Quantification of the Seepage-Induced Internal Erosion

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ABSTRACT
Internal erosion, the migration of fine particles within a soil matrix due to seepage flow, is a critical process that can compromise the stability of geotechnical structures such as dams, levees, and embankments. This study investigates the effects of confining pressure, initial fines content, and flow velocity on the erosion process and the resulting changes in soil properties. Through a series of laboratory seepage tests, the evolution of fines content, void ratio, and volumetric strain under different conditions is analyzed. A predictive equation for fines content is proposed, incorporating the influence of initial fines content, normalized flow velocity, and normalized confining pressure. Additionally, the relationship between cumulative fines loss and erosion-induced volumetric strain is quantified using a hyperbolic tangent function, providing a robust model for predicting volumetric changes. The post-erosion void ratio is estimated by considering both cumulative fines loss and volumetric strain, offering a comprehensive framework for understanding the mechanical behavior of internally eroded soils. The results highlight the significant role of confining pressure in controlling fines loss and the heterogeneity of eroded soils along the flow direction. This study provides valuable insights for assessing and mitigating internal erosion in geotechnical structures, contributing to the development of more resilient infrastructure.

1. Introduction

Internal erosion, the process by which fine particles are transported within a soil matrix due to seepage flow, poses a significant threat to the stability of geotechnical structures such as dams, levees, and embankments (Razavi *et al*., 2020; Liu *et al*., 2025). The migration of fines can lead to changes in soil structure, including alterations in particle size distribution, void ratio, and volumetric strain, which may compromise the integrity of these structures (Wang *et.al*. 2020; 2021). While extensive research has been conducted to understand the effects of factors such as initial fines content (Chang and Zhang, 2013), hydraulic gradient, and flow direction (Marot et al., 2016) on the erosion process, the influence of confining pressure and the quantification of erosion-induced volumetric changes remain less explored. This study aims to address these gaps by providing a detailed analysis of the erosion process in gap-graded soils under varying conditions.
The primary objectives of this research are threefold: (i) to investigate the evolution of fines content during erosion, considering the effects of confining pressure, initial fines content, and flow velocity; (ii) to quantify the relationship between cumulative fines loss and erosion-induced volumetric strain; and (iii) to estimate post-erosion void ratios based on experimental data. A predictive equation for fines content is proposed, incorporating the influence of normalized flow velocity and normalized confining pressure, which accurately captures the key features of the erosion process observed in laboratory tests. Additionally, a hyperbolic tangent function is introduced to model the relationship between cumulative fines loss and volumetric strain, providing a robust framework for predicting the mechanical behavior of eroded soils.

The experimental results demonstrate that confining pressure plays a critical role in controlling the loss of fines, with higher pressures resulting in reduced erosion. Furthermore, the heterogeneity of eroded soils along the flow direction is observed, with fines loss and void ratio changes varying linearly with distance from the top of the specimen. These findings underscore the importance of considering confining pressure and flow direction in the assessment of internal erosion. The proposed predictive models offer practical tools for engineers to evaluate the stability of soils subjected to seepage flow, contributing to the development of more resilient geotechnical structures.

2. Predictive equation of the fines content during the erosion process

When the seepage flow is applied to the unstable soils, the variation of fines contents under different confining pressures against elapsed time could be obtained. As shown in Fig. 1(a), the fines contents decrease with the continuing inflow and tend to converge to certain values (Ke and Takahashi, 2014). It can also be observed that a specimen under higher confining pressure has less loss of fines. Figure. 1(b) shows that final fines content is a monotonic increase function of the mean effective stress (soils with 35% initial fines content under different confining pressures 50 kPa, 100 kPa, and 200 kPa). The fines will be difficult to be eroded when the confining pressure is high. Fines are expected to be eroded mostly when the confining pressure is close to zero, under which there is no external constraining force preventing fines from transporting.

