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Applicability of shear wave velocity to evaluate state of granular materials with fines

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ABSTRACT

Evaluation of state of cohesionless soils is a key issue in majority of geotechnical projects, especially when the major concern is liquefaction phenomenon. Standard approaches based on static or dynamic penetration tests are of limited or no use in granular materials containing considerable amount of fines. Therefore, there is a need to look for alternative approaches to identify state of materials containing certain amount of fines. One of a conceivable approaches rests on shear wave velocity (V_s) measurement. It results from the fact that V_s reflects state of stress and void ratio as well. A little is known to what extent shear wave velocity might reflect state of soil, especially, when it contains certain amount of fines. The paper concerns possibility of evaluation of state of cohesionless soils containing fines on the basis of shear wave velocity. The approach is based on hybrid approach i.e. laboratory and field measurement of shear waves velocity which enables projection of correlation between void ratio and normalized shear wave velocity measured in the laboratory for a given soil on a field profile. Thus, the determined void ratio profile is used to determine undrained shear strength on the basis of steady state line determined in the laboratory for the same soil batch. This procedure was carried out for two kinds of soil containing 10 and 36% of fines, which represent a sand-like material and transition zone behavior. Comments concerning applicability of this approach are given in the conclusion.

Keywords: Liquefaction; sands with fines; steady state shear strength; shear wave velocity.

1. Introduction

Soil liquefaction has been a subject of intensive research over decades, especially after the devastating earthquakes at Niigata and Alaska in 1964. Although catastrophic liquefaction failures are usually associated with cyclic loading released during earthquakes, there are also reported flowslides for which no considerable source of cyclic loading has been detected. Terzaghi (1956), describes several cases of submarine flowslides where no external trigger could be identified. He refers to this phenomenon of sudden liquefaction without presence of cyclic shear stresses as a spontaneous liquefaction. Such failures of natural or man-made slopes are believed to be initiated by minor stress changes such as ground water table fluctuation or toe erosion, hence essentially by static loading. In analysis of soil liquefaction caused by cyclic or static loading a key issue is relative density of analyzed sand. Standard approaches based on static or dynamic penetration tests are applicable mainly to analysis of cyclic liquefaction caused by earthquakes. In case on static liquefaction resolution of these tools are not sufficient especially when soil profile is not homogeneous and cohesionless soils contains certain amount of fines. This situation is often encountered in tailings dams. Therefore, there is a need to look for alternative approaches to identify a state of materials containing certain amount of fines. One of a alternative approaches to evaluation of soil state rests on shear wave

velocity (V_s) measurement. The advantage of this method rests in possibility of V_s measurement in laboratory and in field as well, thus enables to project correlation derived from the laboratory on in situ soil profile. In addition, nondestructive measurement of shear wave velocity allows to avoid to some extent problems of sample disturbance.

However, it should be pointed out that amount of fines can considerably change the response of material to undrained monotonic loading. The paper presents how the approach based on hybrid (laboratory and field) shear wave velocity is applicable to evaluation of the state and undrained shear strength profile for material having 10 and 36% of fines.

2. Undrained response to monotonic loading

The concept of collapse liquefaction dates back to Casagrande in 1936 whose pioneering work was accomplished using direct shear tests. Several decades later Castro (1975) formulated definition of liquefaction on the basis of data from undrained monotonic triaxial tests. Since that time, large body of experimental data was published showing that precise identification of void ratio prior to shearing is a key issue in identifying response of sand.

Over the entire range of states that can be tested on a particular sand, the observed stress-strain behaviour can

be characterised by one of three response types: strain hardening SH, strain softening SS and limited strain softening LSS (Robertson and Fear 1995). The last two are both strain softening responses which can lead to collapse liquefaction and partial or limited liquefaction, as illustrated in Fig. 1. A sand which behaves in this manner is said to be contractive.

Type SS response exhibits a marked strain softening behaviour, i.e., after the peak is reached in the stress-strain diagram, which occurs at a small strain, there is a marked reduction in resistance until the stress stabilises at an ultimate or residual strength (Alarcon-Guzman et al. 1988). The reduction in strength is usually termed „flow deformation” and the residual strength as „steady-state strength”

Type LSS response represents a transition stage in which the strength of the specimen decreases to a residual value and then gains strength (strain hardens). Strain hardening coincides with the onset of dilation and as a consequence, reduction in pore-water pressure (Vaid and Chern 1985). Also characteristic to type LSS response is an „elbow” in the stress path which separates strain softening from strain hardening and corresponds to the minimum deviatoric stress. This state of minimum shear stress is called the state of phase transformation (Ishihara et al. 1975). The temporary stage of strain softening can be referred to as partial or limited liquefaction.

