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Probabilistic modelling of the temporal evolution of the stability of rooted slopes

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ABSTRACT: The increasing focus on sustainability fosters the adoption of nature-based solutions (NBS) in geotechnical slope stabilization practices. Although NBS are recognized for their capability to improve slope stability over time, their geotechnical characterization and modelling involve considerable uncertainty. Probabilistic methods offer a well-established means for addressing uncertainties across spatial and temporal domains. This study illustrates synthetically the main features of a probabilistic approach to the estimation of the stability of rooted slopes. An example application is provided.

Keywords: Slope stability; nature-based solutions; rooted soils; uncertainty; probabilistic modelling

1 INTRODUCTION

The favourable effect of vegetation and root reinforcement on slope stability is extensively recognized and has been studied using physically based approaches such as Limit Equilibrium Method (LEM), including the infinite slope model (Masi et al., 2023). Because root reinforcement exhibits both spatial and temporal variability, incorporating its effects into stability analysis warrants a non-deterministic approach. Previous studies (Bischetti et al., 2016; Hammond et al., 1992) have addressed this issue by applying sensitivity analyses or probabilistic models, while others (Pisano and Cardile, 2023) have represented root reinforcement uncertainty as an additional apparent cohesion, without however accounting for its temporal evolution. This study provides a contribution to the advancement of dynamic probabilistic approaches to address uncertainty in root reinforcement variables and their evolution over time. Specifically, it explores the effect of roots crossing the lateral and basal failure planes and quantifies the impact of uncertainty on the temporal evolution of the factor of safety and the probability of failure.

2 MATERIALS AND METHODS

2.1 *Infinite slope model with root reinforcement*

The factor of safety FS against the instability of a rooted infinite soil mass with depth of the failure plane D, finite width W and slope angle α, accounting for basal and lateral root-induced cohesion can be estimated using limit equilibrium theory, as expressed in Equation (1) (Hammond et al., 1992; Waldron, 1977; Wu et al., 1979):

$$FS_{ISM} = \frac{\tau_f}{\tau_{mob}} \tag{1}$$

where

$$\tau_f = (\sigma - u) \tan \phi' + c_{tot} \tag{2}$$

$$\sigma = [(1 - m_w)\gamma_d + m_w \gamma_{sat}] D \cos^2 \alpha \tag{3}$$

$$\tau_{mob} = [(1 - m_w)\gamma_d + m_w \gamma_{sat}] D \sin \alpha \cos \alpha \tag{4}$$

$$u = m_w D \gamma_w \cos^2 \alpha \tag{5}$$

In Equations (2-5) τ_f denotes the shear strength of the soil along the failure plane, τ_{mob} the mobilized shear strength, and u the pore water pressure. The parameter ϕ' is the effective friction angle, γ_d and γ_{sat} are the dry and saturated unit weight of the soil, respectively. Here, partial saturation effects are not accounted for. In principle, moist unit weight could be used in lieu of the dry unit weight for soils located in the vadose zone. The depth of the failure plane D is expressed in relation to the position of the water table z_w through the dimensionless factor m_w ($0 < m_w \leq 1$) such that $D = z_w / m_w$. In Equation (2), the total cohesion c_{tot} is given by

$$c_{tot} = c'_s + c_{r,b} + (2D/W) c_{r,l} \tag{6}$$

in which c'_s is the cohesion of the non-rooted soil; $c_{r,b}$ and $c_{r,l}$ represent the basal and the lateral root-induced cohesion, respectively. The term $2D/W$ accounts for the contribution from the two lateral planes of the finite-width slide. This extension of the infinite slope concept to a finite-width soil volume allows for lateral root reinforcement consistent with observed three-dimensional root effects (e.g., Schwarz et al., 2010b). The model is