Figure 2 shows the change of fines content with different initial fines contents (15%, 25%, 35%) under 50 kPa confining pressure (Ke and Takahashi, 2014). The fines content decreases with the elapsed time and finally tended to converge to a certain value. The erosion rate depends on the initial fines content. The specimen with larger initial fines content has a larger erosion rate.

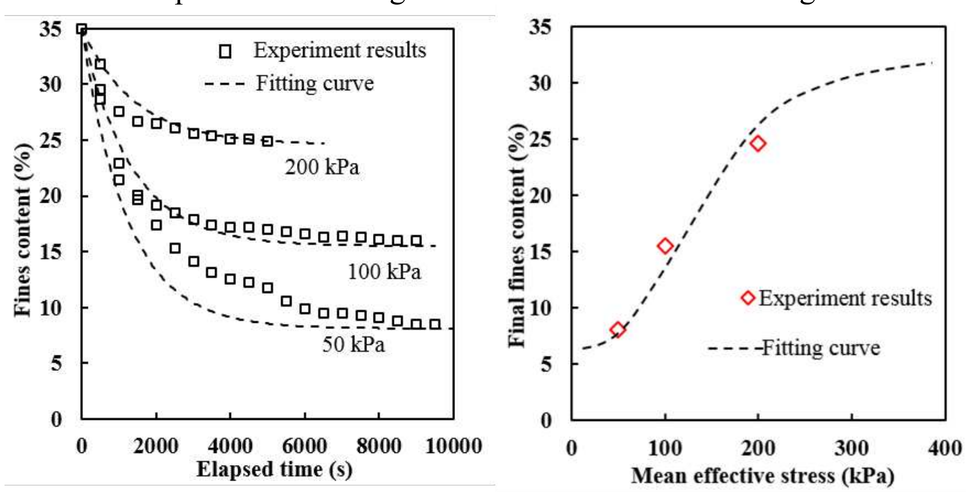

Figure 1 Change in fines contents (a) along with time under different confining pressures (b) under different confining pressures (Experimental data from Ke and Takahashi, 2014)

Cividini *et al.* (2009) regarded the decrease of non-dimensional density of fines as the loss of fines. The long-term non-dimensional density of fines was a function of the initial non-dimensional density of fines and hydraulic gradient. The change of fines can be expressed as the variation of the fines content, the equation of the final fines content considering the effect of initial fines content and the hydraulic gradient is proposed as:

$$FC_\infty(FC_0, i) = FC_0 \cdot [(1 - d_1)exp(-a_1 \cdot i^{b_1}) + d_1] \quad (1)$$

where a_1, b_1, and d_1 are fitting parameters.
However, in this model, the effect of confining pressure has not been considered. Therefore, in this study, the effect of the hydraulic gradient is replaced by the effect of flow velocity. Also, the effect of confining pressure on the variation of fines content is studied. In the formulation, the flow velocity is normalized by reference velocity, and confining pressure is normalized by reference confining pressure, where reference velocity v_{ref} equals to 0.0001 m/s and reference confining pressure p_{ref} equals to 1 kPa. Final fines content $FC_\infty(FC_0, v_{nor}, p_{nor})$ is constructed as a function of initial fines content FC_0, normalized flow velocity v_{nor}, and normalized confining pressure p_{nor}.

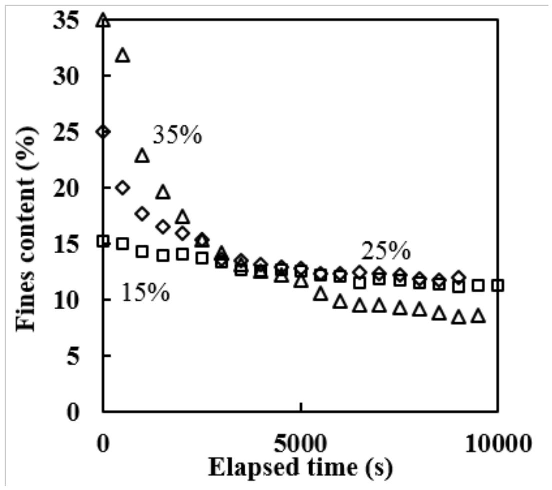

Figure 2 Fines content against elapsed time with different initial fines contents (15%, 25%, 35%) under 50 kPa confining pressure (Data from Ke and Takahashi, 2014)

