Fundamentals of Laminar Flow In Pipes

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ABSTRACT

This paper provides insight in the piping failure mechanism and deals with the specific properties of the pipes in the sand layer beneath clayey dikes. Briefly, the basic concepts of geometric, hydraulic and friction conditions are discussed in order to comprehend the interaction between the pipe height, hydraulic radius, pipe velocity, pipe discharge, and the mean hydraulic gradient in the pipes for different forms. The well-known theories of Darcy-Weisbach, Shields, Ohm, Hagen-Poiseuille and Newton are briefly summarized in order to understand the driving and resistance forces in the pipe erosion process.

PIPING FAILURE MECHANISM

Piping is a three dimensional phenomenon since pipes braid (or meander) and are irregular in shape. The relevant parameters that influence piping are the hydraulic head and the material characteristics of both the sand layer and the clayey dike. During high stream-stage flows, seepage through sand layers occurs, with the flow rate depending on the type of soil and how well the soil layer is compacted. Seepage can lead to piping and when the ditch is connected to the river through pipes the dike will eventually fail as the pipe flow changes from a laminar to a turbulent flow regime (Figure 1).

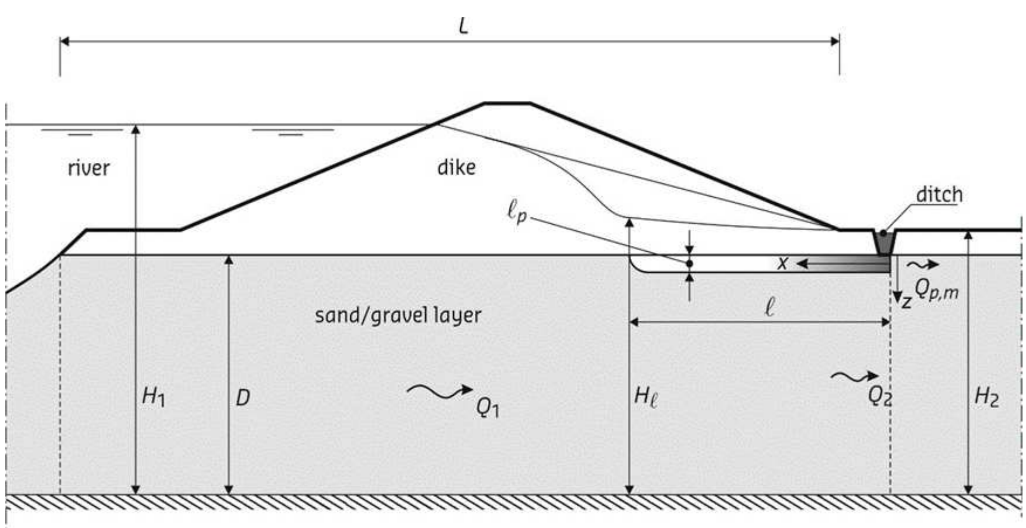

Figure 1. Definition sketch of piping (not to scale).

The load is represented by the hydraulic head, which is usually the difference between the flow level in rivers (H_1) and the surface level on the landside (H_2), (see also Figure 1). The seepage length, the permeability (or particle size and porosity), and grading of the particles in the sand layer are determinant for the erosion resistance. The seepage length (L) is the distance between the entry point for seepage through the sand layer on the riverside and the exit point on the landside.

As long as ($H_1 - H_2$) is sufficiently small, particle-free water will exit, resulting in a wet ground surface. If however ($H_1 - H_2$) and hence the flow velocities in the pores increase, particles could deposit around boils or craters that occur downstream of the dike toe. In such situations the lift, drag and frictional forces acting on the particles may exceed the critical forces for initiation of particle motion and small pipes form at the top of the sand layer directly under the clayey dike, which may grow in length in the direction of the river. This process is called backward erosion.