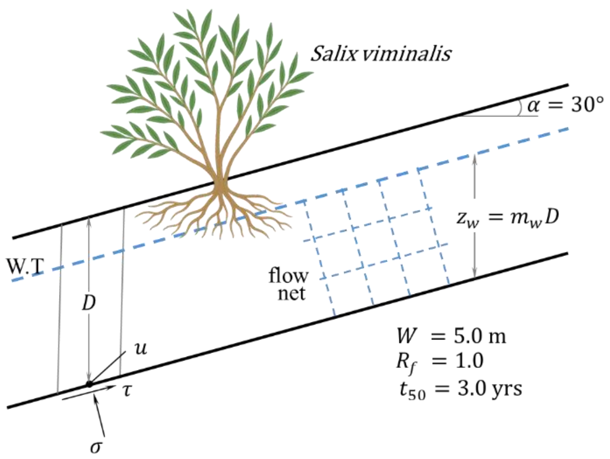

Figure 1. Infinite slope model with root reinforcement

shown graphically in Fig. 1, indicating the values used in the example application discussed in Section 3. The analysis assumes fully saturated conditions and excludes partial saturation or suction effects. It considers only the mechanical reinforcement by roots, without accounting for hydraulic or hydrological influences of vegetation.

2.2 *Modelling of basal and lateral root cohesion*

The contribution of roots crossing the basal and lateral failure planes is commonly represented by the modified Wu-Waldron model (Pollen and Simon, 1995; Waldron, 1977; Wu et al., 1995):

$$c_{r,\odot} = R_f \chi \bar{\sigma}_t RAR_{\odot}(z,t) \quad (7)$$

where the subscript $\odot$ denotes either the basal (b) or lateral (l) contribution. The parameter R_f is an orientation factor accounting for non-vertical root orientation at the failure plane, $\bar{\sigma}_t$ is the mean mobilized root tensile stress at time t, and $RAR_{\odot}(z,t)$ is the root area ratio at the basal or lateral shear plane. The reduction coefficient χ accounts for progressive root breakage or pull-out, which was not considered in the original Wu-Waldron model but introduced later after comparison with more advanced approaches such as the Fiber bundle (FBM) and Root bundle (RBM) models (Pollen and Simon, 2005; Schwarz et al., 2010a).

The term $RAR_{\odot}(z,t)$ represents the evolution of root density with depth and over time. It expressed as the product of two terms as shown in Equation (8):

$$RAR_{\odot}(z,t) = \frac{RAR_{\odot max}}{1 + exp[-k_g(t - t_{50})]} \; exp\left(-\frac{z}{h_r}\right) \quad (8)$$

The first terms contains a logistic (sigmoid) function describing root growth over time, defined by the maximum root area ratio at the surface $(RAR_{\odot max})$, the time required to reach 50% of $RAR_{\odot max}$ (t_{50}), and the logistic growth rate (k_g). The second term is an exponential function representing the decrease in root density, characterized by the rooting depth scale parameter (h_r). Equation (8) combines existing models of temporal and depth-dependent root development into a unified spatio-temporal expression (e.g., Jackson et al., 1996; Bischetti et al., 2005; Giadrossich et al., 2017).

The mean mobilized tensile stress of roots $\bar{\sigma}_t$ is given in Equation (9). This expression provides a simplified representation of root tensile strength and its temporal evolution, defined by the maximum tensile strength (σ_{max}) and the rate parameter (k_T).

$$\bar{\sigma}_t = \sigma_{max}[1 - exp(-k_T\, t)] \quad (9)$$

2.3 *Probabilistic modelling*

Aleatory and epistemic uncertainties play a key role in slope stability analysis, and they become even more significant when vegetation is involved (Masi et al., 2023). Aleatory uncertainty arises from the spatial and temporal variability of natural and man-made materials, as well as fluctuations in pore water pressures caused by groundwater changes. Epistemic uncertainty, in contrast, is related to the limited amount of available data, inaccuracy in parameter transformation, measurement errors, and the limited capacity of slope stability models to fully represent the underlaying process (Baecher and Christian, 2003).

Probabilistic modelling provides a robust framework to address both aleatory and epistemic uncertainty in slope stability analysis. In this study we use Monte Carlo Simulations (MCS) to estimate the factor of safety and the temporal evolution of the probability of failure by modelling marginal probability distributions and correlations among random variables. Aleatory uncertainty and epistemic parameter uncertainties are jointly accounted for through the selection of the parameters of the sampling distributions of input parameters to be used in Monte Carlo simulations (see Table 1). The true but unknown factor of safety FS_{true} can be estimated as

$$FS_{true} = \theta_{ISM}\, FS_{ISM} \quad (10)$$

In Equation (10), FS_{ISM} is modelled probabilistically through the propagation of the uncertainties in the parameters concurring to its estimation using Equations (1-9) during MCS. Epistemic model uncertainty in the ISM, stemming from the simplification of real physical complexity, is represented aggregately by the model factor θ_{ISM}. This parameter serves as a scaling factor for model bias and is treated as a random variable between 0.5 and 1.0 (Table 1), reflecting the tendency of the infinite slope model to overestimate stability (Hammond et al., 1992).