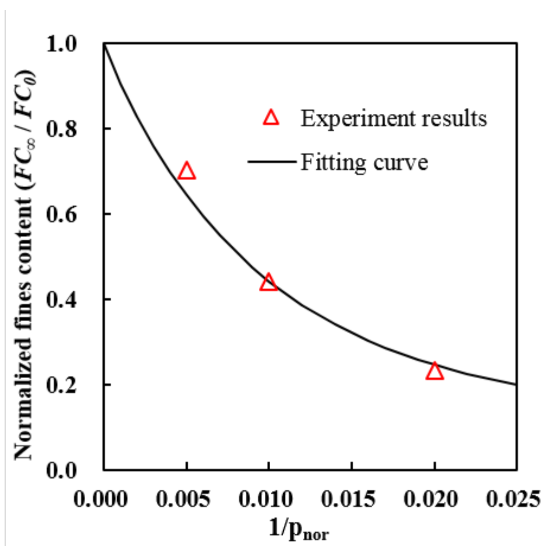

Figure 3 Normalized finial fines content along the reciprocal of normalized confining pressure (Experimental data from Ke and Takahashi, 2014)

Experimental data in Fig. 3 show the change of the normalized final fines content FC_∞/FC_0 along with the parameter $1/p_{nor}$ for specimens with 35% initial fines content and different confining pressures (50 kPa, 100 kPa, and 200 kPa). In the figure, the normalized final fines content decreases with the increase of $1/p_{nor}$, and the normalized final fines content equals one when

$1/p_{nor}$ converges towards zero (which means the confining pressure is very high). The equation of the final fines content is proposed as:

$$FC_\infty(FC_0, v_{nor}, p_{nor}) = FC_0 \cdot \left[(1-d_1)exp\left(-a_1 \cdot (v_{nor})^{b_1} \cdot (\frac{1}{p_{nor}})^{c_1}\right) + d_1\right] \qquad (2)$$

where a_1, b_1, c_1, and d_1 are fitting parameters. From the fitting of the experimental data (Fig. 3), a_1, b_1, c_1, and d_1 are taken as 6.5, 0.95, 0.95, and 0.12 for the soils with 35% initial fines contents under different confining pressures (50 kPa, 100 kPa, and 200 kPa).

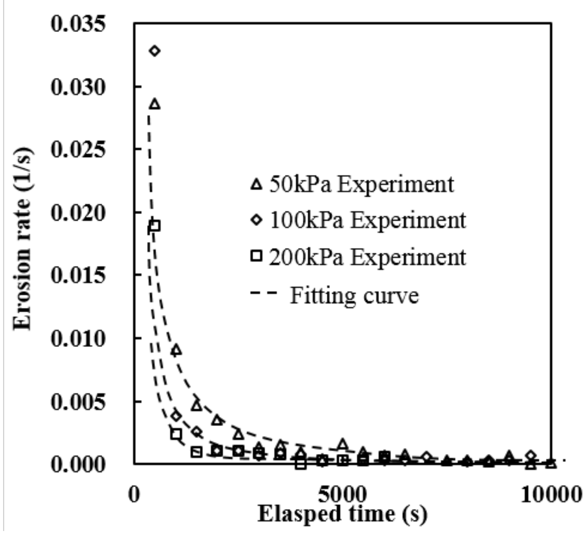

Figure 4 Trends of erosion rate with elapsed time under different confining pressures (Experimental data from Ke and Takahashi, 2014)

