

An approach to modelling roots in a boundary value problem

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ABSTRACT: Anthropogenic climate change is increasingly causing higher incidences of extreme weather events such as storms and droughts. This affects the stability and resilience of earth structures, as it disturbs their water balance by dual action of rainfall infiltration into the soil and evapotranspiration through vegetation roots and leaves. With the development of advanced coupled numerical tools to model soil-vegetation-atmosphere interaction in saturated and unsaturated soil conditions, it is possible to simulate the complete lifecycle of earth structures, such as infrastructure embankments, slopes, and reservoir earthfill dams. This paper focuses on the numerical modelling of roots as contributors to a water-balancing mechanism in the soil that controls the pore pressures / suctions in the soil, and hence the evolving soil strength due to suction changes. It then uses an embankment case study to demonstrate the performance of this modelling approach in the assessment of an infrastructure embankment, both as part of a lifecycle analysis and a resilience analysis that takes into consideration antecedent conditions in a changing climate and their influence on the stability of earthfill structures.

Keywords: roots; vegetation; geotechnics; numerical modelling

1 BACKGROUND

It is well-recognised today that the growing vegetation and seasonal variations in weather patterns can have substantial effects on the serviceability and stability of earth infrastructure (e.g. Vaughan, 1994; O'Brien, 2013). The dynamics of the water balance in natural and cut slopes, and in earthfill embankments and dams, is driven by atmospheric conditions (temperature, rainfall precipitation), vegetation growth (evapotranspiration) and soil's hydro-mechanical behaviour (permeability, water retention, strength, stiffness), e.g. Smethurst et al. (2015); Guo et al. (2023).

In the soil-vegetation-atmosphere (SVA) interaction the vegetation roots draw water from depths in the ground, which transpires into the atmosphere through leaves. This function of roots has been employed in computational modelling through root water uptake models (e.g. Indraratna et al., 2006; Nyambayo & Potts, 2010). More recently, roots have also been considered as contributors to soil mechanical reinforcement, through both their tensile capacity and enhanced properties of rooted soils (e.g. Zhang et al., 2024).

The current paper provides an overview of a modelling approach in which roots are contributing to variations in the seasonal water balance and consequential changes in the pore pressure regime and strength in the soil, without explicitly modelling rooted soils. The effectiveness of this approach is demonstrated on a case of SVA interaction in an infrastructure embankment, using the finite element (FE) platform ICFEP (Potts &

Zdravkovic, 1999) and its thermo-hydro-mechanical formulation for saturated and unsaturated soils, together with appropriate mechanical and hydraulic constitutive modelling and boundary conditions.

2 ROOT WATER UPTAKE MODEL (RWUM)

The rate of water uptake at depth below the ground surface is primarily governed by the root density, conductivity of the soil-root system and the availability of water. In coupled FE analyses, this can be implemented as a sink term in the continuity equation of fluid flow. The sink term represents the volume of water extracted per unit volume of soil, per unit time.

The RWUM implemented in ICFEP (Nyambayo & Potts, 2010) adopts the linear function of Prasad (1988) which relates the maximum extraction rate of water, Q_p , to root depth, r (Figure 1). Conditions associated with Q_p correspond to potential transpiration, T_p , which defines how much water can be taken out of the ground if there was an unlimited supply of moisture (Equation 1):

$$Q_p = \frac{2 \cdot T_p}{r_{max}} \left(1 - \frac{r}{r_{max}} \right) \quad (1)$$

where r_{max} is the prescribed maximum root depth. In field conditions the supply of soil moisture is not unlimited and the actual transpiration rate, Q_{ac} , is lower than Q_p . The RWUM is further extended to simulate water extraction more realistically by adopting Feddes et al. (1978) modification for Q_p , as shown in Equation (2):

$$Q_{ac} = \alpha \cdot Q_p = \frac{2 \cdot T_p}{r_{max}} \left(1 - \frac{r}{r_{max}}\right) \quad (2)$$

In the above, α is a matric suction (s)-dependent function shown in Figure 1. Roots are assumed unable to function (extract water) at suctions higher than the wilting point (s_4) and lower than the anaerobiosis point (s_1 , corresponding to water-logged conditions). The water extraction is assumed a maximum (i.e. $\alpha = 1.0$) between suction values s_2 and s_3 .