GEOMETRIC CONDITIONS

Although the pipes under the clayey dike do not realistically advance in a line from downstream to upstream, in the following model the pipes have a variable length in the x direction. At the laboratory scale, Van Beek et al. (2008, 2011) demonstrated that, at the beginning of the erosion process, the pipe formation is similar to that of braided rivers (Figure 2). Because of horizontal pressure fluctuations, a braided river consists of a network of small pipes separated by small and often temporary islands called braid bars (e.g. Jansen et al. 1979).

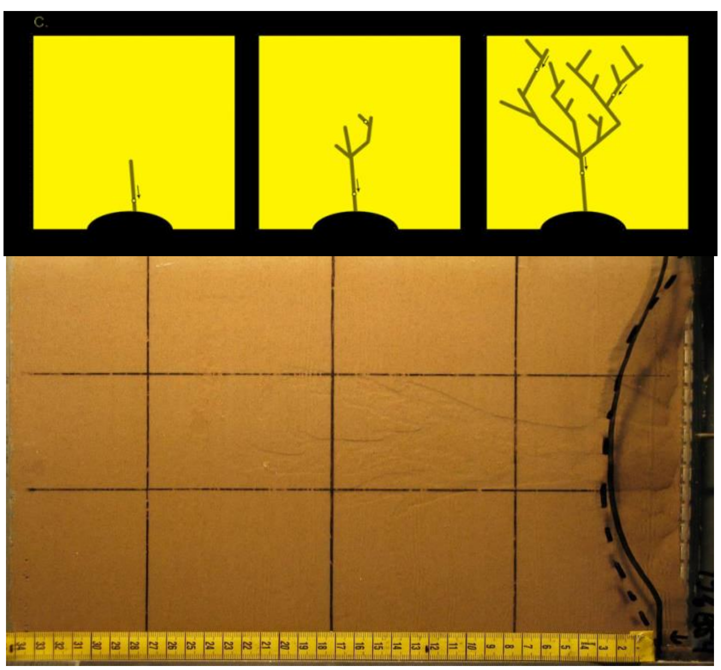

Figure 2. Braiding of pipes (Van Beek et al. 2008, 2011)

Bonelli et al. (2007) deduced a time scale for breaching in hydraulic works (dams, dikes) by modelling the pipes as tunnels with a constant pipe height. In this study, the pipe height increases linearly from the entry points to the landside. Hence, the mean pipe height (ℓ_p) halfway along the pipes (in the longitudinal direction) can be given by (Figure 3)

$$\ell_p = \frac{1}{2}(\ell_{p,h} + \ell_{p,m})$$

where the initial pipe height ($\ell_{p,h}$) is correlated to the pipe height ($\ell_{p,m}$) on the landside through the geometry factor (C_ℓ)

$$\ell_{p,h} = C_\ell \ell_{p,m}$$

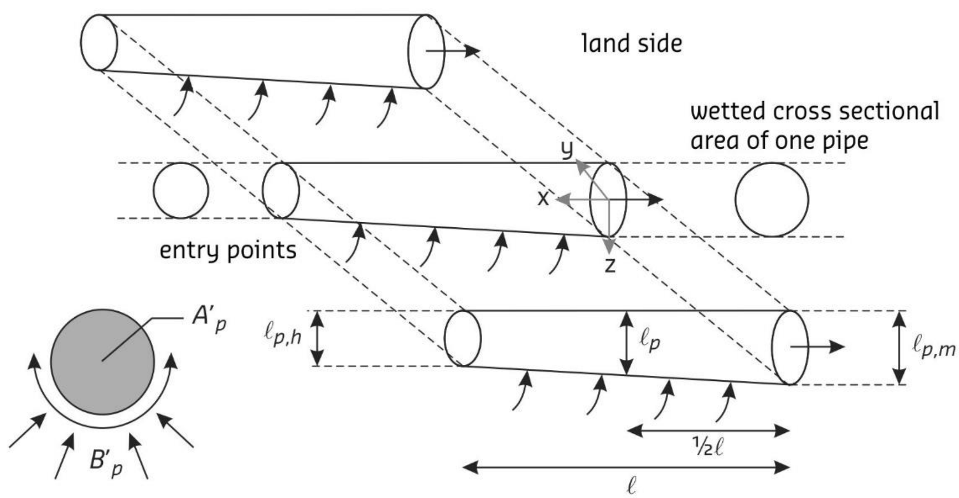

Figure 3. Schematization of circular pipes below clayey dike ($A'_p = A_p/n_p$ and $B'_p = B_p/n_p$)

