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A Coupled MPM-LBM Method for Soil-Water Interactions

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ABSTRACT

The mechanism of internal erosion remains underexplored due to its complexity. Most of numerical methods for internal erosion is on particle level but difficult for field-scale applications. This study proposed an initial approach to combine two numerical methods popularly used in geotechnical engineering: material point method (MPM) and lattice Boltzmann method (LBM). A numerical case of soil column collapse in a water tank is presented to prove the capability of presented algorithm. However, the coupled MPM-LBM needs several improvements to become a complete model for soil-water interactions on continuum scale, but it is expected to be modified in the future for real dam erosion problems.

INTRODUCTION

Internal erosion is a phenomenon when small particles are loosened, detached, and transported through the voids in soils by seepage flow. This process highly threatens the stability of overall constructions. Moreover, internal erosion has been regarded as the main reason for the failures of many water retaining structures including embankment dams, dikes and levees (Zhang et al. 2015). It is noted that more than 40% of dam failures are caused by internal erosion (Foster et al. 2000) with great economic loss all over the world. However, the mechanism of internal erosion still remains poorly understood due to its complexity in interplay between its hydraulic, mechanical and material conditions. The erosion problems are commonly treated as a topic in solving soil-water interactions by numerical tools.

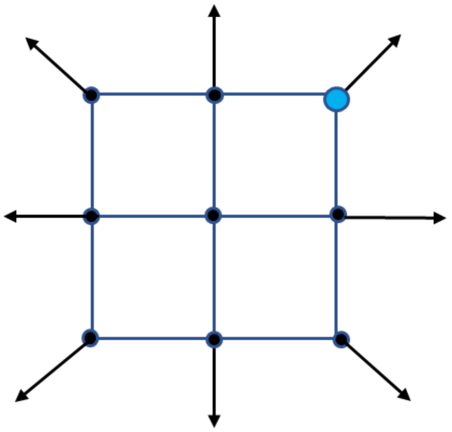
Plenty of numerical methods have been developed to investigate soil-water interactions. Several of them have been applied in modelling different types of internal erosion on particle-level including discrete element method (DEM), coupled discrete element method and computational fluid dynamics (DEM-CFD), coupled discrete element method and lattice Boltzmann method (DEM-LBM), etc. The DEM model is used to analyze the soil susceptibility to suffusion (Shire and O'Sullivan 2013). The model is a dry soil model with no fluid but proved the efficiency of DEM in inspecting soil properties. By coupling DEM with a fluid solver, the DEM-CFD method and DEM-LBM method are both successfully modified to simulate internal erosion. Firstly, the DEM-CFD method is applied in modelling erosion in a soil sample with flow passing upward (Tao and Tao 2017). The progressive stages of erosion can be clearly identified. Furthermore, contact erosion in a soil sample built with two layers of soil particles is modelled by DEM-LBM method (Galindo-Torres et al. 2015). It is found in this paper that small particles are more likely to be pushed into the voids in soil skeletons than large particles due to high buoyancy forces acted on them. These microscale methods are able to capture the initiation and progression of internal erosion. However, when transferring into field-scale applications, the simulation of soil phase deformation is very computational inefficient due to high demand in number of particles in the model. Unfortunately, limited attempts have been done to investigate the internal erosion on continuum=scale. There is currently no model which can be applied in real dam simulations to improve its safety against internal erosion.

In this study, we presented an algorithm to combine Material Point Method (MPM) and Lattice Boltzmann Method (LBM) for soil-water interactions. The two method are coupled by merging background mesh in MPM and lattice nodes in LBM. The soil phase and water phase are modelled by MPM and LBM respectively. A numerical example of soil column collapse in a water tank modelling by MPM-LBM is shown. It should be noted that this paper is aiming to show that the idea of coupling MPM and LBM works. The development of the method is at very beginning stage for various geotechnical problems including internal erosion. However, due to the efficient of MPM in modelling large deformations, we expect this model to be modified in the future for continuum-scale investigation of internal erosion.