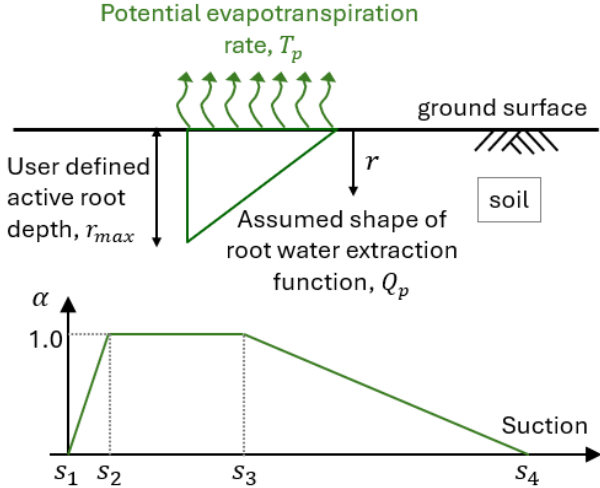


Figure 1. Root water uptake model

The potential evapotranspiration rates are also adjusted for the plant type and climate conditions, such as radiation, wind speed and temperature, by using the FAO Penman-Monteith (Allen et al., 1998) methodology.

3 CASE STUDY

3.1 Problem description

The numerical model considers an old, vegetated railway embankment in Essex, UK, with a cross-section shown in Figure 2. The monitoring of this embankment was carried out from March 2006 to March 2007, to establish baseline measurements of movements and pore pressures while most of the slope was covered with large deciduous trees (along the boundary a-b-c in Figure 2, Smethurst et al., 2015). Trees were removed in part a-b in March 2007, leaving only a grass cover, and monitoring continued to 2011, providing valuable field measurements for validation of numerical models.

The ground conditions comprise a London Clay foundation soil underlain by chalk at around 70m depth, with the ground water level 1.0m below the ground surface. The top 3.0m is classified as Weathered London Clay (WLC), characterised as more fissured than the remaining Unweathered London Clay (UWLC). The original 150-year-old embankment was constructed from loosely dumped clods of London Clay fill (LCF). To maintain the embankment's serviceability over the past decades,

the required rail track elevation was maintained by additional layers of reused locomotive ash (ASH) and more recently by granular ballast (BAL).

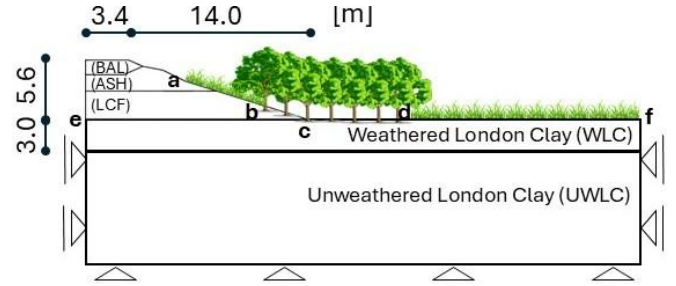


Figure 2. Problem domain

3.2 Material modelling

Mechanical: A numerical model of this embankment, developed by Guo (2021), adopted the unsaturated hydro-mechanically coupled FE framework available in ICFEP (Potts et al., 2021) and constitutive models summarised in Table 1. The WLC and LCF were represented with the unsaturated Imperial College Single Structure Model (ICSSM, Georgiadis et al., 2005), while a Fredlund et al. (1978)-type unsaturated Mohr-Coulomb (UN M-C) model was employed for the ASH. The BAL was simulated with a linear-elastic M-C model and the UWLC with a nonlinear-elastic M-C model. Model calibrations and experimental data for all models are presented in Guo (2021).

Table 1. Summary of constitutive models

Soil	Mech. model	ϕ' (°)	γ (kN/m ³)	k_{sat} (m/s)	SWR
BAL	M-C	40	18.0	Drained	-
ASH	UN M-C	35	11.0	4.0E-5	Mon
LCF	ICSSM	22	18.1	3.7E-8	Hyst
WLC	ICSSM	22	19.1	4.3E-9	Hyst
UWLC	NL M-C	23	19.1	3.7E-10	-

Hydraulic-conductivity: For all unsaturated layers (ASH, LCF, WLC) a dual permeability model was adopted (Potts & Zdravkovic, 1999), allowing reduction of saturated permeability (k_{sat}) due to desaturation (increase in matric suction), or increase of k_{sat} due to desiccation cracking (increase in tensile principal stresses). The BAL was treated as drained, while in the saturated UWLC k_{sat} was modelled as reducing with stress level.

