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Quantifying the effect of aquifer characteristics on 3D backward erosion

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ABSTRACT

Backward erosion piping (BEP) contributes in a large part to the high probability of failure of levees in both the Netherlands and the United States. BEP refers to the regressive erosion of pipes from the inner side to the outer side of the levee, leading to failure. In the Netherlands, the Sellmeijer calculation rule is used to assess BEP, which is based on a 2D schematization. Whereas in a 2D schematization, the pipe is infinitely wide, in 3D situations the pipe has a limited width so flow of water also concentrates from the sides to the pipe, leading to a higher loading. The goal of this research is to investigate the influence of the higher 3D loading on the failure probability. The 3D effects on BEP were analyzed using two types of models: A 3D coupled groundwater flow and erosion model at a small scale and a 3D groundwater model at field scale, where the pipe is modeled as a boundary condition.

The small-scale models show that groundwater flow calculations provide a reasonable estimate of the flow rate in the pipe. The simulations also illustrated that erosion equations can be used to estimate the pipe gradient quite accurately once the flow rate is known. Based on the Sellmeijer model, equations for the critical gradients in 2D and 3D are derived. The field scale models show that the most important parameters affecting 3D effects for BEP are the scale of the model and the width to seepage length ratio. Other parameters such as anisotropy and permeability of the aquifer have a smaller influence on 3D effects.

INTRODUCTION

Backward erosion piping (BEP) occurs due to a water level difference between the inner and the outer side of a clay levee resting on sandy soil or a levee resting on an impermeable layer above an aquifer. Due to this hydraulic gradient, sand from the top of the aquifer can be eroded starting from the inner side to the outer side, leading to regressive pipes below the levee. Their length depends on the hydraulic gradient and characteristics of the soils (Rosenbrand & Wopereis, 2022). Once the gradient is high enough, the pipes reach the water source and high discharges through the pipe leads to widening of the pipe and ultimately failure of the levee. The average hydraulic gradient that, when exceeded, leads to ongoing pipe formation and eventually causes failure, is called the critical hydraulic gradient. In the Netherlands, the Sellmeijer calculation rule is used (Sellmeijer, 1988) to assess this critical gradient, which is the base parameter for assessing the failure probability for BEP.

The Sellmeijer rule is based on 2D groundwater whereas in practice 3D groundwater is more common and leads to a higher hydraulic load on the particles in and around the pipe and thus a lower critical hydraulic gradient. Modelling 3D BEP is time-consuming due to the many iteration steps and required refinement of mesh near the pipe elements. Therefore, this paper attempts to compute the critical gradient based on 3D groundwater flow models instead of BEP models. To achieve this, the following steps are taken:

1. The relationship between flow rate through the pipe and the pipe gradient is investigated.
2. The relationship between the pipe gradient and the critical gradient is investigated.

Furthermore, to better understand 3D groundwater flow towards a pipe, field scale groundwater flow models are used to evaluate the most important parameters that affect a 3D flow towards a pipe.

I - METHOD – MODELLING BACKWARD EROSION PIPING

BEP consists of two types of erosion: erosion in the pipe and erosion at the tip of the pipe (Hanses, 1985; Robbins et al., 2018; Van Beek, 2015). Erosion in the pipe is the erosion of the floor and walls in the pipe due to the shear stress in the pipe. The Sellmeijer model relies on this type of erosion to determining the pipe depth and for the progression of the pipe. The Sellmeijer model does not explicitly account for erosion at the tip of the pipe. The effects of erosion in the pipe, on pipe gradient and progression are discussed below, based on the same assumptions as in the Sellmeijer model. See Figure 1 for the schematization of terms used in this paper.

