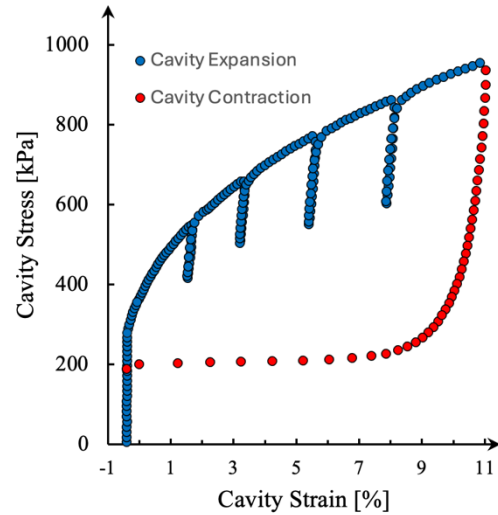


Figure 12. SBPM test in Canvey Sand (Courtesy of Cambridge InSitu)

When the elastic arm of the unloading phase is analysed using the power law method described by Bolton and Whittle (1999), the power-law behaviour will accurately predict the initial elastic portion of the field curve but deviates from the field curve prior to the yield stress. This is likely due to neglecting the softening of sand after stress-reversal as described in (Tatsuoka and Ishihara 1974).

Instead of using the power law relationship, the use of a hyperbolic non-linear elastic model was investigated. A strong hyperbolic stress-strain relationship was observed by Kondner and Zelasko (1963) in sands for triaxial compression tests. This relationship was derived between deviatoric stress and major principal strain. But in the pressuremeter test, for loading and for half portion of unloading, the radial stress remains the major principal stress while we measure cavity-strain, a minor principal strain. Therefore, the Kondner's equation was not directly applicable. By assuming no volume change throughout the elastic region, shear strain is assumed to be twice the cavity strain at the cavity wall.

Prior to the stress reversal, is the shear stress applied at the end of loading is considered as the stress and strain origin to transform the CD portion of curve to a hyperbola with the transformed reference shear stress. The post-stress reversal branch begins from the point of stress reversal and is valid until the yield stress defined as point 'E' in Figure 13. No origin correction is required as the response itself represents a hyperbola.

Use of the hyperbolic, non-linear elastic behaviour for SBPM test data from Canvey sand is shown in Figure 13. The curve is terminated at yield stress determined using power law yield criterion. A coefficient of correlation value of 0.99 is achieved when comparing field and calculated data. Cavity strains from hyperbolic law simulates the field data with considerable accuracy and provides a smooth transition to plastic behaviour leading to a less accurate visual judgement of yield stress and as well as the corresponding yield strain.

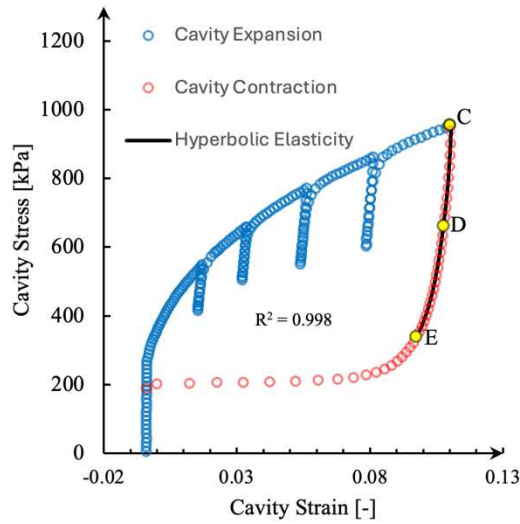


Figure 13. Canvey Sand hyperbolic non-linear elastic fit

The yield stress in unloading is found to be governed by post-stress reversal behaviour leading to incorporation of hyperbolic parameters from post-stress reversal phase. The plastic solution for nonlinear elasticity has been adapted from the Whittle (2010) approach. Results of the adapted analysis are shown and both nonlinear elastic formulations are compared in Figure 14. The abrupt transition to plasticity for power-law can be seen in the cropped section. The accuracy of the model, determined using coefficient of correlation value, is 7.6% higher in case of hyperbolic elasticity compared with power law elasticity.

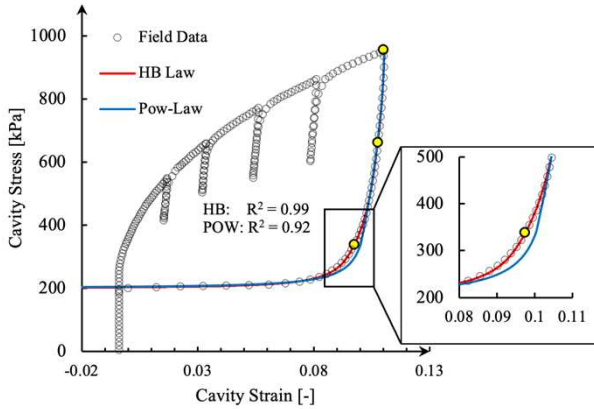


Figure 14. Plastic strain determination for Canvey Sand

Review of the parameters estimated from the loading curve Table 1 summarizes the comparison of the hyperbolic analysis of the unloading curve are consistent.

