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Impact of Nondiagonal Elements of Hydraulic Conductivity Tensor on Seepage through Anisotropic Soils

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ABSTRACT

According to Darcy, Darcy's velocity is proportional to the hydraulic gradient via hydraulic conductivity. Darcy's law was initially developed to calculate the one-dimensional flow velocity through porous media. Darcy's law has since been generalized by the scientific community to analyse 2D and 3D seepage flow. Because hydraulic gradient, and Darcy's flow velocity are vectors hydraulic conductivity needs to be treated as a scalar for isotropic soils and or a tensor for anisotropic soils. In general, the tensor has three categories of major, intermediate, and minor principal hydraulic conductivity scalar values where the minor principal orientation is normal and the major and intermediate principal ones are aligned within the soil stratigraphic plane. Since, the soil is generally axisymmetric on mesoscale, within stratigraphic planes major and intermediate principal hydraulic conductivities are generally equal. In a case where the seepage is analysed in the orthogonal system of coordinates aligned with the three principal orientations of the tensor (e.g., stratigraphic and XY planes are both horizontal), the tensor can be assumed diagonal. In this paper, a 2D finite-difference numerical model developed using the MATLAB interface, capable of simulating seepage through anisotropic soils employing a full nondiagonal hydraulic-conductivity tensor, to examine and analyse the characteristic impacts of considering nondiagonal tensors due to various degrees of stratigraphic tilt with respect to system of coordinates.

Keywords: Seepage flow, nondiagonal hydraulic conductivity tensor, soil stratigraphy, finite-difference scheme, numerical model

1 INTRODUCTION

In nature, soil deposits are inherently anisotropic (He et al., 2022) but within the stratigraphic plane axisymmetric, caused by natural deposition or human activity such as tillage and field traffic (Pulido-Moncada et al., 2021). Anisotropic soils' physical properties, such as hydraulic conductivity, structure, and strength, exhibit changes with the direction of measurement (Peng, 2011). For instance, naturally or artificially induced orientation of soil stratigraphic planes results in a variation in the hydraulic conductivity, commonly referred to as permeability, typically expressed as a tensor. Determination of its correct value is computationally costlier and not straightforward especially if the soil is anisotropic as in this case hydraulic conductivity in one direction could vary from other directions. Therefore, to accurately capture the impact of the orientation of soil stratigraphic planes on the seepage flow pattern-in the case the coordinate axes are not aligned with the stratigraphy-one needs to consider the impact of the relationship between the hydraulic conductivity tensor, the stratigraphic orientation, and the system of coordinates choice. In other words, to realistically simulate the change in the seepage flow pattern through anisotropic soil, a nondiagonal full tensor needs to be used to fully capture the characteristics of the nature of the hydraulic conductivity.

However, in instances, to reduce computational cost, the nondiagonal members may be neglected. This approximation may be accurate in resident column or similar test where the flow is constrained to be one dimensional. Hence, for particular types of problems constrained to be 1D, the directional dependency of hydraulic conductivity has been disregarded, and the hydraulic conductivity has been treated as a scalar. For instance, in a recent study (Iradukunda & Farid, 2022), a 1D seepage numerical model was coupled with another numerical transport model to simulate the fate and transport of Per- and Polyfluoroalkyl substances (PFAS) through vadose and saturated zones within soil, incorporating advection-dispersion, air-water interface, and solid-phase adsorption, where hydraulic conductivity was considered a scalar because the flow was assumed one dimensional. This 1D numerical model was used to simulate various scenarios of seepage and transport of PFAS, and output was validated against the experimental data found in the literature (Guo et al., 2020).

For 2D or 3D cases of anisotropic soil, the impact of this approximation needs to be studied. For example, the fluid-flow equations that haves been used to simulate many commercial reservoirs rely on the assumption that nondiagonal elements of the tensor can be neglected (Ertekin et al., 2001; Fanchi, 2005), which results in erroneous flow calculations (Fanchi, 2008). In the case of reservoir systems, which are influenced by the hydraulic conductivity, Fanchi (2008) estimated the error that appears in the magnitude and direction of flow rate due to neglecting off-diagonal terms of the hydraulic conductivity tensor. Additionally, Fanchi (2008) provided a formula for appropriately calculating a 2D hydraulic conductivity tensor that can be applied for any plane rotation angle.

Other researchers demonstrated that in most realistic settings, including rare fractured reservoir modelling (Gupta et al., 2001), geomechanics (Settari et al., 2001), and up-scaling (Young, 1999), full hydraulic conductivity tensors need to be considered unless the flow is aligned with the coordinate system, e.g., one where the XY plane is aligned within the stratigraphic plane. Even improper consideration of the full hydraulic conductivity tensor could result in inaccurate outcomes. In a recent study, the full permeability (hydraulic conductivity) tensor of coal samples under different effective stress was calculated (Feng et al., 2020) in the sense of Mohr's circles, which may be confusing if not misleading , especially the sign convention for its nondiagonal elements, in the scenario of tilted stratigraphic planes. Therefore, the proper accountability of the relationship between coordinates' orientation and hydraulic conductivity tensor is essential to avoid inaccuracies in results.