Figure 4 shows the trend of erosion rate along elapsed time for the specimens under different confining pressures. The erosion rate denotes the fines content change per unit time. The erosion rate follows the conditions: (a) for all cases with different confining pressures, it decreases monotonically with time and finally tends to zero; and (b) it decreases with the increase of confining pressure. Previous research indicated that the erosion rate also depended on the root of the hydraulic gradient (Cividini *et al.*, 2009). The change of hydraulic permeability was small after the onset of internal erosion, which was assumed to be unchanged during erosion for simplification. Therefore, the hydraulic gradient could be replaced by flow velocity (Bowman and Hunter, 2017). Consequently, the following equation of current fines content $FC(FC_0, v_{nor}, p_{nor}, t)$ is proposed:

$$\frac{\partial FC(FC_0, v_{nor}, p_{nor}, t)}{\partial t} = -e_1 \cdot (v_{nor})^{0.5} \cdot \left(\frac{1}{p_{nor}}\right)^{e_2} \cdot (FC - FC_\infty) \qquad (3)$$

Based on this, the function of FC can be obtained by integration:

$$FC = (FC_0 - FC_\infty) \cdot \exp\left[-e_1 \cdot (v_{nor})^{0.5} \cdot \left(\frac{1}{p_{nor}}\right)^{e_2} \cdot t\right] + FC_\infty \qquad (4)$$

where e_1=0.00035 and e_2=0.12, the predictive equation of Eqn. (4) can capture the features of the experimental results under different confining pressures (see Fig. 1). At the same time, if the final fines content is known for the specimens with 15%, 25%, and 35% initial fines contents under 50 kPa confining pressure, Eqn. (4) can also estimate the variation of the fines content for the soils with different initial fines contents during internal erosion (see Fig. 2). Eqns (2) and (4), considering the effect of flow velocity and confining pressure, are suitable for the prediction of the fines content of the eroded soils obtained through the loss of fines under the seepage flow, but not suitable for the prediction of the fines content of the eroded soils obtained through the salt dissolution.

3. Estimation of the erosion-induced volumetric strain

The erosion-induced change in volume of the soils under seepage flow was found in some experiments (Xiao and Shwiyhat 2012; Ke and Takahashi, 2014, 2015; Chen *et al.*, 2016). However, no volume change also happened when soils were subjected to the seepage flow (Fannin and Slangen, 2014; Li *et al.*, 2020). The possible explanation may be that the soils are constituted by two parts: the stable skeleton (mainly formed by coarse particles) and the migratable particles that do not contribute to the stress transmission (mainly fines). When the cumulative fines loss is small, or the skeleton is competent enough, the volume may be unchanged even the internal erosion occurs due to the seepage flow (suffusion). Contrarily, when the loss of the fines is large, or the skeleton collapses by the large seepage force, the volume may change dramatically (suffosion).
It is important to find the relation between erosion-induced volumetric strain and the cumulative fines loss for the modeling of the internally eroded soil behavior. As the maximum cumulative fines loss exists for any binary mixture under seepage flow and both coarse particles and fines are nearly incompressible, the maximum erosion-induced volumetric strain $\varepsilon_{v\max}^{er}$ may also exist for the soils subjected to the seepage flow.