Table 1. Probabilistic modelling settings for random variables

Parameter	min*	mode	max*	COV	ρ
ϕ' [°]	(14.1)	30.0	(45.9)	0.3	-0.3
c_s' [kPa]	(1.0)	5.0	(9.0)	0.3	
γ_{sat} [kN/m³]	(15.6)	18.0	(20.4)	0.05	
z_w [m]	(0.2)	1.0	(1.8)	0.3	
m_w ** [-]	0.0	-	1.0	-	
$RAR_{b,max}$ [-]	0.006	0.008	0.010	-	0.6
$RAR_{l,max}$ [-]	0.003	0.004	0.005	-	
k_g [-]	0.8	1.2	1.5	-	-
h_r [m]	0.3	0.5	0.6	-	-
σ_{max} [kPa]	10000	15000	25000	-	-
k_T [-]	0.3	0.6	1.0	-	-
χ [-]	0.2	0.4	0.6	-	-
θ_{ISM} [-]	0.5	0.9	1.0	-	-

* min/max values in parentheses were estimated for the PERT distribution based on the mean and COV.
** assigned as uniform distribution $\mathbb{U}$ [0, 1].

3 EXAMPLE APPLICATION

An example application of the proposed probabilistic model is conducted on the infinite slope model shown in Fig. 1. Root cohesion parameters were specified for the *Salix viminalis* species based on published data (Bischetti et al., 2005; Giadrossich et al., 2017; Schwarz et al., 2010a,b). Both parameter and model uncertainties were modelled using PERT probability distribution functions (pdf), which are re-parameterized forms of the beta distribution defined by three parameters: minimum, mode, and maximum. PERT distributions are versatile because their parameters are straightforward to specify, they can capture both skewed or symmetric distributions, and–unlike normal or lognormal distribution– they remain bounded within the defined minimum and maximum values. The PERT distribution parameters adopted for the generation of sampling distributions in the example application are summarized in Table 1. The correlation between c_s' and ϕ' was accounted for by using a Gaussian copula to construct a multivariate distribution from the marginal PERT distributions and the specified correlation coefficient (ρ). The variables α, W, R_f and t_{50} were treated as deterministic with fixed values as shown in Figure 1. In addition, the maximum depth to the failure plane, defined as $D = z_w/m$ was set to 2.0 m.

4 RESULTS AND DISCUSSION

Figure 2 presents the FS distribution for selected years, together with their main sample statistics obtained from 100,000 Monte Carlo simulations. At t=0, the distribution shows a mean of approximately 1.0, a standard deviation of 0.32, and a probability of failure of 0.47. At t=10, the mean increases to 2.3 and the standard deviation to 1.6, confirming the contribution of vegetation growth to both overall stability and uncertainty propagation.

Figure 3 complements these results by presenting the continuous temporal evolution of the FS mean, the 90% credible interval– given by the difference between the 5th and 95th sample percentiles ($CI = P_{95\%} - P_{5\%}$)– and the frequentist probability of failure $p_f = p(FS < 1)$, estimated at time steps of 0.25 years. The FS mean increases slightly during the first two years, followed by a sharp rise until t=6 after which it stabilizes, while CI broadens considerably over time, confirming the progressive increase in variability. The marked changes observed in FS and CI between t=2 and t=6 are directly related to the logistic/sigmoid model in Equation (8) and to the probabilistic model adopted to describe the root density of the plant species.