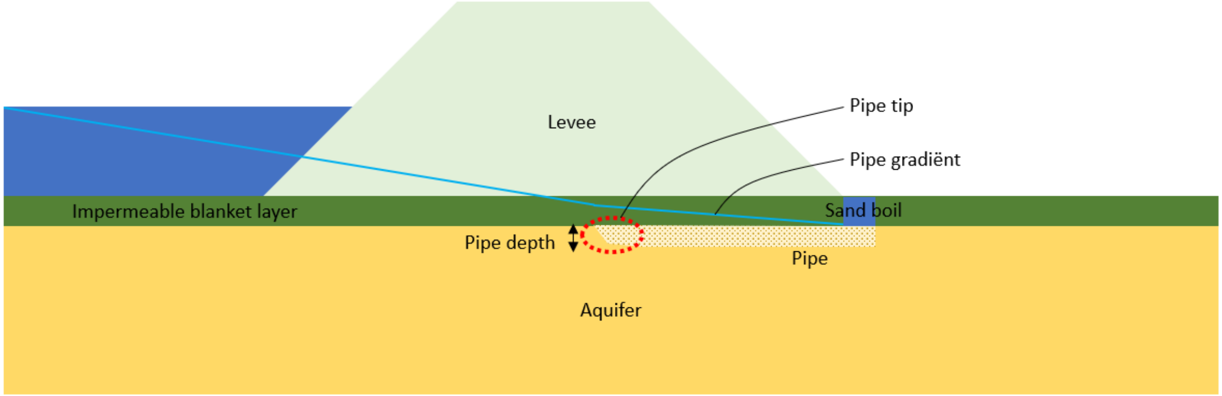

Figure 1 Schematization of terms used in this paper: pipe gradient, pipe tip, pipe depth etc.

Erosion in the pipe

Exceedance of the critical shear stress leads to sand transport. For wide and shallow pipes, the shear stress (τ_w) in the pipe is defined as:

$$\tau_w = \rho_w g \frac{a}{2} \frac{d\varphi}{\mathrm{d}x} \qquad \textbf{Equation 1}$$

With ρ_w the water density, g the gravitational constant, a the pipe depth and $d\varphi/\mathrm{dx}$ the gradient in the pipe. This equation is valid for both 2D situations (infinitely wide pipe) and 3D situations with pipe width to depth ratios approximately higher than 10 (Van Beek, 2015), as it assumes the erosion pipe is infinitely wide. Most BEP pipes have width to depth ratios higher than 30 (Allan 2018; Vandenboer et al. 2019).

The flow through the pipe (Q_{pipe}) is related to the depth of the pipe and the gradient in the pipe through the viscous pipe flow relation for wide ducts. For 2D and 3D situations[1], the equations are:

2D: $\rho_w g \frac{d\varphi}{\mathrm{d}x} a^3 = 12 Q_{pipe} \mu$ **Equation 2**

3D: $\rho_w g \frac{d\varphi}{\mathrm{d}x} a^4 \frac{w}{a} = 12 Q_{pipe} \mu$ **Equation 3**

Combining the above equations illustrates the relation between shear stress, flow rate, and gradient (and pipe width in 3D):

2D: $Q_{pipe} \left(\frac{d\varphi}{dx}\right)^2 = \frac{\rho_w g}{12\mu} \left(\frac{2\tau_w}{\rho_w g}\right)^3 = c_{2D}$ **Equation 4**

3D: $Q_{pipe} \left(\frac{d\varphi}{dx}\right)^3 = \frac{\rho_w g}{12\mu} \left(\frac{2\tau_w}{\rho_w g}\right)^4 \frac{w}{a} = c_{3D}$ **Equation 5**

These equations show that an increase of flow can only lead to a constant shear stress if the pipe gradient is decreased, resulting from depth increase of the pipe. By increasing the depth, the critical shear stress can be met again, at the same head. Therefore, by looking at erosion in the pipe alone, neglecting pipe tip progression, theoretically there is no unique relation between the flow and the head: for each flow rate, the pipe can adjust to meet the erosion criterion in the pipe. A criterion for the tip element is required to find the critical head. The equations also show that the relation between pipe gradient and flow rate is different for a 3D pipe than for a 2D pipe.

Progression of the tip of the pipe

Progression of the pipe can be based on different mechanisms. The traditional mechanism considered for this is the Sellmeijer approach, in which the tip pipe element is opened if the flow is sufficient to find an equilibrium condition for the particles in the pipe. This approach is therefore directly related to the erosion of the pipe mechanism that controls the depth of the pipe. A different approach is proposed by (Robbins & Griffiths, 2018) in which the tip pipe element is opened once a local critical gradient is exceeded, which is generally denoted by primary erosion. In this paper the Sellmeijer method is explored.

The Sellmeijer method evaluates the element upstream of the pipe to see if the critical shear stress can be exceeded, by gradually increasing the depth of the element. If the critical shear stress is exceeded, the element is ‘opened’ and the depth is further increased to find equilibrium. If, for all pipe depths considered, the wall shear stress does not exceed the critical shear stress, the element remains part of the sand body. The highest head drop for which an equilibrium situation

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[1] Note the Q is defined in m^2/s for 2D situations and m^3/s for 3D situations.

can be found is denoted by the critical head. In this situation, the element upstream of the pipe, at the critical length, cannot be opened.