If the pipes are cylinder-shaped then the wetted cross sectional area (A_p) and the effective wetted width (B_p) of n_p pipes (or half the perimeter) can be approximated halfway along the pipes by (see also Figure. 3)

$$A_p = \frac{1}{4}\pi \ell_p^2 n_p \quad \text{and} \quad B_p = \frac{1}{2}\pi \ell_p n_p$$

For rectangular pipes the wetted cross sectional area and the effective wetted width of n_p pipes can be estimated halfway along the pipes by (with χ as the ratio between the width and the height of the pipe; $30 < \chi < 50$)

$$A_{p,rec} = \chi \ell_{p,rec}^2 n_{p,rec} \quad \text{and} \quad B_{p,rec} = \chi \ell_{p,rec} n_{p,rec}$$

For elliptic pipes holds

$$A_{p,e\ell\ell} = \frac{1}{4}\chi\pi\ell_{p,e\ell\ell}^{2}n_{p,e\ell\ell} \quad \text{and} \quad B_{p,e\ell\ell} \approx \frac{1}{2}\sqrt{\frac{1}{2}}\chi\pi\ell_{p,e\ell\ell}n_{p,e\ell\ell}$$

HYDRAULIC CONDITIONS

Various parameters can be used to determine the flow regime, e.g. hydraulic (or pressure) gradient, shear stress, shear velocity, filter velocity or pore velocity. All these parameters can be related to forces. When they are determined by the flow properties only the flow is laminar otherwise it is turbulent.

When there are no pipes, that is, in the equilibrium phase the groundwater flow in the sand layer is fully laminar. The transition to turbulent flow in a porous medium is not defined by a unique Reynolds filter number (Re_f), as this number is influenced by a characteristic particle size (for example by d_{15}), the filter velocity (u_f), and the kinematic viscosity (v). Following Bear (1979) the groundwater is completely laminar if

$$\mathrm{Re}_f = \frac{d_{15}u_f}{v} < 1 \text{ to } 10$$

The Reynolds pipe number (Re) depends on the pipe geometry, i.e., the hydraulic radius (R is the ratio between the wetted area (A_p) and the wetted perimeter (P)), the mean pipe flow velocity (U_p) which is directed in the streamwise direction and the temperature (through v)

$$\mathrm{Re} = \frac{U_p R}{v}$$

For cylinders, rectangular and elliptic pipes the following relations hold $R = \frac{1}{4}\ell_p$, $R_{rec} = \frac{1}{2}\ell_{p,rec}$ and $R_{e\ell\ell} \approx \frac{1}{3}\ell_{p,e\ell\ell}$. Note that the hydraulic radius for a squeezed elliptic pipe with $\chi = 40$ is somewhat larger $R_{e\ell\ell} \approx \frac{3}{8}\ell_{p,e\ell\ell}$ and that $\ell_p \neq \ell_{p,rec} \neq \ell_{p,e\ell\ell}$.

The pipe flow is laminar provided Re is smaller than 500 and is turbulent if Re reaches a value of 2000. The continuity equation with Q_p as the pipe discharge reads

$$U_p = \frac{Q_p}{A_p}$$

Three regimes may be distinguished, namely 1) laminar flow in both the pipes and the sand layer, 2) turbulent flow in both the pipes and the gravel layer, and 3) turbulent flow in the pipe and laminar flow in the sand layer.

According to Den Adel (1986) the flow in a gravel layer is turbulent if the mean particle size is greater than 1 cm. In this study, only the first regime is considered, that is, the mean particle diameter (d_{50}) ranges from 0.1 mm to 0.5 mm (0.1 mm < d_{15} < 0.3 mm with d_{15} is the particle diameter below which 15% of the particles are smaller).