Even some of the well-established numerical approximation methods neglect nondiagonal terms in the hydraulic conductivity tensor in exchange for reduced computational cost. In a previous study (Cao et al., 2019), the effectiveness of two numerical schemes, namely the Two-Point Flux Approximation (TPFA) and MultiPoint Flux Approximation (MPFA), was investigated to model the incompressible single-phase flow in an anisotropic porous medium with a full hydraulic conductivity tensor. It was discovered that, while the TPFA demonstrated greater computational efficiency, failing to account for the off-diagonal terms in the hydraulic conductivity tensor caused an overestimation of the total flux and severe inaccuracies in calculating the pore-water pressure and flux in nondiagonal cases.

In the present study, to simulate the seepage flow for various degrees of stratigraphic tilt with respect to the system of coordinates (XYZ), a 2D numerical model incorporating the finite-difference method has been developed where hydraulic conductivity has been treated as a full nondiagonal tensor. Various scenarios of 2D seepage flow have been simulated for different degrees of orientation of the planes to investigate and analyze the impact of nondiagonal elements of the hydraulic conductivity tensor. Graphical illustrations of the seepage flow in these cases with and without considering nondiagonal tensor elements are also shown in order to demonstrate how disregarding nondiagonal tensor terms replicates erroneous directions and values of seepage flows.

The following sections will discuss some background of seepage and the mathematical development of the full hydraulic conductivity tensor and the mathematical formulations and the development of the 2D numerical seepage model, as well as results for a few scenarios with different degrees of stratigraphic tilt based on assumed constraints to illustrate the impact of nondiagonal components of the hydraulic conductivity tensor.

2 BACKGROUND

The following describes the background of seepage and the technique used to calculate the full anisotropic hydraulic conductivity tensor to enable evaluation of the impact of hydraulic conductivity on the seepage flow.

2.1 Seepage

In the presence of a hydraulic gradient, there will be flow. In the most general case, using conservation of mass, the governing equation of seepage can be found. In addition, assuming the density of water is constant, the conservation of mass turns into the conservation of volume. For an element of soil, this results in the following equation for transient seepage (Fredlund & Rahardjo, 1993).

$$\vec{\nabla} \cdot \vec{v} = -\frac{\partial \theta}{\partial t}, \tag{1}$$

where $\vec{v}$ is Darcy's velocity, otherwise known as discharge velocity, $\theta = nS_w$ is the volumetric water content, where n is the soil porosity and S_w is the degree of water saturation.

Variations in the volumetric water content can be found (Genetti Jr, 1999) as a function of the specific/elastic capacity (i.e., retention) of water, m_v, and temporal variations of the hydraulic head, h resulting in:

$$\vec{\nabla} \cdot \vec{v} = -m_v \frac{\partial h}{\partial t}, \tag{2}$$

where for unsaturated soils, the water-retention characteristics can be approximated to a two-segment polynomial with the two values for its slope, $m_v \approx 0.001\ m^{-1}$ for unsaturated soils and $m_v \approx 0.00001\ m^{-1}$ for saturated soils.

2.2 Full hydraulic conductivity tensor

Darcy's velocity $\vec{v}$ is the discharge velocity of groundwater flow, which can be calculated from Darcy's law:

$$\vec{v} = -k\vec{\imath}, \tag{3}$$

where k is the hydraulic conductivity,$\vec{\imath} = \vec{\nabla}h$ is the hydraulic gradient, and h is the total (hydraulic) head. The negative sign shows that the water moves from the higher total head toward a lower total head. Thus, Equation (3) can be rewritten as follows.

$$\vec{v} = -k\vec{\nabla}h. \tag{4}$$

If the soil is isotropic (k is the same in all directions), then k could be treated as a scalar. If the soil is anisotropic, since hydraulic conductivity is different in different directions, k needs to be treated as a tensor, $\underline{\underline{k}}$.

Hence, if v_x, v_y, and v_z are the velocity component in X, Y, and Z directions, respectively, Darcy's law can be written as follows.

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = -\begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} h, \tag{5}$$

where k_{xx}, k_{yy}, k_{zz} are diagonal elements and k_{xy}, k_{xz}, k_{yx}, k_{yz}, k_{zx}, and k_{zy} are nondiagonal elements of hydraulic conductivity tensor, $\underline{\underline{k}}$. The nondiagonal elements of $\underline{\underline{k}}$ are symmetric.

If the system of coordinates is considered such that one plane (e.g., XY) is aligned with the stratigraphic plane on the mesoscale, the tensor $\underline{\underline{k}}$ can be assumed to be a diagonalized tensor. In other words, in these cases, seepage is analysed in an orthogonal system of coordinates aligned with the three axes of coordinates aligned with the three principal orientations (major: k_1, intermediate: k_2, and minor: k_3) of the tensor $\underline{\underline{k}}$ in which, k_3 is normal to the stratigraphic plane and k_1 and k_2 within stratigraphic planes. In most cases, soils are axisymmetric within the stratigraphic plane; hence, k_1 and k_2 are equal. Since the XY plane is almost always considered horizontal, and most soil stratigraphic planes are horizontal and axisymmetric, this represents most cases.

However, stratigraphic planes can be titled due to several reasons such as tectonic plate movements. In the case of tilted stratigraphic planes, nondiagonal elements of the hydraulic conductivity tensor $\underline{\underline{k}}$ must be considered. In 2D, if α is the angle of stratigraphic tilt, then the hydraulic conductivity tensor $\underline{\underline{k}}$ will be nondiagonal and Darcy's velocity will be:

$$\begin{bmatrix} v_x \\ v_z \end{bmatrix} = -\begin{bmatrix} k_{xx} & k_{xz} \\ k_{zx} & k_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial z} \end{bmatrix} h, \quad (6)$$

where $\underline{\underline{k}} = \begin{bmatrix} k_{xx} & k_{xz} \\ k_{zx} & k_{zz} \end{bmatrix}$,

considering Equation (5), k_{xx} and k_{xz} contribute to v_x through the impact of the X and Z components of the hydraulic gradients, respectively. On the other hand, k_{zx} and k_{zz} contribute to v_z through the impact of the X and Z components of the hydraulic gradients, respectively.