For a 2D situation, numerical simulations have led to the Sellmeijer rule (see Equation 6), that illustrates the relation between particle characteristics (d_{70} and volume density of particles under water, γ_p'), hydraulic conductivity (k), and aquifer geometry (seepage length, L and aquifer thickness, D) (Sellmeijer et al., 2011). The relation between hydraulic conductivity and critical head (H_c) in the 2D situation illustrates that this is a non-linear process: a change in hydraulic conductivity, which in linear groundwater problems results in a linear decrease of flow towards the pipe, does not result in an equivalent decrease in critical head.

$$\begin{gathered}\frac{H_c}{L} = F_R F_S F_G \\ F_R = \eta \frac{\gamma_p'}{\gamma_w} \tan\theta \\ F_S = \frac{d_{70m}}{\sqrt[3]{\kappa L}} \left(\frac{d_{70}}{d_{70m}}\right)^{0.4}, \kappa = \frac{\nu_w}{g} k \\ F_G = 0.91 \left(\frac{D}{L}\right)^{\frac{0.28}{\left(\frac{D}{L}\right)^{2.8} - 1} + 0.04}\end{gathered} \qquad \textbf{Equation 6}$$

II- COMPUTING THE CRITICAL GRADIENT BASED ON FLOW RATE IN THE PIPE

Small-scale numerical model setup

To investigate whether the critical gradient can be computed based on 3D groundwater flow models, in which the pipe is modelled as a boundary condition, 3D numerical BEP models are constructed. The finite element groundwater model DgFlow is used for the analysis. The basis for these calculations is a very simple configuration, a cube of 1 x 1 x 1 m. The 3D calculations are conducted in half-space, which is allowed due to the symmetry of the model. More information can be found in Van Beek & Robbins (2021).

Two types of calculations are made, one using a fixed pipe length and a fixed hydraulic head, to investigate the relation between flow rate and pipe gradient. The fixed pipe has a length of 0.45 m (x=0.50 m), the erosion is artificially stopped by assigning a tenfold particle weight to the line elements upstream of the pipe. The second one evaluates pipe progression in a model that computes the critical pipe length and the critical head, to investigate the relation between pipe gradient and critical head. For this analysis, when applicable, the hydraulic conductivity, upstream head and model width are varied. The properties for the 3D models are provided in Table 1. The combination of parameters results in a critical shear stress of 0.327 Pa, according to the equation for critical shear stress in the Sellmeijer model (Sellmeijer et al, 2011).

Table 1. Cube calculation characteristics (basic configuration in bold)

Property	**Value**
Width cube (halfspace) [m]	0.25, **0.5**, 1, 2
Height cube [m]	1
Pipe length [m] (only for fixed pipe length)	0.45
Hydraulic head [m] (only for fixed pipe length)	0.10, 0.15, **0.20**
Sand hydraulic conductivity [m/s]	4.00E-4, 2.00E-4, **1.00E-04**, 5.00E-5, 1.00E-5

Sand intrinsic conductivity [m2]	4.08E-11, 2.04E-11, **1.02E-11**, 5.10E-12, 1.02E-12
Water viscosity [Pa s]	1.00E-03
Water density [kg/m3]	1000
Particle density [kg/m3]	1650
g [m/s2]	9.81E+00
Theta [-]	37
Eta [-]	0.25
d70 [mm]	0.2
w/a (halfspace)	10
M	1000
Modelfactor	1
Initial pipe depth [m]	2.00E-05

A constant head boundary is applied to the inflow area (in red) and outflow line (green). The downstream head was set to 0 and the upstream head is variable. The remaining seepage length is 0.95 m. A structured mesh was selected with mesh size of 0.05 m (Figure 2).

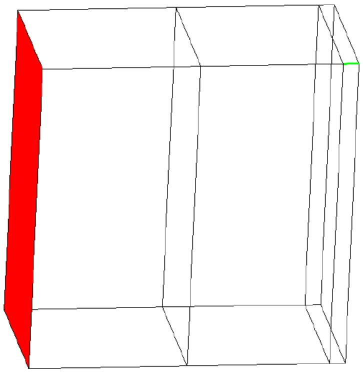
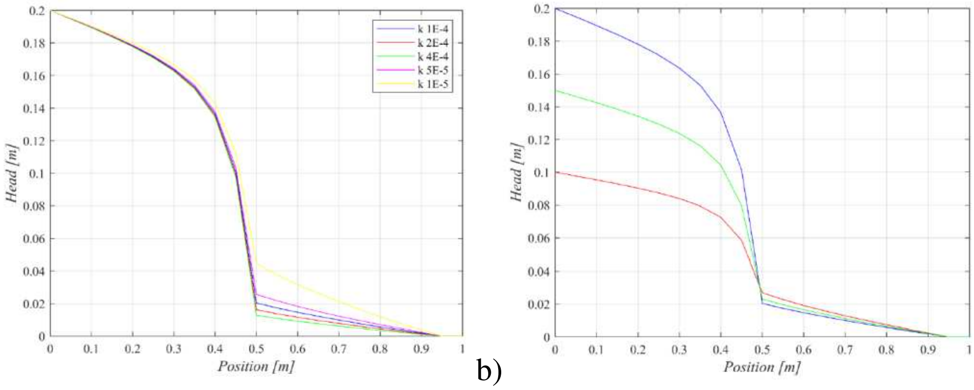