Hydraulic-water retention: A monotonic van Genuchten (1980)-type soil water retention (SWR) model was adopted for the ASH, while a hysteretic SWR model (Tsiampousi et al., 2013) was adopted for the LCF and WLC. The relevant data and calibrations are detailed in Guo (2021).

3.3 Boundary conditions (BCs)

A lifecycle analysis of this embankment simulated the history of its construction and vegetation growth and removal. Starting with a flat ground surface which was agricultural land with grass cover, a 0.1m root depth was assumed in the model along the boundary e-c-d-f in Figure 2. Appropriate average monthly evapotranspiration rates (calculated as described in Section 2) and precipitation rates were cycled over 5 years to initialise stresses and pore pressures in the foundation soil. The meteorological data for average monthly precipitation were obtained from the raingauge station nearest to the site and available for period 2006-2020.

The embankment was then constructed in layers over one year, removing the evapotranspiration BC along the e-c boundary, followed by 25 years of applying the same average monthly precipitation. The evapotranspiration rates were appropriately adjusted to allow an incremental growth of tree roots to a maximum 2.0m depth over the boundary a-b-c-d (Figure 2), as observed on site in March 2006. The felling of trees over the boundary a-b in March 2007 was represented by reducing both the root depth to 0.1m and the evapotranspiration rate, to reflect the grass cover. Measured monthly precipitation rates were then applied from 2007 to 2020, followed by projected precipitation rates from 2021 to 2080. The latter were derived with statistical and stochastic modelling detailed in Guo (2021).

4 RESULTS AND DISCUSSION

4.1 Seasonal variation of movements

As explained above, the modelling approach applied in the present study does not introduce discrete roots into the discretisation of the boundary value problem (BVP), nor does it account for the mechanical properties of the soil containing roots (i.e. rooted soil). The effect of roots for different plant types is instead represented by the RWUM and evapotranspiration rates, controlling, together with precipitation rates, the water balance in the embankment body, which consequently changes the pore water pressures / suctions in the soil. As the embankment layers are modelled with unsaturated constitutive models, these changes in suction directly induce changes in strength in the embankment materials, including the rooted zone.

As a matter of the FE model validation, Figure 3 demonstrates good agreement between measured and predicted vertical displacements in the mid-slope section from March 2006 to March 2007, before the removal of trees. In the summer months (May to Sep) evapotranspiration from the trees exceeds precipitation in the same period, resulting in the embankment shrinkage and an increase in settlements. Transitioning into the wet period (Oct to Mar), the embankment heaves and largely recovers the vertical movements by March 2007.

Assessing these results in conjunction with contours of mobilised suctions in the embankment body in Figure 4, it is evident that dominant evapotranspiration induces high suctions and a depressed phreatic surface by the end of summer (Sep 2006). The subsequent dominant precipitation reduces suctions by the end of the wet season in March 2007.

The felling of trees in March 2007 leads to practically unchanged low suctions in the embankment body until Sep 2007 (Figure 5), due to a much reduced water demand by the grass roots remaining on that part of the slope. The suctions, however, increase around the toe where the tree roots are still present.

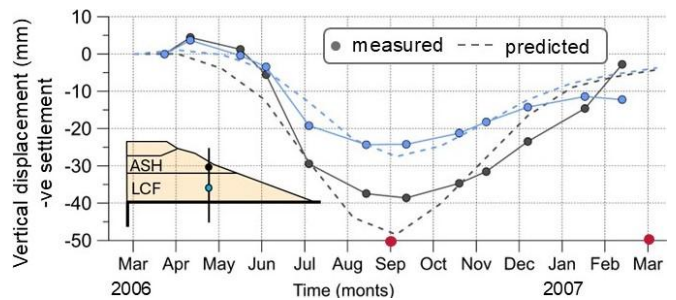


Figure 3. Annual vertical displacements in the embankment

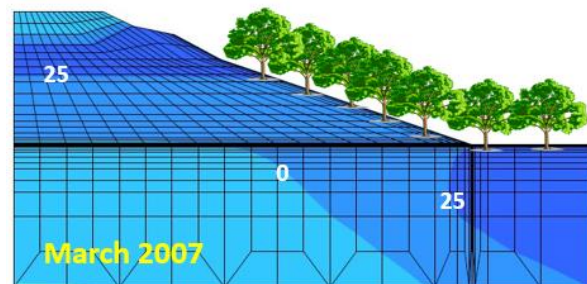
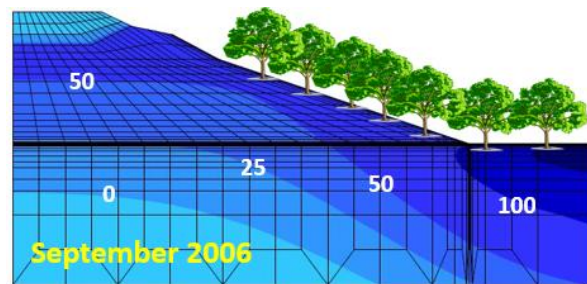