METHODOLOGY

The material point method (MPM) is a Lagrangian method which can be also regarded as an updated finite element method (FEM). It can directly apply many advanced features established in FEM (Nguyen 2014). However, compared to classical FEM, MPM can avoid the errors owing to severe mesh distortion in modeling the behavior involved with large deformations. This is why MPM is chosen here to model the dynamics of soil phase in erosion problems since it enables the evaluation of the failure of those water-retaining structures caused by internal erosion. As a fluid solver, the lattice Boltzmann method (LBM) is superior in efficiency through solving Boltzmann equation rather than typical Naiver-Stokes equations. Moreover, LBM is more suitable for flow in porous media (Girimaji 2013).

In MPM, a background grid is set behind the continuum body which has been discretized into particles with valuable information. At first, the information of particles within the influence range is accumulated on the grid nodes. Then the momentum of the nodes is updated on the nodes. Finally, information of nodes is mapped back to its neighbor particles which is the end of one time step. The background mesh is always recovered to its original configuration at each step. In LBM, the fluid is regarded as a large number of small particles moving with random motions. The macroscopic parameters of one fluid particle including velocity and density can be calculated by distribution functions. There are two main computational steps in LBM: streaming and collision. At streaming, the distribution function moves to other lattice nodes according to its direction. For collision, every distribution function is recalculated according to equilibrium function and relaxation time towards equilibrium.



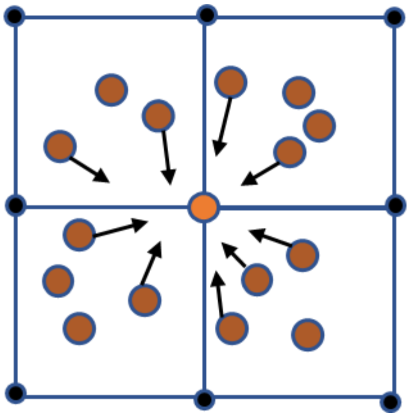


Figure 1 Streaming Step in LBM

Figure 2 Particle to Nodes Mapping in MPM

The fundamental idea to couple two particle-based methods is to combine background mesh in MPM and lattice nodes in LBM. As shown in Fig 1 and Fig 2, after the “streaming “step in LBM and “particles to nodes mapping” step in MPM, the density and velocity of fluid particle can be obtained (blue node in Fig 1) as well as the solid particle (orange node in Fig 2). After merging, the conversation and momentum balance for the nodes on one coupled grid can be written as:

$$\mathrm{\rho_{cI} = \bar{\rho}_s + \bar{\rho}_f} \tag{1}$$

$$\mathrm{\rho_{cI} v_{cI} = \bar{\rho}_s v_s + \bar{\rho}_f v_f} \tag{2}$$

where the $\mathrm{\rho_{cI}}$ is the density on the coupled nodes, $\mathrm{\bar{\rho}_s}$ and $\mathrm{\bar{\rho}_w}$ are the effective density for solid and fluid particles respectively, $\mathrm{v_{cI}}$, $\mathrm{v_s}$ and $\mathrm{v_f}$ are the velocity for the coupled nodes, solid particles and fluid particles. For fluid saturated soil with porosity $\emptyset$, the relationship between effective density for soil phase is calculated by $\mathrm{\bar{\rho}_s = \emptyset \rho_s}$ and effective density for water phase is computed by $\mathrm{\bar{\rho}_w = (1 - \emptyset)\rho_w}$.