FRICTION CONDITIONS

The friction in the pipes can be expressed with the Darcy-Weisbach equation

$$\tau_0 = \frac{1}{8} f_{DW} \rho U_p^2$$

in which ρ represents the density of water and τ_0 is the (mean) wall (or bed) shear stress. The Darcy-Weisbach friction factor (f_{DW}) is not a constant and depends on the parameters of the pipe and the flow velocity. Figure 4 shows results of non-Newtonian fluids, including aqueous suspensions of bentonite and kaolin, and aqueous solutions of synthetic polymer carboxymethyl cellulose (CMC). The 16/Re line represents also laminar water (clean sand).

$$f_{DW} = \frac{16}{Re}$$

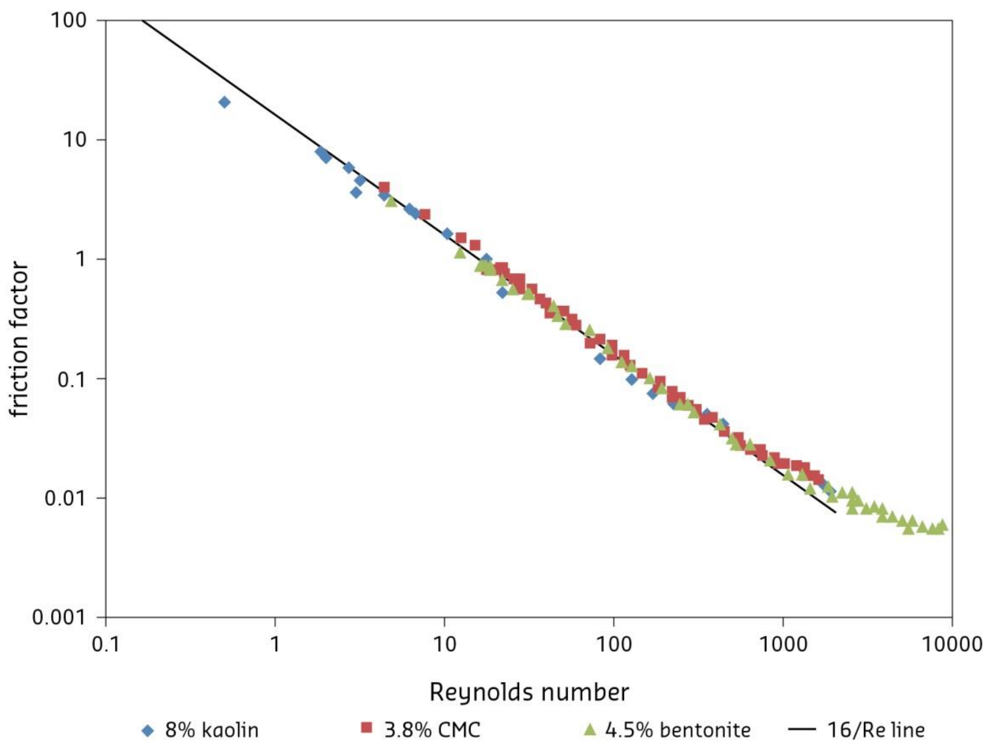

Figure 4. Friction factor as function of Reynolds number for 3 different fluids (Haldenwang, 2010)

Following Chow (1959) the coefficient 16 represents a mean value as it ranges from 14 (for triangle pipes) to 24 (for rectangular pipes). By using the Shields (1936) approach the wall shear stress for laminar flow can be written as

$$\tau_0 = \Psi_{\ell am}(\rho_s - \rho) g d_{50}$$

where g is the acceleration of gravity, ρ_s is the density of the sediment and $\Psi_{\ell am}$ represents the Shields parameter for laminar flow conditions.