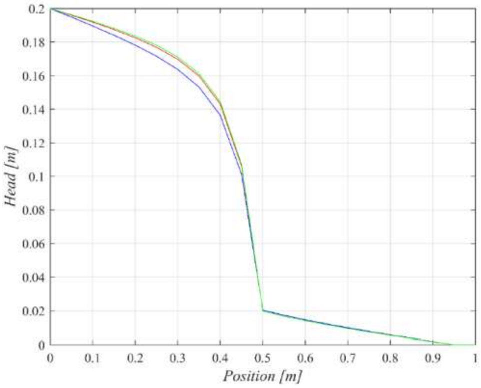
Figure 2. Geometry with boundary conditions for 3D model (half-space width 0.5 m)

Results – Is the groundwater flow in the pipe dependent on the pipe gradient and can the pipe gradient be predicted using erosion and pipe flow equations?

To answer this question, the calculations with fixed pipe length are used. The hydraulic conductivity, head drop and model width are varied to obtain different pipe gradients. Having different pipe gradients allows to investigate the relation between head drop, pipe gradient and flow rate. All the hydraulic heads along the pipe path are plotted in Figure 3.



a) b)



c)

Figure 3. Head profile with varying hydraulic conductivity (a), varying head drop (b) and varying width (blue – basic configuration, red – width increased with factor of 2, green – width increased with factor of 4) (c). The critical shear stress used in these DgFlow calculations is 0.327 Pa.

As opposed to the coupled erosion-groundwater model, in the groundwater model, the pipe is assigned as a boundary condition without head loss. In these calculations the average flow rate to the pipe (Q_{aveg}), which is assumed to be 0.5 times the total flow rate through the pipe, is linearly related to hydraulic conductivity and applied head drop. Table 2 illustrates that the 'normalized' pipe flow in BEP calculations ($Q_{aveg,norm}$), which is corrected for changes in hydraulic conductivity and head, is nearly the same for all simulations with the same geometry, despite differences in pipe gradient. This means that flow rates obtained with simpler simulations, in which the pipe is represented by a boundary condition, will provide flow rates that are similar to those obtained in BEP calculations. This is because the resistance in the pipe is much smaller than that in the sand body.

For the situations studied, the equilibrium of particles in the pipe is achieved, the pipes are all fully developed. This means that Equation 5 should be valid. Using a critical shear stress of 0.327 Pa, Equation 5 gives a constant c_{3D} of 3.23E-10 m^3/s. With this constant, the expected pipe gradient (p_{pred}) can then be computed using the numerical average flow rate over the pipe. The estimated pipe gradient is validated using the pipe gradient found in the numerical models ($\overline{p_{num}}$), see Equation 7. With l the pipe length and $\varphi(x = l)$ the hydraulic head at the tip of the pipe can be calculated conform Equation 7.

$$\overline{p_{num}} = \frac{\varphi(x=l)}{l}$$ **Equation 7**

Table 2 shows that Equation 5 provides a reasonable estimate of the pipe gradient ($\overline{p_{num}} \approx p_{pred}$) given that the flow rate through the pipe is known. This analysis has also been performed with 2D BEP models, the same conclusions were found.

Table 2. Results of predicted and numeric pipe gradient

	Head drop [m]	Hydraulic conductivity [m/s]	Width (full-space) [m]	$Q_{aveg,num}$ [m^3/s]	$Q_{aveg,num,norm}$ [m^3/s]	$\overline{p_{num}}$ [-]	p_{pred} [-]
Cube1	0.2	1E-4	1	3.58E-06	3.58E-06	0.044	0.045
Cube2	0.2	2E-4	1	7.25E-06	3.63E-06	0.036	0.035
Cube3	0.2	4E-4	1	1.46E-05	3.65E-06	0.029	0.028

Cube4	0.2	5E-5	1	1.77E-06	3.54E-06	0.058	0.057
Cube5	0.2	1E-5	1	3.36E-07	3.36E-06	0.100	0.099
Cube6	0.1	1E-4	1	1.63E-06	3.26E-06	0.060	0.058
Cube7	0.15	1E-4	1	2.61E-06	3.48E-06	0.051	0.050
Cubew2_1	0.2	1E-4	2	3.94E-06	3.94E-06	0.044	0.043
Cubew4_1	0.2	1E-4	4	4.01E-06	4.01E-06	0.044	0.043

Results – Can the critical head be determined based on pipe gradient?