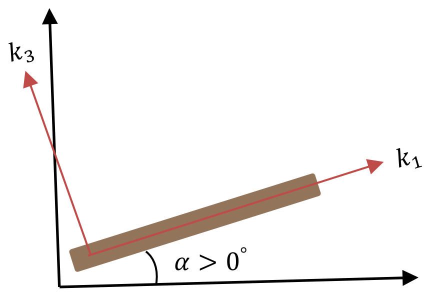


Figure 1. *The schematic oblong rectangle (▬) shown on the figure represents the stratigraphic plane with a positive slope (used $\alpha > 0^{\circ}$); k_1 aligns with the length of the schematic rectangle, representing the stratigraphic plane orientation.*

In a 2D system of coordinates and the stratigraphic plane with a positive slope α shown in Figure 1, positive values of $\left(-\frac{\partial h}{\partial x}\right)$ and $\left(-\frac{\partial h}{\partial z}\right)$ can be projected on the k_1 and k_3 axes. Then, the flow velocities along the 1 and 3 directions can be simplified as follows.

$$v_1 = k_1 \left[\left(-\frac{\partial h}{\partial x}\right)\cos\alpha + \left(-\frac{\partial h}{\partial z}\right)\sin\alpha\right], \quad (7a)$$

$$v_3 = k_3 \left[\left(-\frac{\partial h}{\partial x}\right)\sin\alpha + \left(-\frac{\partial h}{\partial z}\right)\cos\alpha\right]. \quad (7b)$$

If the resultant of two vectors v_1 and v_3 are found and projected onto the X and Z directions, v_{xx} and v_{zz} can be computed and simplified as follows.

$$v_{xx} = v_{1x} + v_{3x} = -\left[\left(\frac{k_1+k_3}{2}\right) + \left(\frac{k_1-k_3}{2}\right)\cos 2\alpha\right]\frac{\partial h}{\partial x} - \left[\left(\frac{k_1-k_3}{2}\right)\sin 2\alpha\right]\frac{\partial h}{\partial z}. \quad (8a)$$

$$v_{zz} = v_{1z} + v_{3z} = -\left[\left(\frac{k_1-k_3}{2}\right)\sin 2\alpha\right]\frac{\partial h}{\partial x} - \left[\left(\frac{k_1+k_3}{2}\right) + \left(\frac{k_1-k_3}{2}\right)\cos 2\alpha\right]\frac{\partial h}{\partial z}. \quad (8b)$$

Comparing Equations (8) and (6), k_{xx}, k_{zz}, k_{xz}, and k_{zx} for a 2D case, can be written in terms of k_1 and k_3, which agrees with what was found by (Fanchi, 2008).

$$k_{xx} = \left(\frac{k_1+k_3}{2}\right) + \left(\frac{k_1-k_3}{2}\right)\cos 2\alpha, \quad (9a)$$

$$k_{xz} = k_{zx} = \left(\frac{k_1-k_3}{2}\right)\sin 2\alpha, \quad (9b)$$

$$k_{zz} = \left(\frac{k_1+k_3}{2}\right) - \left(\frac{k_1-k_3}{2}\right)\cos 2\alpha. \quad (9c)$$

3 METHODOLOGY

3.1 Mathematical formulation

Substituting Darcy's law into Equation (2), the governing equation of seepage can be found.

$$\vec{\nabla} \cdot \left(-\underline{\underline{k}}\vec{\nabla}h\right) = -m_v \frac{\partial h}{\partial t}. \tag{10}$$

Then, for a 2D case, Equation (10) takes the following form.

$$-\begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} k_{xx} & k_{xz} \\ k_{zx} & k_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial z} \end{bmatrix} h = -m_v \frac{\partial h}{\partial t}, \tag{11}$$

where hydraulic conductivity tensor $\underline{\underline{k}}$ whose elements for any orientation angle α of soil stratigraphic planes are given by Equation (9). Equation (11) can then be simplified as follows.

$$-\left[\frac{\partial k_{xx}}{\partial x}\frac{\partial h}{\partial x} + k_{xx}\frac{\partial^2 h}{\partial x^2} + \frac{\partial k_{xz}}{\partial x}\frac{\partial h}{\partial z} + k_{xz}\frac{\partial^2 h}{\partial x \partial z} + \frac{\partial k_{zx}}{\partial z}\frac{\partial h}{\partial x} + k_{zx}\frac{\partial^2 h}{\partial z \partial x} + \frac{\partial k_{zz}}{\partial z}\frac{\partial h}{\partial z} + k_{zz}\frac{\partial^2 h}{\partial z^2}\right] = -m_v \frac{\partial h}{\partial t}. \tag{12}$$

3.2 Numerical Modelling

Using the MATLAB interface, a 2D numerical model was developed to solve the governing equation of transient seepage, Equation (10) to simulate the impacts of the nondiagonal $\underline{\underline{k}}$ tensor on the seepage flow for soils with tilted stratigraphic planes (i.e., there is a tilt between the X and k_1 axes and, in turn, Z and k_3 axes), types, and different orientations of soil stratigraphy.