Table 1. Comparison of Loading and Unloading Analyses

	ϕ_p (deg)	ϕ_{cv} (deg)	Ψ (deg)
Loading	38	30	9.5
Unloading	40	30	7

4.1. G_0 Prediction

The last topic that will be covered is the estimation of the small strain shear modulus, G_0 from reloading cycles. The analysis is presented here in terms of undrained tests

solely due to simplicity. The analysis works well for drained tests provided that the non-linear shear moduli are normalized to the mean effective stress (Whittle and Liu, 2013) or for intermediate geomaterials.

In recent years, attempts (Lopes et al., 2021; Byrne and Whittle, 2023; Contreras et al., 2023) have been made to reconcile the differences between triaxial test derived stiffness decay and those obtained from pressuremeter tests. Use of power law and hyperbolic curve fitting methods were used to fill in gaps within the stiffness decay plot obtained from the pressuremeter at shear strains less than 10^{-4} . Lopes et al. (2021) attempted to estimate the small-strain modulus from the non-linear behaviour of the initial unloading portion of an unload-reload cycle. Byrne and Whittle (2023) attempted to fit data using a hyperbolic curve to ensure compliance with current non-linear elasto-plastic models and thereby make the results directly relatable to current modelling software.

The calculated non-linear modulus functions will plot as one function regardless of stress state. This behaviour is demonstrated in data acquired from the Clearwater Shale in the Athabasca Oil Sands region in Northern Alberta shown on Figure 15.

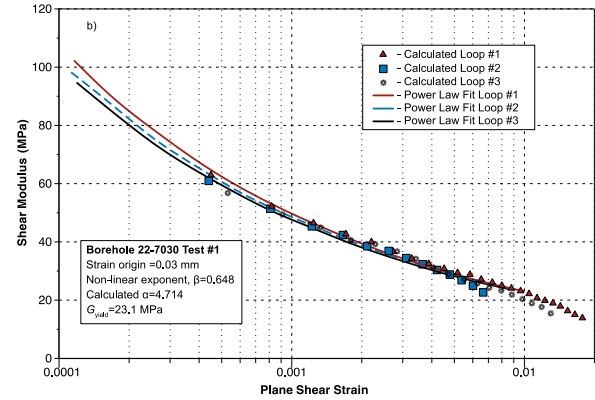


Figure 15. Shear modulus from Clearwater Shale

It is observed that when the non-linear shear modulus above is plotted versus the inverse of shear strain, γ , the resulting data takes the form of a logarithmic function. The inverse shear strain versus shear modulus data for Figure 15 is shown below in Figure 16. Once the data has been plotted, the data can be fitted with a logarithmic function and the corresponding equation for the line of best fit is determined.

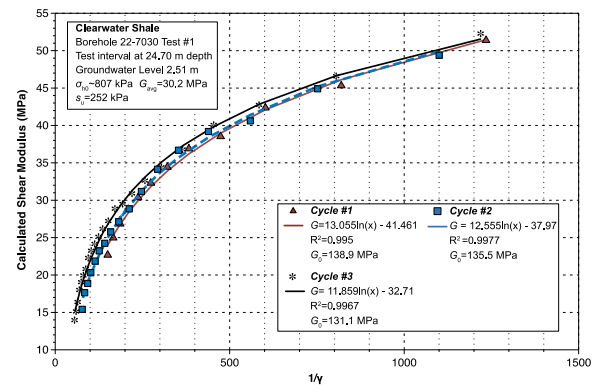


Figure 16. Inverse strain moduli

Simply by using the logarithmic equation fit to the field data and using 10^{-6} as the shear strain increment of

interest, a G_0 value can be obtained. When the G_0 values obtained from the inverse strain method are compared to the shear modulus obtained from vertical seismic profiling, the difference is approximately 0.4 to 4.9% depending on the chosen test interval.

5. Conclusions

The intent of this paper is to briefly summarize the work carried out at the Universities of Saskatchewan and Alberta in Canada. Most of this research is nearing completion but is still ongoing. Our findings indicate that direct measure pressuremeters effectively measure the transversely anisotropic behaviour of soils and soft rocks. They provide vital tools for determining the operational behaviour of discontinuous formations, which might otherwise yield misleading results in a laboratory environment.

The development of our non-linear elastic unloading model is not peer reviewed but is quite promising in providing parameterization of sands. This is key in that it may no longer be necessary to analyze self-bore data or a fully displaced model. This has profound impacts on determination of the in-situ lateral stress for pre-bored tests as well as provides insight to liquefaction potential in fine sands. Lastly, when the above methods are combined with analyses of the loading arms of unload-reload cycles, a reliable small strain shear modulus, G_0 can be apparently predicted. This information can be used to assess whether the ground will fail either in shear (non-linear elastic) or in tension (linear elastic).

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