We have seen before that once the flow rate is known, the pipe gradient can be determined, see Equation 5. To analyze if this pipe gradient can then be related to the critical gradient, the calculations with pipe progression are used. For groundwater calculations, the flow rate through the aquifer (Q) is linearly related to the hydraulic head and the hydraulic conductivity by Equation 8. A_{eff} is a fictive area, that represents the flow area for the situation in which flow lines are parallel, H is the head drop and L is the seepage length. The effective area (A_{eff}) can be determined based on a simple 3D groundwater flow calculation.

3D: $$Q = k\frac{H}{L}A_{eff}$$ **Equation 8**

Numerical simulations are made for different hydraulic conductivities and model widths to simulate different pipe gradients. For each model, the relationship between pipe gradient and critical gradient is analyzed. Figure 4 shows the results, for the numerical models made, the pipe gradient (p) and the overall critical gradient (i_c) ratio is approximately constant (c_2). This leads to Equation 9.

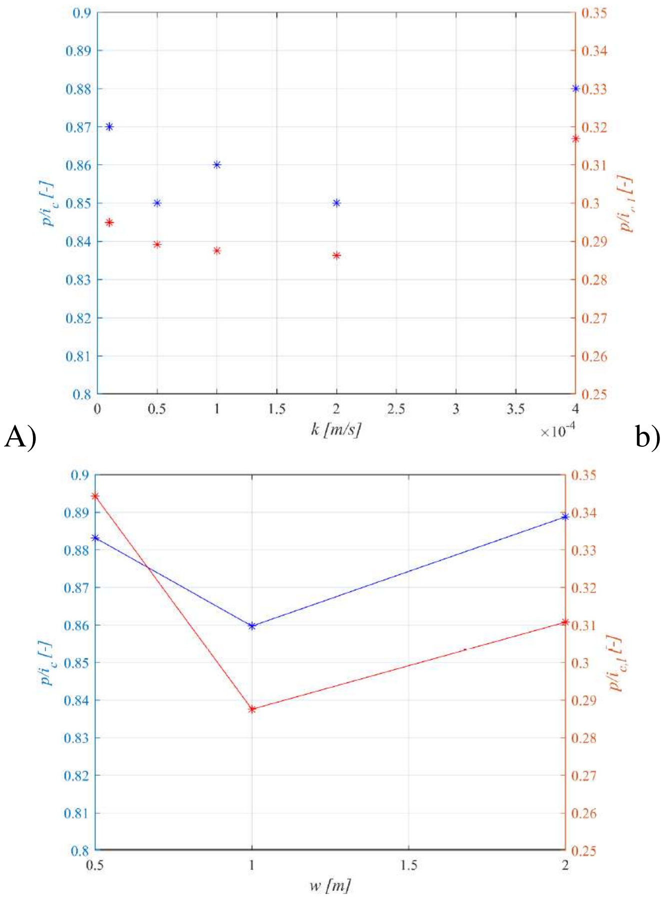


Figure 4. Pipe gradient divided by overall critical gradient and by local critical gradient for different hydraulic conductivity (a) and different widths (b).

$$\frac{p}{i_c} = c_2$$ **Equation 9**

It is possible that the value of c_2 is similar as the considered configurations are also similar. The value of c_2 in this equation will likely depend on the geometry. Although not studied here, it is expected that variations in leakage length will also influence c_2. Assuming that the value of c_2 is known, combining Equation 5 and 9 leads to Equations 10. Equations 10 was used to calculate the critical gradient for the cases considered, as shown in Figure 5.