For our seepage model, two types of boundary conditions were considered: Neumann boundary conditions on most of the domain boundary, which stimulates impermeable boundaries, and Dirichlet boundary conditions at inlets and outlets, where hydraulic heads are constants: H_1 and H_2, respectively, to allow flow.

The finite difference (FD) with the forward difference in time derivatives and first-order space derivatives and central difference in higher-order space derivatives to discretise the time and space domains in order to linearize Equation (12) to the following form. In Equation (13), superscripts (t_k, t_{k+1})represent time steps and subscripts (i, j) represent space discretization. The implicit scheme was applied to h on the left side of Equation (12) and all h terms are considered at time t_{k+1}. Thus, $h_{i,j}^{t_k}$ represents the hydraulic head at Row i, Column j, i.e., Node (i, j) in the space, and at time instance t_k; $h_{i,j}^{t_{k+1}}$ is the hydraulic head at Node (i, j) at time $t_{k+1}$and so on.

$$\frac{k_{xx(i,j)}}{(\Delta x)^2} h_{i,j-1}^{t_{k+1}} - \left(\frac{k_{xx(i,j+1)} - k_{xx(i,j)}}{(\Delta x)^2} + 2\frac{k_{xx(i,j)}}{(\Delta x)^2} + \frac{k_{xz(i,j+1)} - k_{xz(i,j)}}{\Delta x \Delta z} + 2\frac{k_{xz(i,j)}}{\Delta x \Delta z} + \frac{k_{zx(i+1,j)} - k_{zx(i,j)}}{\Delta z \Delta x} + 2\frac{k_{zx(i,j)}}{\Delta z \Delta x} + \frac{k_{zz(i+1,j)} - k_{zz(i,j)}}{(\Delta z)^2} + 2\frac{k_{zz(i,j)}}{(\Delta z)^2} + \frac{m_{v(i,j)}}{\Delta t}\right) h_{i,j}^{t_{k+1}} + \left(\frac{k_{xx(i,j+1)} - k_{xx(i,j)}}{(\Delta x)^2} + \frac{k_{xx(i,j)}}{(\Delta x)^2} + \frac{k_{zx(i+1,j)} - k_{zx(i,j)}}{\Delta z \Delta x}\right) h_{i,j+1}^{t_{k+1}} + \left(\frac{k_{xz(i,j)}}{\Delta x \Delta z} + \frac{k_{zx(i,j)}}{\Delta z \Delta x}\right) h_{i-1,j-1}^{t_{k+1}} + \frac{k_{zz(i,j)}}{(\Delta z)^2} h_{i-1,j}^{t_{k+1}} + \left(\frac{k_{xz(i,j+1)} - k_{xz(i,j)}}{\Delta x \Delta z} + \frac{k_{zz(i+1,j)} - k_{zz(i,j)}}{(\Delta z)^2} + \frac{k_{zz(i,j)}}{(\Delta z)^2}\right) h_{i+1,j}^{t_{k+1}} + \left(\frac{k_{xz(i,j)}}{\Delta x \Delta z} + \frac{k_{zx(i,j)}}{\Delta z \Delta x}\right) h_{i+1,j+1}^{t_{k+1}} = -\frac{m_{v(i,j)}}{\Delta t} h_{i,j}^{t_k}, \tag{13}$$

Diagonal elements of $\underline{\underline{k}}$: $k_{xx(i,j)}$, $k_{zz(i,j)}$, $k_{xx(i,j+1)}$, and $k_{zz(i+1,j)}$ are at Nodes (i, j), $(i, j+1)$and $(i+1, j)$ correspondingly, to account for the flow velocity in one direction based on the hydraulic gradient in the same direction whereas nondiagonal elements $k_{xz(i,j)}$, $k_{zx(i,j)}$, $k_{xz(i,j+1)}$, $k_{zx(i+1,j)}$ are at Nodes (i, j), $(i, j+1)$and $(i+1, j)$ accordingly to account for the flow velocity in one direction based on the hydraulic gradient in the orthogonal direction. Here, $i = 1,2, \cdots\cdots, M$ and $j = 1,2, \cdots\cdots, N$ are the number of nodes in the vertical and horizontal directions, respectively. Values M and N are selected to be odd to allow a midpoint row to accommodate symmetry and calculated based on the horizontal length, L, and vertical length (thickness of soil), T.

The model simulates seepage flow for a domain consisting of two soil types deposited in two layers stacked up together with the same length, which can also be used to simulate the flow for a single layer of soil considering the fact that hydraulic conductivity for both of the types is the same. In addition, the code is flexible to simulate other scenarios, e.g., soil layers stacked either horizontally or vertically, and there could be any degree of tilt, α, of the soil stratigraphic planes with respect to the X (in this case horizontal) axis.

However, for the selected values of hydraulic conductivity along the major principal orientations, k_1, and minor principal orientations, k_3, of $\underline{\underline{k}}$ for both soils, the hydraulic conductivity tensor elements were calculated using Equation (9). To ensure relative proper selection of space and time discretization resolution, the following mathematical expressions are used to approximate the horizontal space grid size, Δx, and vertical space grid size, Δz, for a given time step Δt.

$$\Delta x = -k_{xx}\frac{H_2-H_1}{L}\Delta t, \text{ and } \frac{\Delta x}{\Delta y} = \sqrt{\frac{k_1}{k_3}}, \tag{14}$$

where H_1 and H_2 are constant hydraulic heads at inlets and outlets. In the case of two types of soils, the values k_{xx} , k_1, and k_3 are assumed to be the average of the corresponding values for the two soil types.