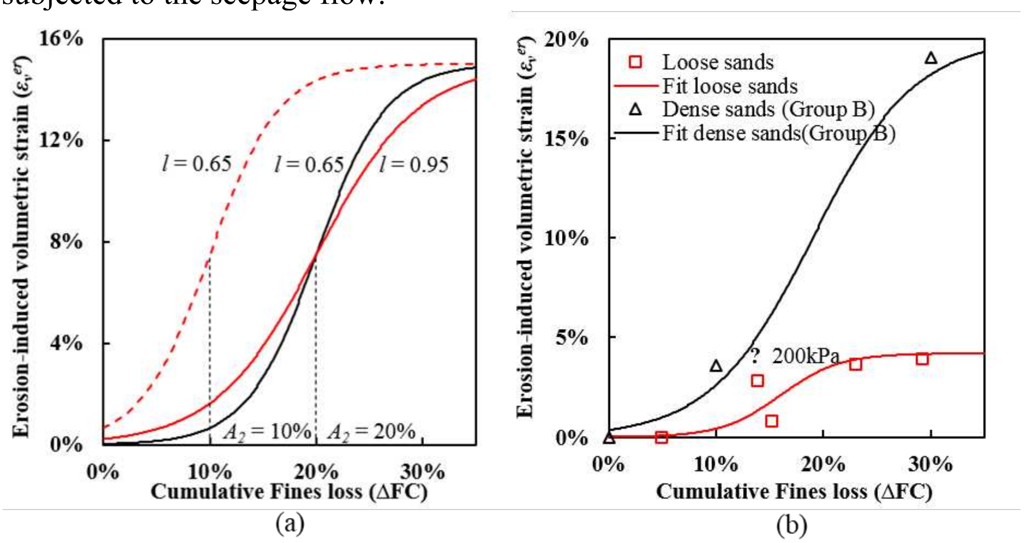

Figure 5 Erosion-induced volumetric strain against cumulative fines loss (a) Change of the hyperbolic tangent function with A_2 and l (b) Fitting of erosion-induced volumetric strains of both the loose and dense soils (Experimental data of loose sand from Ke and Takahashi, 2014; Experimental data of dense sand from Chen *et al.*, 2016)

In this paper, the variation of the erosion-induced volumetric strain from two cases is investigated. For the loose soils, the experiments conducted by Ke and Takahashi (2014) were analyzed and for dense soil experiment by Chen *et al.* (2016) were investigated. Generally, the erosion-induced volumetric strain of dense soils is expected to be smaller than that of the loose soils. However, the erosion-induced volumetric strain of dense soils is much larger than that of the loose soils in this study (Fig. 5). The explanation is that the salt is used to mimic the erosion of the dense soils and the salt dissolution can also cause a decrease in the soil volume. Here, it is assumed that depending on the cumulative fines loss ΔFC, the erosion-induced volumetric strain varies from 0 to the maximum volumetric strain $\varepsilon_{v\max}^{er}$. The equation of the erosion-induced volumetric strain is proposed as:

$$\varepsilon_v^{er} = \frac{1}{2}\varepsilon_{v\max}^{er}\left(1+\tanh\left(\frac{1}{l}(\Delta FC - A_2)\right)\right) \tag{5}$$

where A_2 is the threshold, l is a parameter deciding the smoothness of the fitting curve, the curve is much smoother when the value of l is larger (Fig. 7(a)). From the fitting of the experimental data (see Fig. 7(b)), $\varepsilon_{v\max}^{er}$, A_2, l are taken as 20%, 19%, and 0.095 for dense soils (Group B, Chen *et al.*, 2016); 4.2%, 16%, and 0.055 for loose soils (Ke and Takahashi, 2014). The range of the fitting parameter l is suggested to be $0 < l < 0.1$. When $l > 0.1$, it is difficult to predict the phenomenon that volume does not change when the cumulative fines loss is small through Eqn. (5). The internal erosion occurs when the soils are unstable, which suggests that Eqn. (5) is suitable for the most gap-graded soils and the eroded soils obtained through salt dissolution.
The erosion-induced volumetric strain is also affected by the confining pressure, but this effect is not considered in Eqn. (5). From the fitting curve, the erosion-induced volumetric strain is almost zero when the cumulative fines loss is less than 5% for loose soils. When the cumulative fines loss is more than 25%, the volumetric strain of loose soils shows almost the greatest value but becomes insensitive to the amount of the loss of fines. The change of volumetric strain for dense soils has a similar trend.