Figure 4. Seasonal pore water pressures (suction +ve) [kPa]

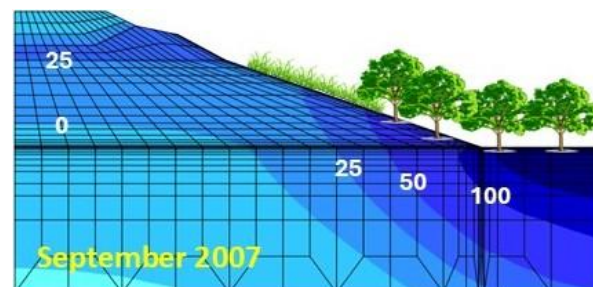


Figure 5. Pore water pressure after tree removal [kPa]

The predicted vertical displacements (Figure 6) in the same mid-slope section until March 2011 show continuous heaving and capture the measurements reliably.

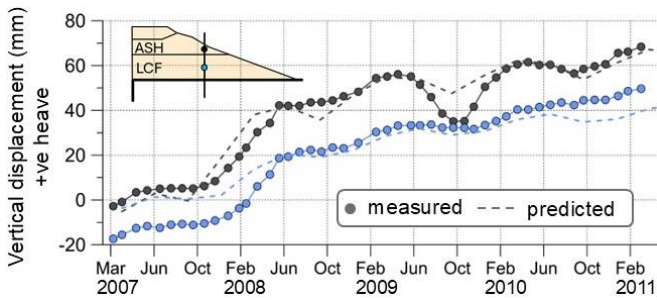


Figure 6. Embankment displacements post-tree felling

4.2 Resilience to storm

Figure 7 shows the contours of suction and magnitudes of the corresponding factor of safety, F_S , before and after a storm event, recorded as 95mm daily rainfall. The F_S was determined using the methodology developed by Potts and Zdravkovic (2012) and available in ICFEP. The storm was applied to embankment conditions in the driest summer (Aug 2069) from the lifecycle analysis. While the $F_S=1.66$ remains satisfactory after the storm, the suctions in the slope are substantially depleted.

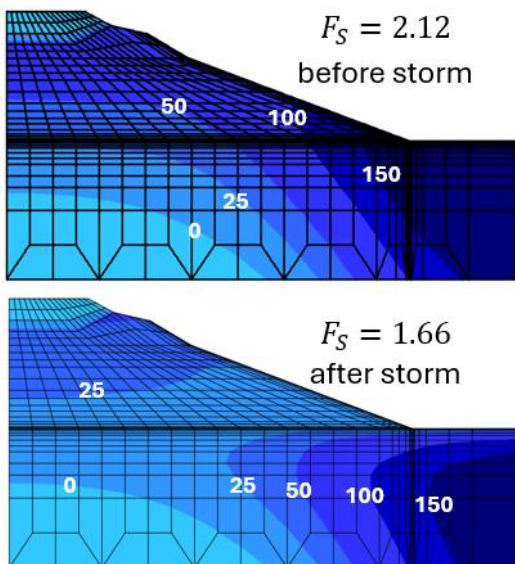


Figure 7. Changes in suction and F_S due to storm [kPa]

5 CONCLUSIONS

This paper introduces a numerical approach to modelling the effects of roots in a BVP, without directly discretising roots and simulating the behaviour of rooted soils. Different root depths and plant types are accounted for by the root water uptake model and evapotranspiration boundary condition which, together with the applied precipitation rates, control the changes in pore pressure/suction in the soil. This in turn controls the changes in soil strength, enabled by the unsaturated soil constitutive modelling. The approach is demonstrated to work well and to efficiently substitute a direct inclusion of roots in a numerical model.

6 ACKNOWLEDGEMENTS

The work presented here was sponsored by the EPSRC Centre for Doctoral Training in Sustainable Civil Engineering (EP/L016826/1) at Imperial College London, and the Geotechnical Consulting Group LLP.

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