3D: $$i_c = \frac{1}{c_2}\sqrt[3]{\frac{c_{3D}}{Q_{pipe,aveg}}}$$ **Equation 10**

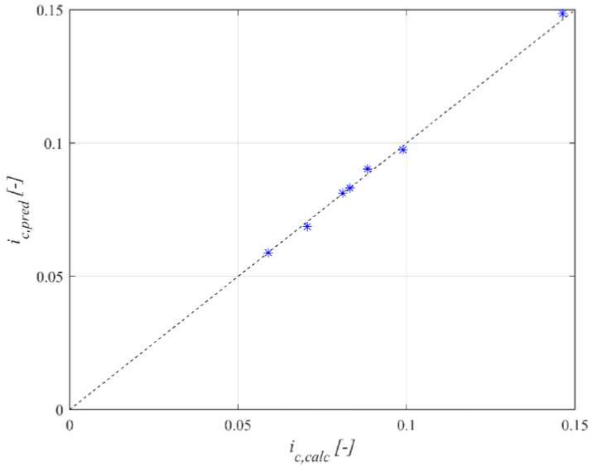


Figure 5. Predicted critical gradient versus calculated critical gradient

Combining equation 5, 8 and 10, and if the average flow rate through the pipe equals half of the total flow rate through the pipe used in Equation 8, leads to Equation 11. In analogy to what is done for 3D configurations, the same can be done for 2D configurations, leading to Equation 12. For 3D configurations, like the 2D Sellmeijer rule, the critical shear stress linearly relates to the critical gradient. In contrary to the 2D rule, the effect of intrinsic permeability is included through an inverse quadratic root, rather than an inverse cube root. The influence of the effective area suggests that the model width plays a role as well. Finally, the w/a ratio has an important effect on the critical gradient.

3D: $$i_c = \frac{2\tau_w}{\rho_w g \sqrt[4]{\frac{1}{\kappa A_{eff} 6 {c_2}^3}\frac{w}{a}}}$$ **Equation 11**

2D: $$i_c = \frac{2\tau_w}{\rho_w g \sqrt[3]{\kappa D_{eff} 6 {c_2}^2}}$$ **Equation 12**

Therefore, considering the parameter c_2, which is unknown, the critical gradient cannot be directly determined based on flow rate from groundwater models. Equations 11 and 12 do provide insight into the effect of parameters on critical gradient according to the 3D Sellmeijer model.

III - FIELD-SCALE GROUNDWATER FLOW MODELS

Numerical model

It is shown that the average discharge in the pipe affects the critical gradient, therefore, it is further investigated what parameters affect the discharge to the pipe given a 3D configuration. The effect of aquifer thickness, model width, scale, aquifer permeability, anisotropie and hinterland permeability are investigated using a 3D groundwater model with DgFlow and a boundary condition to stimulate the pipe. The pipe has a fixed length which is 30% or 60% of the seepage length. The effect of scale is investigated by scaling the model by a factor 0.35, 2, 10, 40 and 100 smaller. More information can be found in Wopereis et al. 2022.

2D and 3D groundwater flow models were constructed to get an overview of the so-called 3D effect ($= Q_{pipe,3D}/Q_{pipe,2D}$). The basis for these calculations is a typical field scale configuration found in practice, see Figure 6. The 3D models are half-space models. The following mesh properties are used: 0.1 m at the tip of the pipe, 0.25 m at the outflow, 1 m at the hinterland and 5 m everywhere else. The properties of the models are provided in Table 3.

Table 3. Characteristics of groundwater flow models (basic configuration in bold)

Property	**Value**
Model width (half-space) [m]	25, **50**, 105, 200
Aquifer thickness [m]	10, **25**, 40, 80
Sand hydraulic conductivity [m/day]	15, **30**, 60
Seepage length [m]	**100**
Pipe length in percentage of seepage length [%]	**0.3 or 0.6**
Anisotropie [-]	**1**, 2, 5, 10
Length of outflow well	**1**
Width of well (half-space) [m]	**0.5**
Hinterland length [m]	**600**
Hinterland thickness [m]	**2**
Hinterland leakage length [m]	50, 200, **100.000**
Hydraulic head [m]	**1**

A constant head boundary is applied to the inflow area (blue) and outflow (red) (Figure 6). The downstream head was set to 0 and the upstream head to 1 m. The seepage length is 100 m.