The interaction between solid and liquid phase in this study only considers the influence of drag force. The drag force $\mathrm{F_d}$ is defined as the body force acting on one phase by the other with unit N·m^{-3} in this model. Referring to (Baumgarten and Kamrin 2019), it can be assumed to be depended on the relative velocity $\mathrm{(v_s - v_f)}$ between viscous fluid around and within solid particle for an immiscible mixture with porosity $\emptyset$ between 0.1 to 0.6, and Reynold number Re < 1000 given as:

$$\mathrm{F_d = \frac{18\emptyset(1-\emptyset)\eta}{d^2} \hat{F}(\emptyset, Re)(v_s - v_f)} \tag{3}$$

Where d is the particle diameter, η is the dynamics viscosity of water, and the $\mathrm{\hat{F}(\emptyset, Re)}$ is a function of dimensionless parameter. After the drag force on coupled nodes are updated, it should be mapped back to both fluid particles and solid particles as indicated by the arrows in Fig 3. For water phase, the forces distribution function as the influence of drag force is added as an additional collision term in LBM (Afra et al. 2018). Similarly, the updated force $\mathrm{F_{cI}}$ on coupled nodes in MPM has the following form:

$$\mathrm{F_{cI} = S(x,t)(f_b + f_d + \sigma_p)} \tag{4}$$

where the $\mathrm{F_b}$ and $\mathrm{\sigma_p}$ are the body force and internal stress respectively, $\mathrm{S(x,t)}$ is the shape function for interpolation in MPM (Nguyen 2014). After the drag force on each coupled node is calculated and mapped back, the subsequent computation in MPM and LBM is same as its original scheme going into “nodes to particles mapping” and “collision” step as shown in Fig 3.

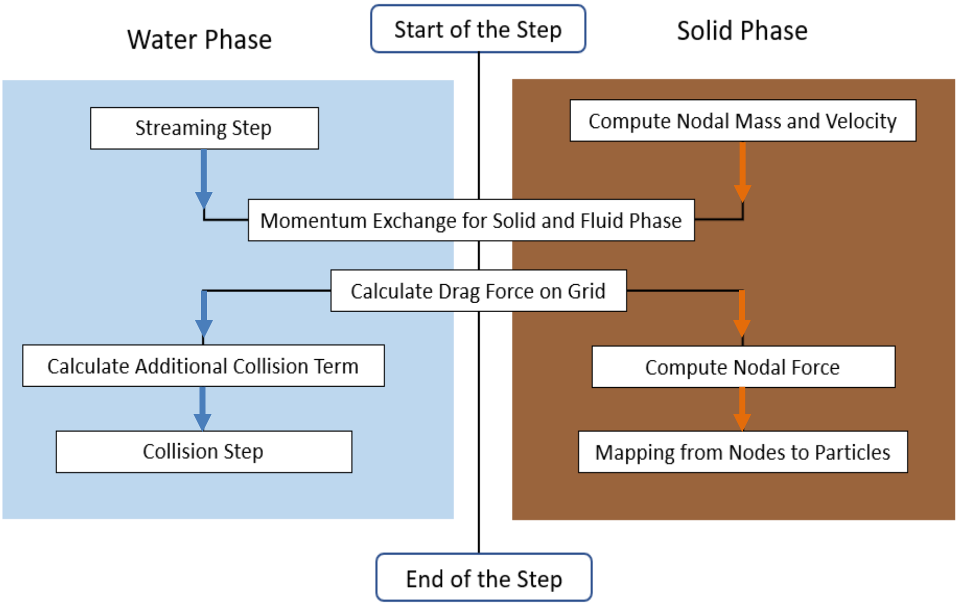


Figure 3 Coupled MPM-LBM Algorithm

RESULTS AND DISCUSSIONS

The collapse of a soil column in water tank is simulated by proposed algorithm to test the capability of it in combining the two methods. The soil phase and water phase are modelled by MPM and LBM respectively as described in last section. The simulation set-up is shown in Fig 4 where the rectangular soil column is initially rested at the left bottom corner of the tank. All the parameters used in simulation is summarized in Table 1. The soil phase is deformed by gravity force. A constitutive model of soil to update stress and strain in each time step can refer to a return mapping method (Calausen et al. 2017). There is no force acted on the fluid phase, so the velocity profile variation is completely caused by interaction between soil and water phase. All the boundary conditions for water phase is set to be no-slipping boundary conditions. The left and bottom side of soil column is slip boundaries with no frictions.