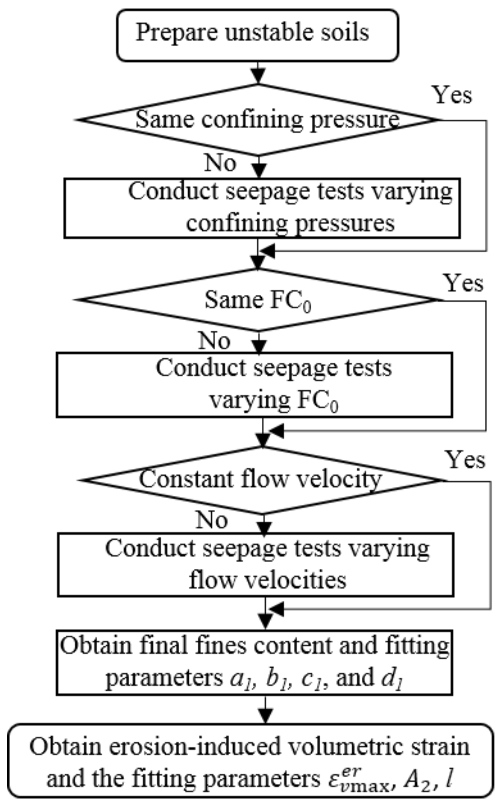

Figure 6 Flowchart for determining required seepage tests and erosion parameters

Figure 6 shows the seepage tests required for the determination of the erosion parameters. The number and type of seepage tests depend on the many conditions (e.g., confining pressure, initial fines content, and flow velocity). When only confining pressures are different, a series of seepage tests under different confining pressures with the same initial fines contents and constant flow velocity need to be conducted. However, when initial fines contents and flow velocities change, more seepage tests considering the variations of initial fines contents and flow velocity need to be conducted.

4. Estimation of the post-erosion void ratio

The void ratio of the specimen increases after the seepage test. Sterpi (2003) divided the total specimen into voids and solid and proposed three hypotheses about the variation of void ratio and volumetric strain after the seepage test: (1) the total volume of the specimen kept constant, which noted that the volumetric strain was zero. Eroded fines could cause the increase of voids and the decrease of the solid; (2) all eroded fines were washed out while the voids did not change, which caused the variation of volumetric strain; (3) the void ratio of the specimen were unchanged with the loss of both the voids and the solid (Fig. 7). The specific gravities of both fines and coarse particles are assumed to be the same, and then the percentage by volume of eroded particles can be expressed as the percentage by mass of eroded particles (ΔV_s=ΔFC).

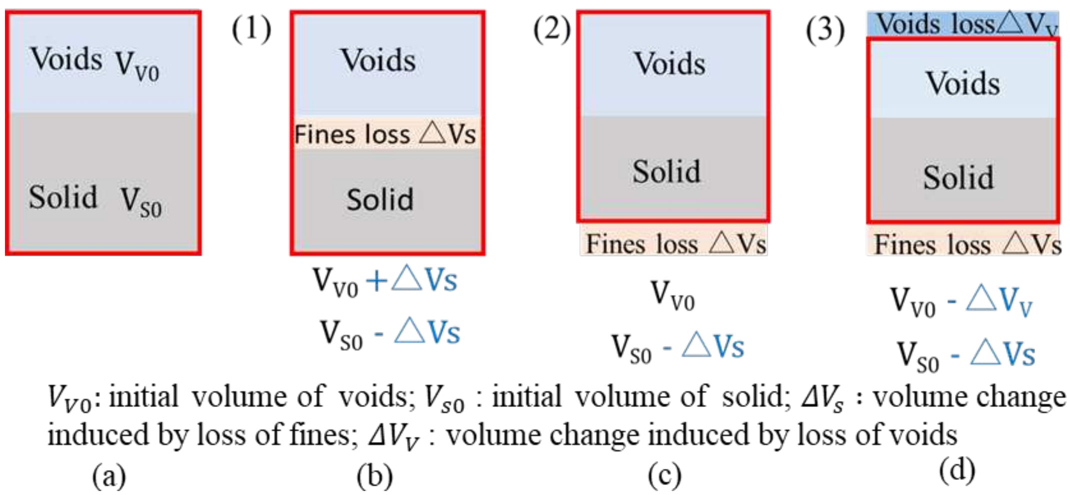

Figure 7 Change in void ratio and strain (a) initial condition (b) based on hypothesis 1 (c) based on Hypothesis 2 (d) based on Hypothesis 3