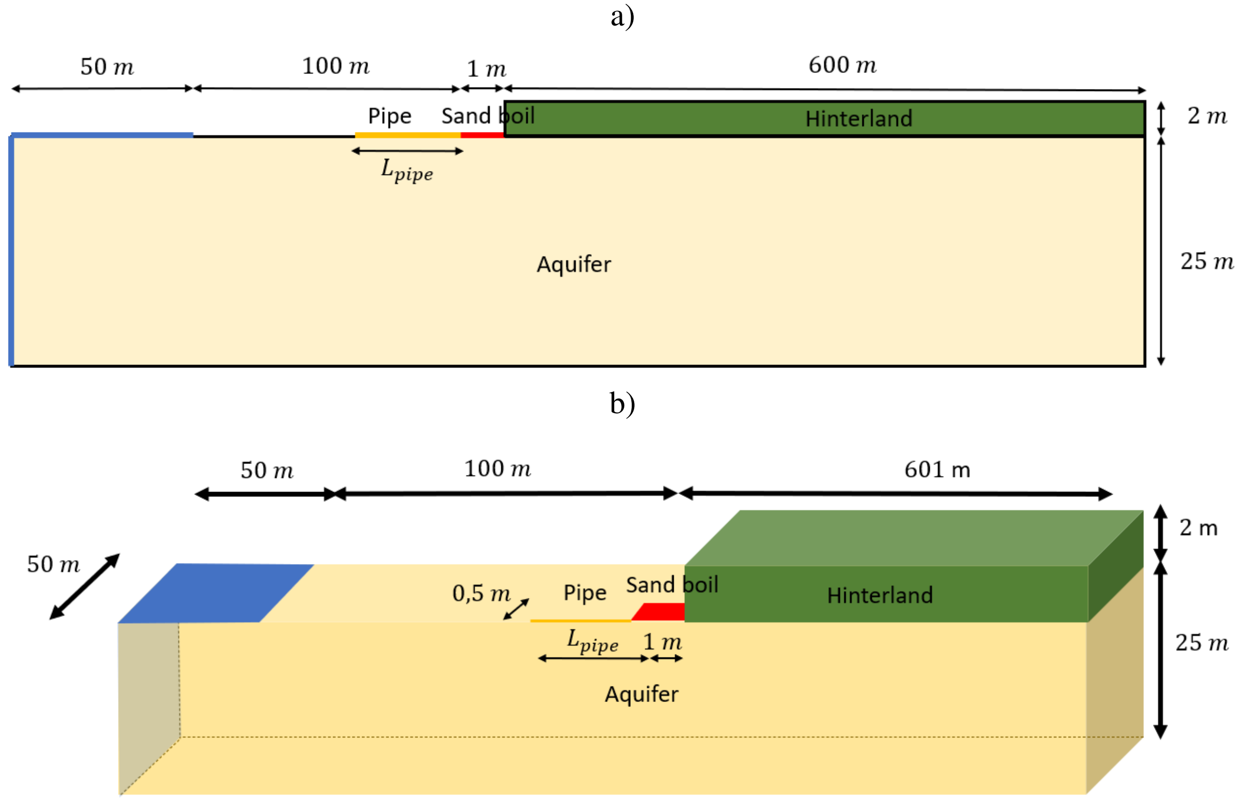
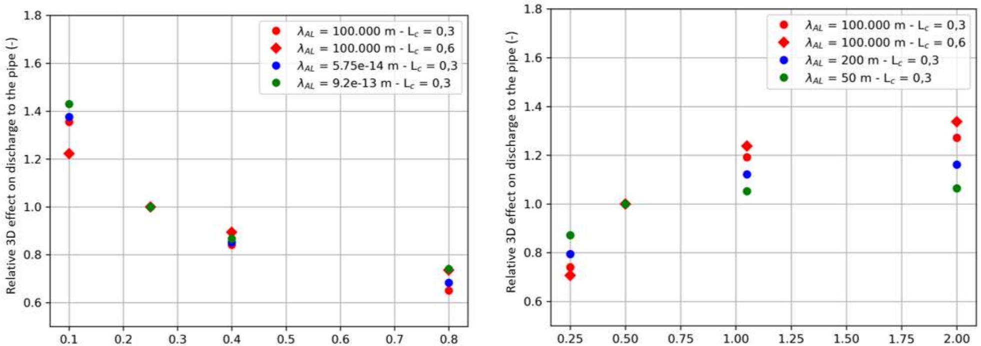