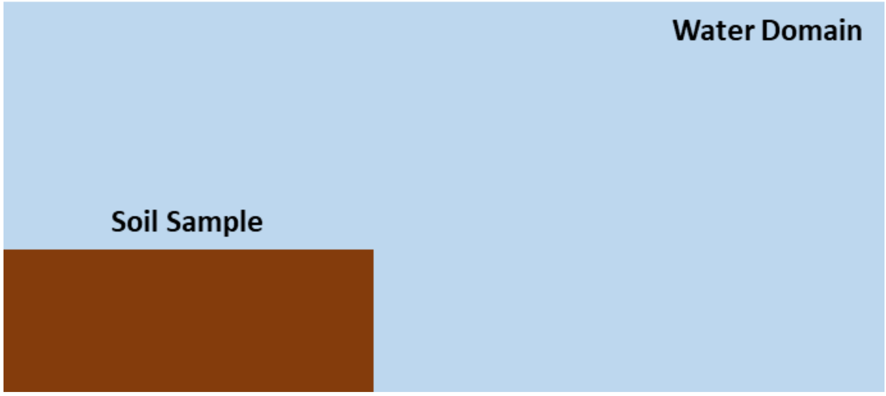


Figure 4 Schematic of fully coupled case

Parameter	Value
Soil particle density	2039 kg/m^3
Gravity acceleration	9.8 m/s

Young’s modulus	75 Gpa
Poisson ratio	0.3
Cohesion coefficient	0 Pa
Angle of internal friction	45^o
Angle of dilatation	0^o
Soil sample	$20 \times 10 \times 4m^3$
Flow domain	$80 \times 50 \times 4m^3$

Table 1 Coupled MPM-LBM simulation parameters

It has to be noted that the presented case is simulated by a one-way coupled algorithm. It means the drag force exerted by soil-water interactions initiates the movement of fluid but does not affect the dynamics of soil phase. The variation of velocity profile of water phase around boundary between soil particles and fluid is shown in Fig 5. With the deformation of soil sample, increasing velocity of soil particles leads to greater drag force calculated by Equation (3). It then results in higher velocity for water phase around interface between soil and water which is clearly shown in Fig 5 (2) to Fig 5 (3). The results here indicate that the drag force is computed on combined background in MPM-LBM by velocity of soil particle and fluid nodes. It then changes the velocity distribution of water phase.

However, although the simulation results discussed above can prove the capability of proposed algorithm in this paper to combine two methods, there are several limitations to be identified here. Firstly, the presented result is a one-way simulation that the influence of drag force on behaviors of soil phase has not been considered. It is questionable if the fully coupled model will have problems in numerical stability. Secondly, it is argued that a buoyancy force should be considered for immiscible mixtures (Drumheller 2000). This will change the form of governing equations both for soil and water phase. Thirdly, the typical lattice Boltzmann method is developed to solve Naiver-Stokes equations. It has been pointed out that flow in a porous system either with constant porosity of variable porosity is governed by a modified form of Naiver-Stokes equation (Guo and Zhao 2002). Therefore, the LBM part in the coupled method should be updated for simulating flow from porous medium to open channel for erosion problems. Finally, to improve the presented MPM-LBM model for internal erosion, the detachment and movement of soil particles should be included by specific equations.