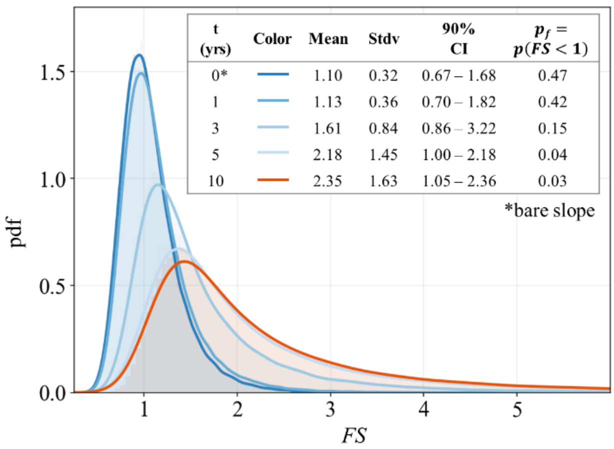

Figure 2. Temporal evolution of the FS distribution at selected timesteps t=0,1,3,5, and 10 years

The black line in Fig. 3 illustrates the marked decrease in p_f over time. At t=0, although the mean FS is slightly above 1.0, p_f is approximately 0.5, indicating that the slope is likely unstable. With root reinforcement development, p_f decreases to 0.1 by t=4, and further to 0.03 by the end of the period analysed. This evolution reflects the combined effect of increasing stability and broadening uncertainty: although the scatter in FS samples increases over time, the likelihood of reaching an unstable condition decreases significantly due to root reinforcement.

Figure 4 presents the continuous evolution of soil, basal, and lateral root cohesion, as well as the relative contribution of each component to overall cohesion. Soil cohesion remains constant over time since no change was assumed, whereas basal and lateral root cohesion increase as defined by Equations (7-9) and the probabilistic model described in Table 1. By the end of the investigated period, basal and lateral reinforcement contributed on average nearly 70% of the overall cohesion. This result confirms the key role of vegetation growth

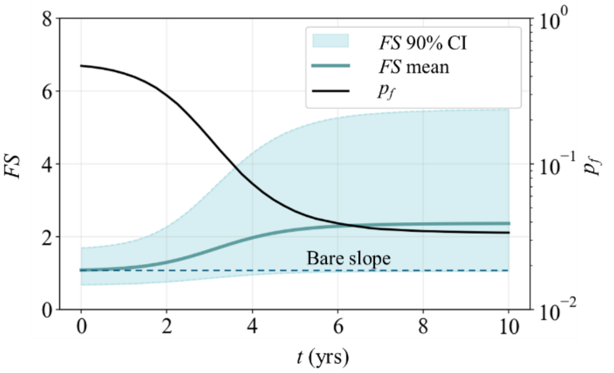

Figure 3. Temporal evolution of the mean and 90% interval of FS and probability of failure

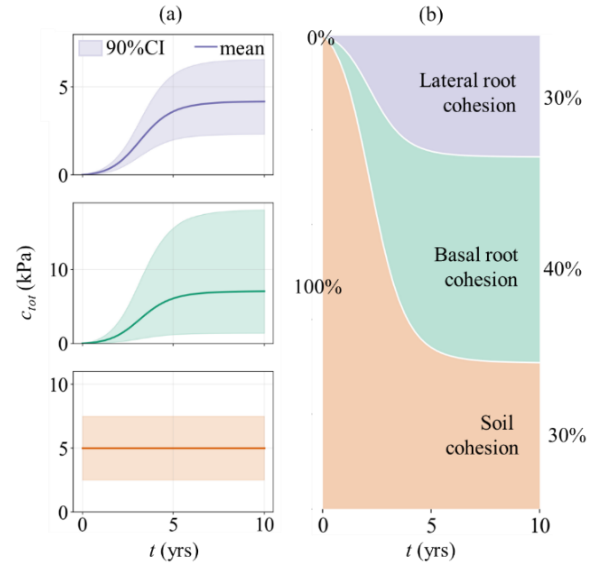

Figure 4. (a) Sample mean and 90% credible interval of soil, basal, and lateral root cohesion; and (b) relative contributions to the overall cohesion

both in improving stability and increasing uncertainty in the prediction of stability of slopes through the infinite slope model.

5 CONCLUSIONS

This study developed a probabilistic model to evaluate the influence of vegetation growth and root reinforcement on shallow slope stability, accounting for both basal and lateral root reinforcement contributions to cohesion. The results of an example application show that, although the dispersion in sample of FS increases with time, root reinforcement improves stability and reduces the probability of failure. These findings emphasize the dual role of vegetation in the stability of shallow slopes: enhancing slope stability over time while remaining associated with considerably uncertainty into the analysis.

6 ACKNOWLEDGMENTS

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