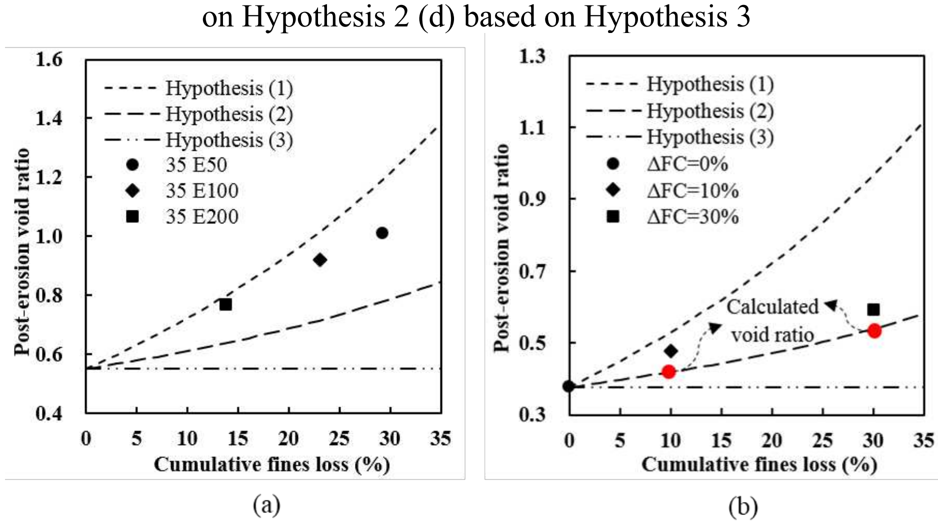

Figure 8 Change in the post-erosion void ratio with different initial fines contents along with the cumulative fines loss (a) loose sand with initial 35 % fines content (Experimental data from Ke and Takahashi, 2014) (b) dense soils (Group B, Experimental data from Chen *et al.*, 2016)

As mentioned above, the specimens with higher confining pressure have fewer eroded fines, which results in relatively smaller fines content variation. The trends of the void ratio change of the internally eroded soils along different cumulative fines loss are plotted based on three hypotheses (Fig. 8).

The experimental results drop between prediction curves obtained through hypotheses (1) and (2), while hypothesis (3) shows that no change of void ratio happens with the increase of the cumulative fines loss. Compared with the prediction curve by hypothesis (2), experimental results are closer to that by hypothesis (1) for most cases. The equation used in hypothesis (1) is as below:

$$\mathrm{e}_{H1} = \frac{e_c + \Delta FC}{1 - \Delta FC} \tag{6}$$

where e_c is the void ratio after the consolidation, e_{H1} is the post-erosion void ratio calculated based on hypothesis (1). For the case (Group B, Chen *et al.*, 2016), the experimental results are closer to prediction results calculated by hypothesis (2), which are underestimated (see Fig 8(b)).

The prediction curves for hypothesis (1) are closer to the experimental results but overestimate for most cases, which results from the ignorance of the effect of the volumetric strain. If we know the erosion-induced volumetric strain, we can estimate the post-erosion void ratio (Ke and Takahashi, 2014). Then the equation considering the effect of the volumetric strain is as follow:

$$e_{er} = (1 - \varepsilon_v^{er})\left(\frac{e_c + \Delta FC}{1 - \Delta FC}\right) - \varepsilon_v^{er} \tag{7}$$

where ΔFC is also regarded as a percentage by volume when the specific gravities of both the coarse particles and fines are the same. The post-erosion void ratios from both experimental and prediction results calculated by Eqn. (7) are plotted in Fig. 9, from which we can know that Eqn. (7) can be used to estimate the post-erosion void ratios by considering the cumulative fines loss and erosion-induced volumetric strain, i.e., Eqn. (7) is suitable for the prediction of post-erosion void ratio for the gap-graded soils and the eroded soils obtained after the salt dissolution. For the dense soils with 30% cumulative fines loss, the calculated post-erosion void ratio is larger than that from the experiment (question mark in Fig. 9(a)). This discrepancy could be attributed to the post-erosion void ratio of soils with 30% cumulative fines loss was closer to the prediction curve calculated by hypothesis (2) (Fig. 8(b)). The seepage scenario of the hypothesis (2) is that the loss of fines (dissolution of the salt) does not increase the voids dramatically, but decreases the solid.