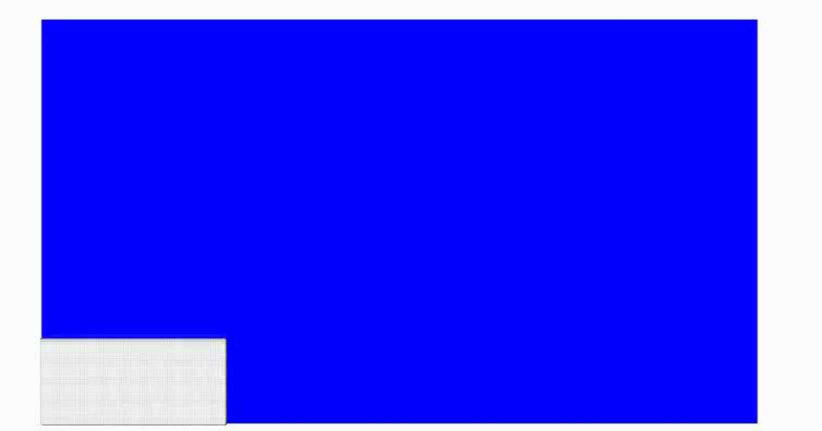


Figure 5(1) One-Way Coupled MPM-LBM simulation at 0s

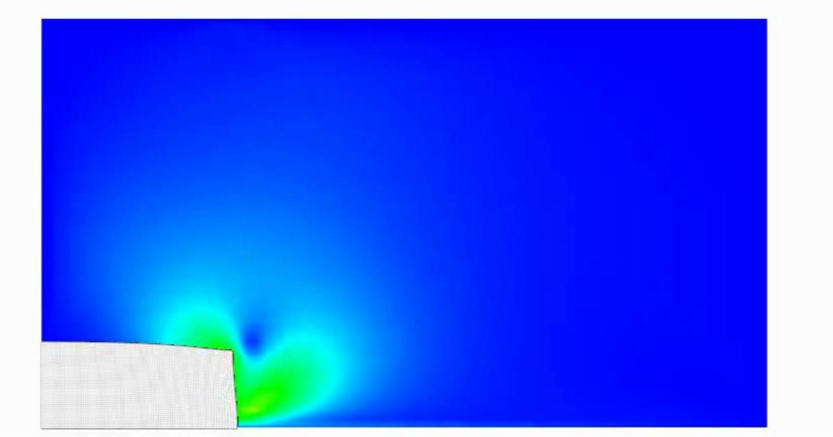


Figure 5(2) One-Way Coupled MPM-LBM Simulation at 3s

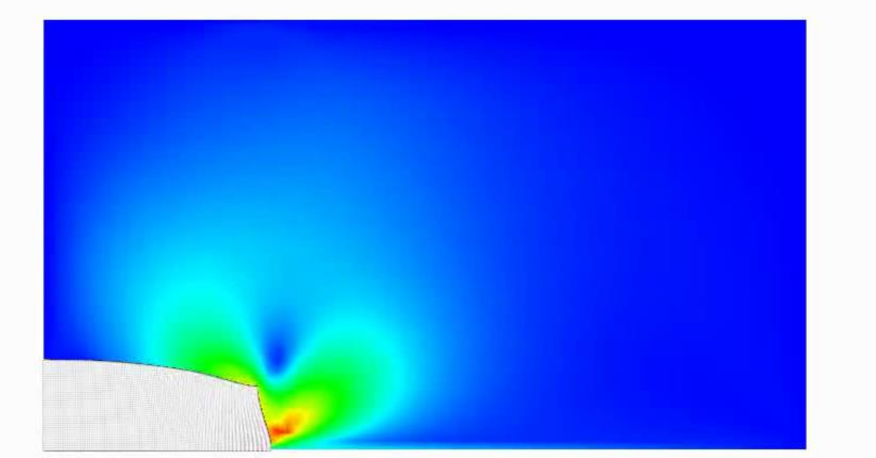


Figure 5(3) One-Way Coupled MPM-LBM Simulation at 6s

CONCLUSIONS

In this paper, an initial approach to couple two numerical methods: Material point method (MPM) and lattice Boltzmann method (LBM) is presented. The key idea is to merge the background grids in MPM and the lattice nodes in LBM as a same mesh. A numerical example in simulating the collapse of soil sample in flow domain has been shown to prove its ability in combining MPM and LBM. However, only the drag force is considered in this numerical study to govern soil-water interactions. It needs several improvements to be developed as an efficient numerical tool for solving soil-water interactions on continuum scale. Due to the advantages of MPM and LBM, this coupled method has lots of potential to be built a tool in simulating real dam internal erosion.

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