PIPE RESISTANCE

The law of Ohm (1872) states that the current through a conductor between two points is directly proportional to the potential difference across the two points and inversely proportional to the resistance between them. The mathematical equation is

$$I = \frac{V}{\Omega}$$

where I is the current through the conductor, V is the potential difference measured across the conductor and Ω represents the resistance of the conductor. Similar to Ohm's law the Hagen-Poiseuille equation can be written as

$$Q_p = \frac{S}{\Omega_p}$$

with Ω_p as the horizontal pipe resistance (of one pipe)

$$\Omega_p = \frac{2\nu}{g A_p R^2}$$

The mean energy slope (S) is the slope of the hydraulic grade line. In open channel flow, it is the slope of the water surface. In pipes under pressure, it is the slope of the water pressure (or the mean hydraulic gradient). For laminar flow conditions S is

$$S = \frac{2\nu U_p}{g R^2} = \frac{2 U_p^3}{g\, Re^2\, \nu}$$

which is the equation of Hagen-Poiseuille, i.e., the law that the pipe velocity is directly proportional to the hydraulic gradient and the square of the hydraulic radius.

FORCE BALANCE

Newton's Second Law states that the acceleration of a control volume times the mass of that volume is equal to the sum of all forces acting on it. If the control volume is the water in a short reach of steady, uniform flow, the volume undergoes no accelerations and the forces acting on it have to be balanced. These forces (per unit width) are the downslope component of the weight of water in the reach ($F_A = \rho g A_p S$) and the total boundary shear force, which is resisting the flow ($F_R = \tau_0 P$). Equating these forces ($F_A = F_R$) the depth-slope product can be deduced which is used to calculate the shear stress at the bed of an open channel (e.g. Chow, 1959)

$$\tau_0 = \rho g R S$$

Though this equation is widely used in river engineering, stream restoration, sedimentology, and fluvial geomorphology, thus for turbulent flow, it can also be applied for piping if the flow is laminar. It is the product of the hydraulic radius and the mean hydraulic gradient, along with the acceleration due to gravity and density of the water. The local hydraulic gradient can be written with x is the longitudinal distance as

$$S(x) = \frac{\tau_0(x)}{\rho g R(x)}$$

For very fine sand there is hardly a vertical inflow from the aquifer to the pipes due to the very low permeability of the sand. In such cases, the hydraulic radius of the pipes is almost constant in the streamwise direction. If the intrinsic soil properties in the sand layer do not change then the wall shear stress is also constant and thus the hydraulic gradient on the entry points equals approximately the hydraulic gradient on the landside. These conditions are comparable with open-channel flow.

Usually there is a vertical inflow as a result of the (high) permeability. Then the pipe dimensions increase gradually towards the landside. Consequently, the hydraulic gradient in the pipes decreases slowly to the polder. However, occasionally the pipe flow decelerates which is observed temporarily. Then particles can block the pipes because the wall shear stresses are smaller than the critical ones. Particles are only transported to the landside provided the wall shear stresses exceed their critical values at every location in the pipes.

If the pipes grow to the riverside then the hydraulic gradient near the entry points can increase to large values, theoretically to infinity (Sellmeijer 1988 and Van Beek 2015), since at these locations the local hydraulic radius is about nil. In the streamwise direction the local hydraulic gradient decreases enormously due to an increasing hydraulic radius.

CONCLUSIONS

This paper describes the erosion in the pipes which governs partly the failure mechanism piping. For predicting the latter mechanism a resistance equation is needed that describes not only the pipe resistance (this paper) but also the seepage resistance in the sand layer (e.g. Hoffmans and Van Rijn, 2017).

In this study the wall shear stress is expressed threefold. These three equations refer to the critical pipe velocities (Darcy Weisbach), initiation of motion (Shields) and the critical pipe dimensions (through the hydraulic radius) and the critical hydraulic pipe gradient (Newton). For describing erosion in the pipes these conditions must be met.

Typically in geotechnical engineering the hydraulic gradient which expresses the driving force, is often used to describe the groundwater flow in aquifers. However, for the description of the flow in pipes the product of the hydraulic gradient and the hydraulic radius is needed or the wall shear stress (second law of Newton).

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