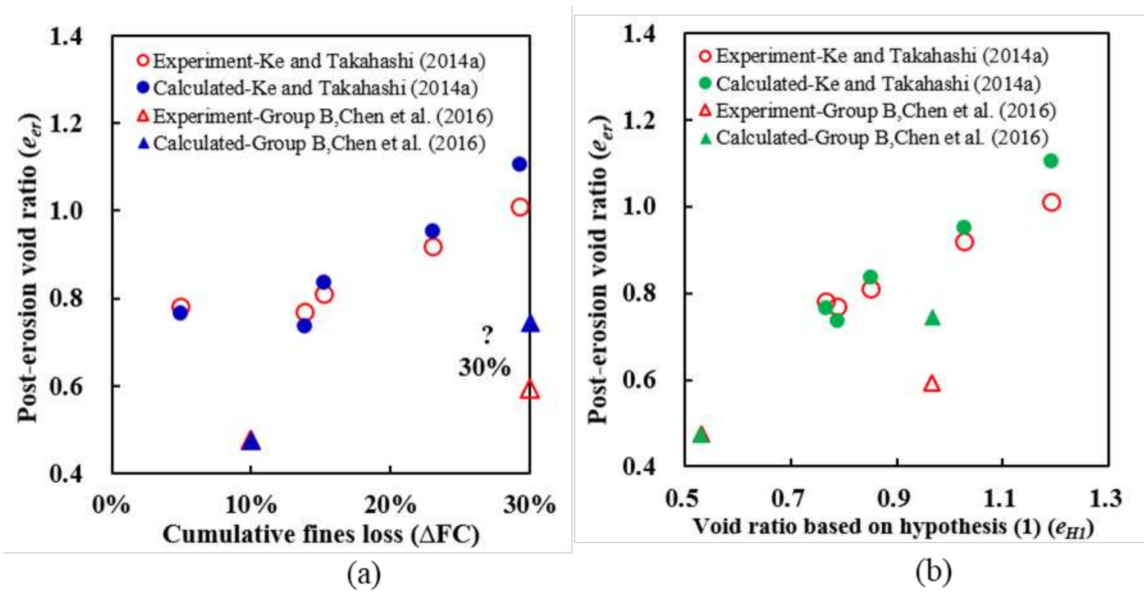

Figure 9 Post-erosion void ratios comparison between experimental and calculated results (a) Variation with cumulative fines loss (b) Variation with void ratio e_{H1}

5. Conclusions

This study has provided a comprehensive analysis of the internal erosion process in gap-graded soils, focusing on the effects of confining pressure, initial fines content, and flow velocity on the evolution of soil properties. Through a series of laboratory seepage tests, the following key conclusions can be drawn:

i. Higher confining pressure reduces fines loss, limiting particle migration.
ii. A fines content equation incorporating initial fines, flow velocity, and confining pressure accurately predicts erosion behavior.
iii. A hyperbolic tangent function models erosion-induced volumetric strain, showing negligible strain at low fines loss and saturation at high loss.
iv. Post-erosion void ratio is reliably estimated using cumulative fines loss and volumetric strain.
v. Fines loss and void ratio vary linearly along the flow direction, with gravity influencing particle movement.

In conclusion, this study advances our understanding of the internal erosion process in gap-graded soils, offering new insights into the effects of confining pressure, initial fines content, and flow velocity on soil properties. The proposed predictive models for fines content, volumetric strain, and post-erosion void ratio provide a robust framework for analyzing and mitigating the risks associated with internal erosion in geotechnical engineering applications. Future research should focus on further validating these models under a wider range of conditions and exploring their applicability to other types of soils and erosion mechanisms.

6. References

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