In this model, in the case of unsaturated soil, with each time step, both diagonal and nondiagonal components of the hydraulic-conductivity tensor are updated using the following soil water-retention formula.

$$k_{p,q} = \frac{k_{0_{p,q}}}{1+a_1\left|h_{p,q}-z_{p,q}\right|^{a_2}}, \tag{15}$$

where $k_{0_{p,q}}$ is the saturated hydraulic conductivity for the soil; two constant coefficients $a_1 = 1$ and $a_3 = 3$ are used in this formula; $k_{p,q}$, represents k_{xx}, k_{zz} , k_{xz}, and k_{zx} at Nodes $p = i-1, i, i+1,\ q = j-1, j, j+1$, and $z_{p,q}$ is the elevation head at any Node (p, q).

The term $h_{p,q}$ in the denominator of Equation (15) can be considered at either of Times t_k, t_{k+1}, or an average of the two. In here, a scheme similar to the Crank-Nicholson scheme(Chávez-Negrete et al., 2018) is used, and the average of $h_{p,q}^{t_k}$, and $h_{p,q}^{t_{k+1}}$, was used in Equation (15).

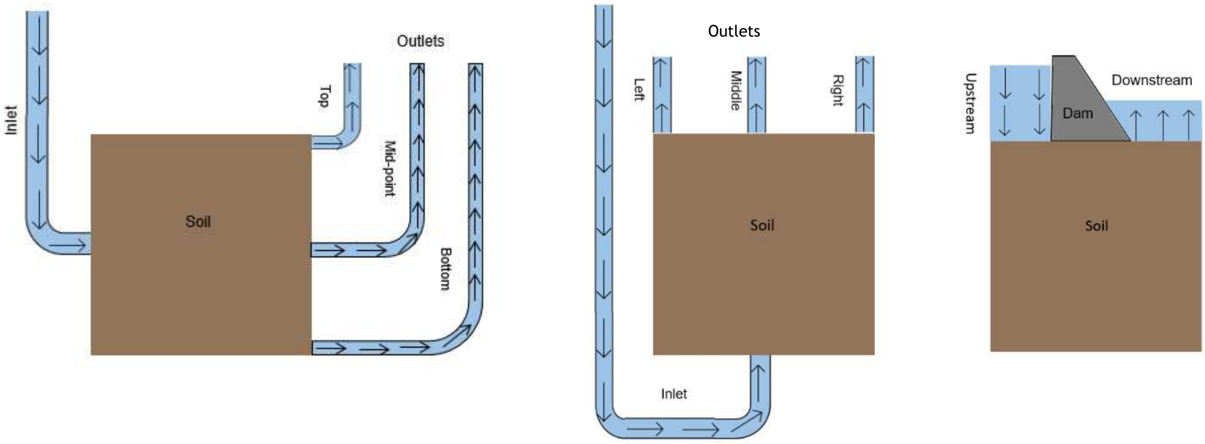
$$k_{p,q} = \frac{k_{0_{p,q}}}{1+a_1\left|\frac{h_{p,q}^{t_k}+h_{p,q}^{t_{k+1}}}{2}-z_{p,q}\right|^{a_2}}. \tag{16}$$

Due to the presence of the unknown $h_{p,q}^{t_{k+1}}$ in the denominator of Equation (16), if Equation (16) is substituted in Equation (13), the resulting equation will be nonlinear, which will defeat the purpose of linearization to enable solving the system of linear equations. Hence, instead of solving this resulting equation, all coefficients $k_{p,q}$ need to be found separately and substituted as known values in Equation (13) to maintain the linearity of Equation (13). Since neither $h_{p,q}^{t_{k+1}}$ nor $k_{p,q}$ are known, $k_{p,q}$ (k based on average values of h for the time increment between t_k and t_{k+1}) are found using a successive iteration scheme. Basically, for each time step, initially, $k_{p,q}$ is found using only $h_{p,q}^{t_k}$, the solver is used to find $h_{p,q}^{t_{k+1}}$, then, $k_{p,q}$ is updated using this newly calculated $h_{p,q}^{t_{k+1}}$, which is then used to solve and find updated $k_{p,q}$.This successive iteration is continued until the code converges to the best answer for $h_{p,q}^{t_{k+1}}$ and in turn, the best $k_{p,q}$. Only then, the code is ready to advance (i.e., "march to the next step in time") using $h_{p,q}^{t_{k+1}}$ through another successive iteration that converges to the best $h_{p,q}^{t_{k+2}}$ and corresponding $k_{p,q}$ (average for the increment between t_{k+1} and t_{k+2}). This applies to all elements of the hydraulic conductivity tensor. The code then advances to all following time steps. The total head at each time and space node is used to calculate Darcy's velocity and pore-water pressure values.

4 RESULT

4.1 Problem setup

The developed 2D transient seepage numerical model outlined above was used to analyse the impacts of nondiagonal components of the hydraulic conductivity tensors in an anisotropic soil on seepage flow for various stratigraphic tilts. Several scenarios were simulated based on variations of inlet and outlet combinations as well as single and double layers of soil where each of the scenarios has been simulated for both considering and neglecting nondiagonal elements of the hydraulic conductivity tensor to visualize the error in the directions of flows associated with disregarding the tensor's nondiagonal terms. Figure 2 shows the schematic diagram of the model with different inlet and outlet combinations on the boundary walls.