Figure 6. Geometry with boundary conditions for 2D and 3D model

Results - Field scale models

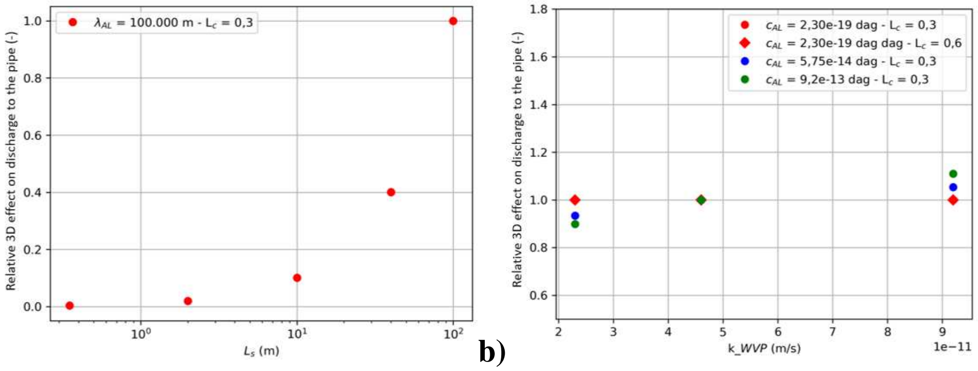
To enable immediate comparison between the effect of different parameters, a so called 'relative effect' is computed for each variant. The relative effect is the 3D effect on flow rate of a variant divided by the 3D effect of the base configuration, see Equation 13. Figure 7 gives the relative 3D effect of different aquifer thickness and model width (for model width the base configuration has a half-space width of 200 m instead of 50 m). Figure 8 gives the results for scale (plotted based on the seepage length) and aquifer permeability. Finally, Figure 9 for anisotropie and hinterland permeability.

$$\frac{\left(Q_{pipe,3D}/Q_{pipe,2D}\right)_{variant}}{\left(Q_{pipe,3D}/Q_{pipe,2D}\right)_{base}} \qquad \textbf{Equation 13}$$



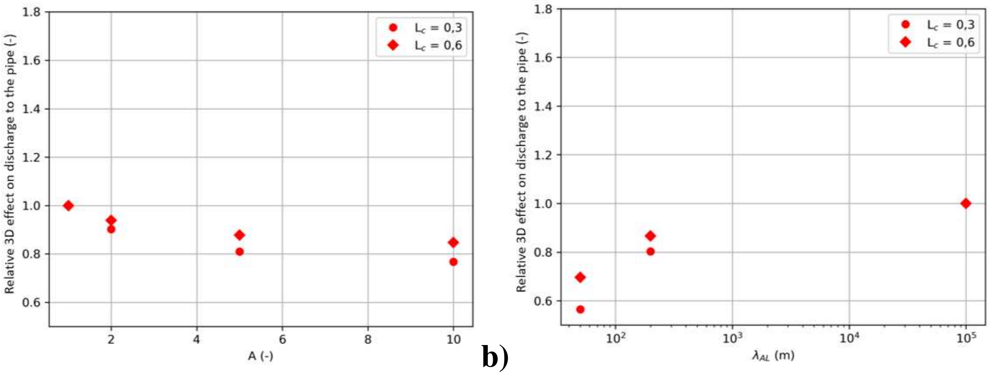
a) **b)**

Figure 7. Relative 3D effect of aquifer thickness (a) and model width (b)



a) **b)**

Figure 8. Relative 3D effect of scale represented by L (constant D/L) (a) and aquifer permeability (b)



a) **b)**

Figure 9. Relative 3D effect of anisotropie (a) and hinterland leakage length (b)

Figure 7, 8 and 9 illustrate that anisotropy, hydraulic conductivity and pipe length have a relatively small effect on the relative 3D effect. The effect of aquifer geometry (scale and shape) and leakage to the hinterland have a large influence on the relative 3D effect.

CONCLUSION

2D and 3D small-scale numerical simulations were conducted with the Sellmeijer model to relate pipe gradient and critical head. For different hydraulic conductivities and model widths, the relation between pipe gradient and critical head, according to the Sellmeijer model was found to be constant. Using this concept and underlying theory, the effect of material properties on the critical head as is present in the 2D rule was confirmed and an equation was derived for 3D models. However, in this equation three unknowns are present, the value of c_2, that represents the ratio of critical gradient and critical pipe gradient, w/a, which represents the ratio of pipe width and depth, and the effective area. The value of c_2 was found to be similar for the considered cube simulations, but probably depends on geometry and leakage length. The value of w/a is still subject of investigation. In this report a value of 20 (full model width) was chosen, based on simulation of experiments. The effective area can be determined based on a simple 3D groundwater flow calculation.

The derived equations presented here are applicable only when the 3D Sellmeijer model is valid, for which no evidence is yet available (Van Beek et al., 2022). It is recommended to investigate the effect of the use of a pipe tip progression criterion based on a local critical gradient. The use of a local critical gradient will yield significantly different results since the critical head is not dictated solely by the flow rate. It is recommended to investigate the value of c_2 for other configurations (depth, width, leakage length etc.).

The field scale models show that the most important parameters affecting 3D effects for BEP are the scale of the model and the width to seepage length ratio. Other parameters such as anisotropy and permeability of the aquifer have a smaller influence on 3D effects.

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