(a) (b) (c)

Figure 2. *Schematic of diagram of the model for combinations of (a) a mid-point inlet on the left vertical boundary and three outlets on the right vertical boundaries (one at the midpoint and two at the top and bottom corners); (b) a mid-point bottom inlet on the left boundary, and three outlets at mid- and both endpoints of the top boundary, (c) an upstream (inlet) and an downstream(outlet) at the top boundary.*

The following is considered for all of the scenarios: (i) initially soil is unsaturated with total head, $h = 0$ (capillary rise and suction exists at all elevations); (ii) except for inlets and outlets, all other boundaries of soil are impermeable; (iii) initial hydraulic heads at the inlet(s) $H_1 = 7\ m$ and at outlet(s) $H_2 = 3\ m$, (iv) each scenario was analysed for 1000 seconds at time step of $\Delta t = 10$ seconds; (v) the horizontal length L and vertical length (thickness) T of soil assumed to be $2.1\ m$ and $2\ m$, respectively.

In the following subsections, more specifics for these scenarios will be described followed by results and analysis.

4.2 Simulation result

Due to limited space, the following only shows a couple of scenarios though the model was tested for numerous other scenarios.

4.2.1 Scenario 1: single layer soil with no stratigraphic tilt

The first case is a simple one, where seepage flow is simulated through an anisotropic soil with no tilt between the soil stratigraphic plane and the horizontal X axis, i.e., $\alpha = 0^{\circ}$, with a major principal hydraulic conductivity value of $k_1 = 5.5 \times 10^{-3}\ m/s$ and a minor principal hydraulic conductivity value of $k_3 = 1.5 \times 10^{-3}\ m/s$ of the hydraulic conductivity tensor $\underline{\underline{k}}$. Here, a mid-point inlet on the left vertical boundary and three outlets on the right vertical boundaries (one at the midpoint and two at the top and bottom corners) were considered.

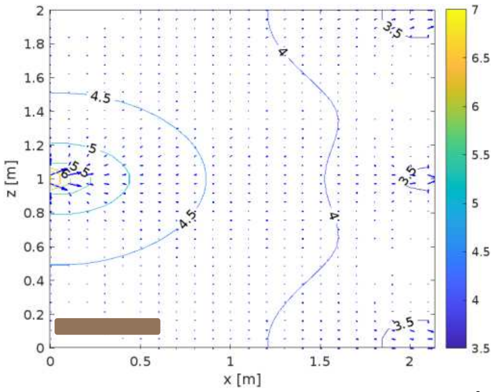


Figure 3. Flow *velocity vectors and equipotential lines when $\alpha = 0^{\circ}$, hence, nondiagonal hydraulic conductivity, i.e., $k_{xx} = k_1, k_{zz} = k_3$, and $k_{xz} = k_{zx} = 0$. A midpoint inlet on the left boundary and three outlets on the right-side boundary. The schematic oblong rectangle ( ) shown on the figure represents the stratigraphic plane with slope $\alpha = 0^{\circ}$; k_1 aligns with the length of the schematic rectangle.*

In the case of Figure 3, there is the hydraulic conductivity tensor is actually diagonal; hence, the nondiagonal elements are actually zero ($k_{xz} = k_{zx} = 0$) when $\alpha = 0^{\circ}$.

4.2.2 Scenario 2: single layer soil with a tilted stratigraphic plane

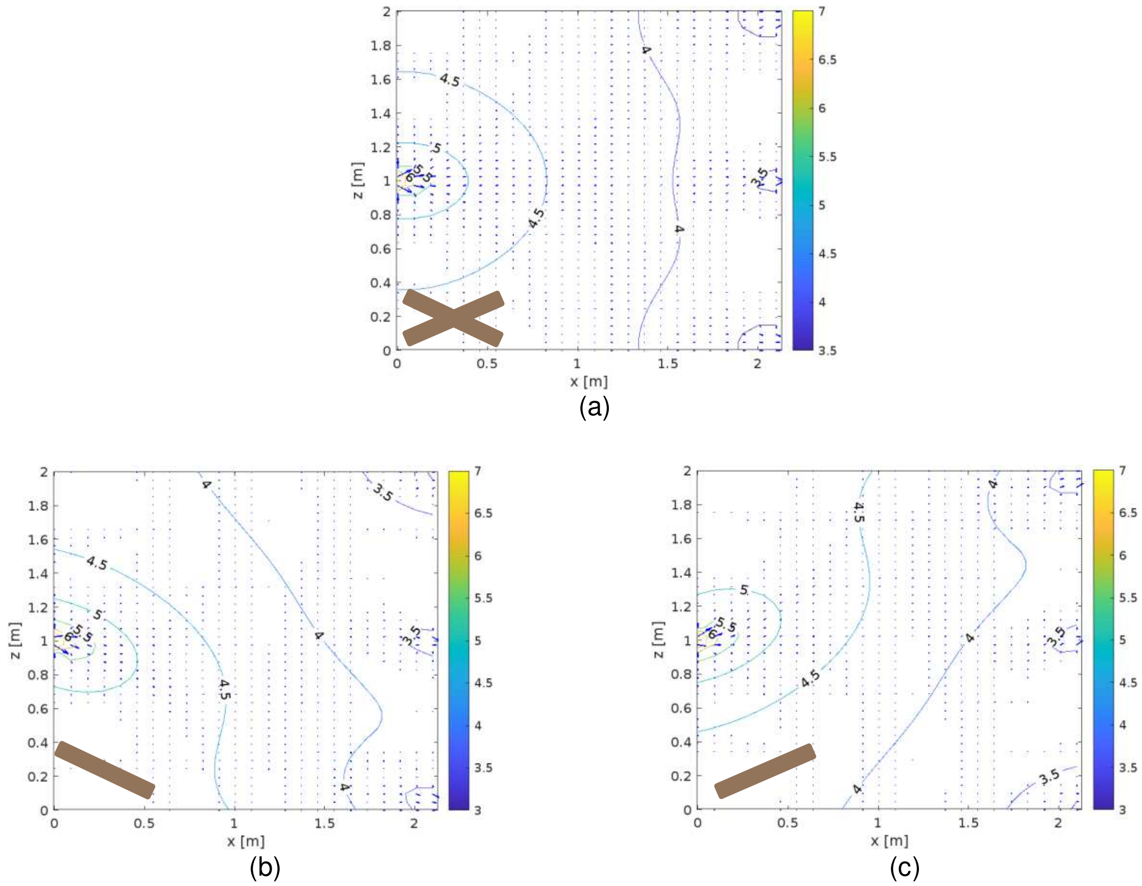


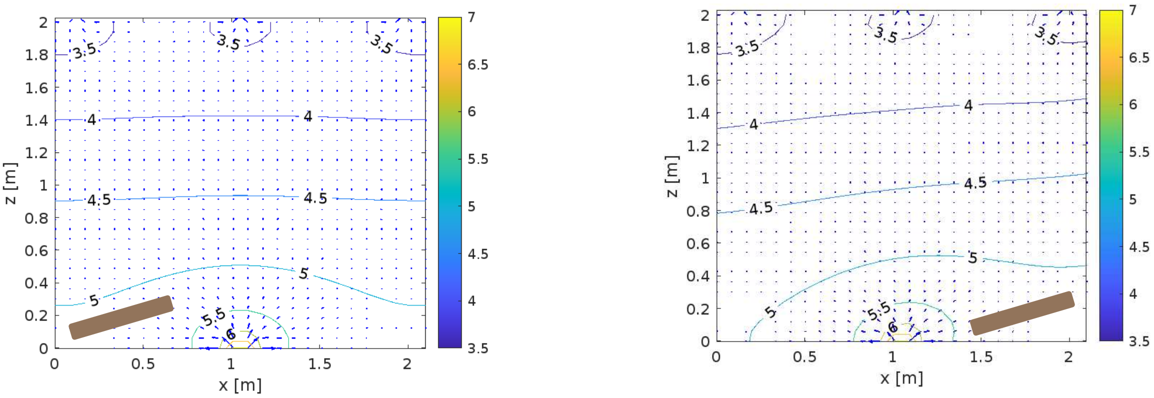
Figure 4. Flow *velocity vectors and equipotential lines for (a) erroneously diagonalized hydraulic conductivity tensor when $\alpha = -20^{\circ}$ and 20°; (b) correct nondiagonal hydraulic conductivity tensor used for the case with $\alpha = -20^{\circ}$ (c) correct nondiagonal hydraulic conductivity tensor used for the case with $\alpha = 20^{\circ}$.* A *midpoint inlet on the left boundary and three outlets on the right-side boundary. The schematic oblong rectangle ( ) shown on the figures represent the stratigraphic plane with their corresponding slope α; k_1 aligns with the length of the schematic rectangle.*

In this scenario, there is a tilt for all cases; hence, the hydraulic conductivity tensor where $k_{xx} < k_1$, $k_{zz} > k_3$, and $k_{xz} = k_{zx} \neq 0$. Figures 2b and 2c show the results for $\alpha = 20^{\circ}$ and $\alpha = -20^{\circ}$, respectively where all nonzero values of nondiagonal elements (k_{xz} and k_{zx}) for the two cases are correctly considered nonzero. In these two cases, k_{xx} and $k_{zz}$are the same. Hence, if the cases are erroneously diagonalized (k_{xz} and k_{zx} are forced to zero), and the same erroneous result is created for both cases.

As seen from Figure 4, in the case of a tilt above the horizon ($\alpha = 20^{\circ}$) and down ($\alpha = -20^{\circ}$) titled stratigraphic plane, disregarding nondiagonal tensor terms (erroneously forced $k_{xz} = k_{zx} = 0$) leads to an erroneous seepage flow pattern that is symmetric across a horizontal axis of symmetry while the soil stratigraphy in neither case guarantees symmetry. However, correcting considering nonzero nondiagonal elements correct seepage pattern on both cases that are asymmetric following the stratigraphic tilt, i.e., flow tilted downward for $\alpha = -20^{\circ}$ and tilted upward for $\alpha = 20^{\circ}$. The values of the nondiagonal elements are $k_{xz} = k_{zx} = -1.3 \times 10^{-3}\ m/s$ and $k_{xz} = k_{zx} = 1.3 \times 10^{-3}\ m/s$respectively.

4.2.3 Scenario 3: tilted single layered soil stratigraphic plane with inlet at the bottom and outlets at the top

A case with one soil type with a stratigraphic plane at ($\alpha = 15^{\circ}$) with $k_1 = 4.5 \times 10^{-3}\ m/s$ and $k_3 = 2.5 \times 10^{-3}\ m/s$ with a midpoint bottom inlet and three outlets at mid- and both endpoints of the top boundary is considered. For this problem setup, the full hydraulic conductivity tensors' diagonal elements are $k_{xx} = 4.4 \times 10^{-3}\ m/s$ and $k_{zz} = 2.6 \times 10^{-3}\ m/s$, whereas nondiagonal elements are $k_{xz} = k_{zx} = 5.0 \times 10^{-4}\ m/s$. Based on forced diagonalized (erroneous approximation) and exact nondiagonal, i.e., full hydraulic conductivity tensors $\underline{\underline{k}}$, the following simulations are generated.



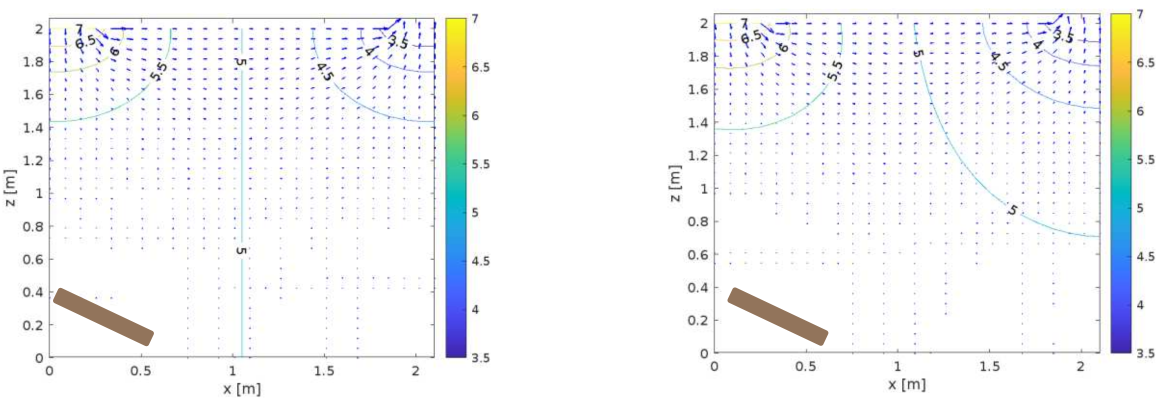
(a) (b)

Figure 5. *Flow velocity and equipotential lines for the seepage flow considering, (a) forced diagonalized (erroneous approximation); (b) nondiagonalized (exact) hydraulic conductivity tensor with $\alpha = 15^{\circ}$, a mid-point bottom inlet on the left boundary, and three outlets at mid- and both endpoints of the top boundary. The schematic oblong rectangle ( ) shown on the figures represent the stratigraphic plane with their corresponding slope α; k_1 aligns with the length of the schematic rectangle.*

It is evident that as the stratigraphic plane is slanted above the horizontal plane, seepage flow contour lines should also be slanted. As a result, the flow patterns depicted in Figure 5(a) are inaccurate because only diagonal elements are considered (while nondiagonal elements are omitted), while Figure 5(b) is accurate since it takes into account both diagonal and nondiagonal elements of the tensor.

4.2.4 Scenario 4: concrete dam with single soil type with a tilted stratigraphy

For this scenario, a concrete dam with an upstream lake and a downstream lake as inlets and outlets at the top and a stratigraphic plane of soil tilted below the horizon with an angle $\alpha = -10^{\circ}$ and hydraulic conductivity values along the major principal orientation k_1 and minor principal orientation k_3 of the hydraulic conductivity tensor $\underline{\underline{k}}$ are the same as the previous scenario. Illustrations based on the neglecting and considering nondiagonal elements of $\underline{\underline{k}}$ are shown in Figure 6.



(a) (b)

Figure 6. *Flow velocity and equipotential lines for the seepage flow considering, (a) a forced diagonalized (erroneous approximation) and (b) nondiagonalized (exact) hydraulic conductivity tensor with $\alpha = -10^{\circ}$. The schematic oblong rectangle ( ) shown on the figures represent the stratigraphic plane with their corresponding slope α; k_1 aligns with the length of the schematic rectangle.*

Since the soil stratigraphic plane is tilted below the horizon, the seepage flow directions should not be symmetric. Thus, it is clear from Figure 6(a) that off-diagonal elements, $k_{xz} = k_{zx} = -3.4202 \times 10^{-4}\ m/s$ of the tensor $\underline{\underline{k}}$ must be considered to simulate the flow directions correctly to result in Figure 6(b).

5 CONCLUSIONS

In this paper, a 2D numerical modelling framework for simulating seepage flow was developed. Employing this 2D numerical model, several scenarios of seepage flow through unsaturated soil considering both the approximated diagonalized and exact nondiagonal (full) hydraulic conductivity tensors based on variations in the degree of the soil stratigraphic plane(s) tilt, the number of inlets and outlets and their positions, the values of hydraulic conductivity along the minor and major principal orientations, and the orientations of soil layers. All these scenarios demonstrate how ignoring nondiagonal terms results in incorrect seepage flow simulation. Unless the flow is aligned with a diagonalized coordinate system (i.e., X and/or Z axes are aligned with k_1 or k_3), a full hydraulic conductivity tensor that includes both diagonal and nondiagonal elements must be taken into account to accurately simulate seepage flow, as in the case of flow simulation for reservoir systems (Fanchi, 2008). While this 2D model and the enforced assumptions allow for a demonstration of how the nondiagonalized hydraulic conductivity tensor impacts seepage flow, a 3D model is necessary to simulate more realistic circumstances.

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