The above two responses of loose saturated sands loaded undrained represent a state (at the end of movement) of constant void ratio, deviatoric stress and mean effective stress. This condition has been termed steady state by Castro (1975) and Poulos (1981). Some researchers (Ishihara 1993) emphasize the difference between SS and LSS what results in a lower position of quasi steady state line QSSL than steady state line SSL. In any case the key parameter in these considerations is the void ratio. This characteristic has been long recognised as an important parameter controlling undrained response of sand. All parameters developed for scaling state of sand are based on void ratio (e.g. relative density D_r , state parameter ψ as defined by Been and Jefferies (1985), state index I_s proposed by Ishihara (1993) after Verdugo's work in 1989. Engineers intuitively know that if a sand has a value of the relative density corresponding to loose or even medium dense state (say lower than 50%), it might be prone to liquefaction when subjected to undrained cyclic loading. However, in analysis of factors pertaining to undrained behaviour of sand it is vital to try to separate void ratio contribution to overall behaviour. As an illustration of its importance, in Fig. 2 an example is given in which comparison of shearing characteristics of two triaxial tests is shown. Two specimens of low compressibility coarse sand prepared by moist tamping and consolidated isotropically to the same effective stress 200 kPa are compared. The only parameter which differed these two specimens was the void ratio value. However, the difference in e between two tests at the end of consolidation determined on the basis of internal measurements of soil deformation was 0.026 (sic!). As a result, from the characteristics shown in Fig. 2, this small difference was sufficient enough to change entirely the response of soil from dilative to contractive. In terms of

shear stresses at 30% of vertical strain, it means ten times decreased value of shear strength from 130 kPa to 13 kPa. Pore pressure changes in both specimens until vertical strain of around 1.5% were substantially the same. Pore pressure having achieved certain value of around 130 kPa in contractive specimen continued increasing, while in the dilative sample started to decrease. This phenomenon is reflected very well in the effective stress paths, which until a certain point are very similar, but when the denser specimen achieves phase transformation, both go in the opposite direction.

In terms of large strain behaviour it is worth to noticing that the contractive specimen achieved the quasi steady state at 25% of vertical strain while for the slightly denser specimen 33% was not enough to reach a steady state. In order to emphasize the importance of relation between void ratio and shearing response of the tested sand the effective stress paths and stress strain characteristics for contractive and dilative response were shown respectively above and below the steady state line depicted in Fig. 2.

Certainly, such a small difference in the void ratio might make a difference in response only in sand of low compressibility.

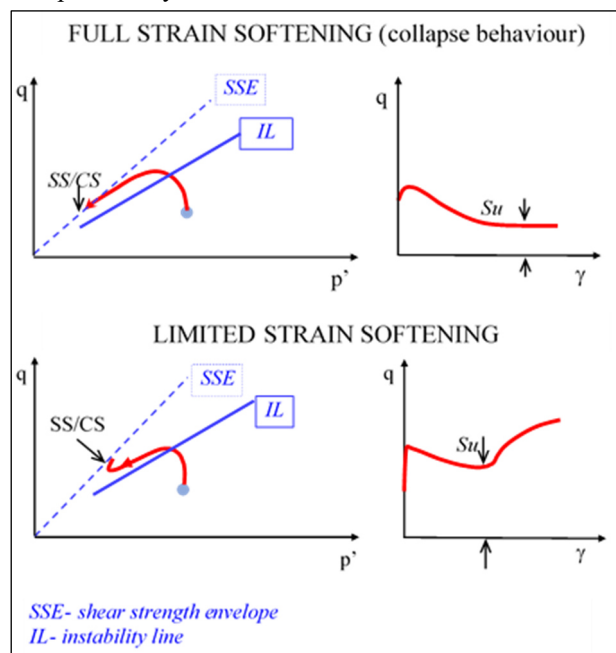


Figure 2. Types of undrained response of soil to monotonic loading leading to liquefaction.

3. Shear wave velocity as a soil state indicator

In the last 60 years a great interest can be observed in the correlation of body wave velocity to many geotechnical parameters. Majority of them concern the use of shear wave velocity to determination of initial stiffness of soil since it has attractive background in theory of elasticity. However, this kind of waves has a potential to evaluate state of soil. Early work by Hardin and Richart (1963) who first measured shear wave velocity in the laboratory showed that V_s is primarily a function of both the void ratio e and the mean normal effective stress p' and can be expressed by the following equation:

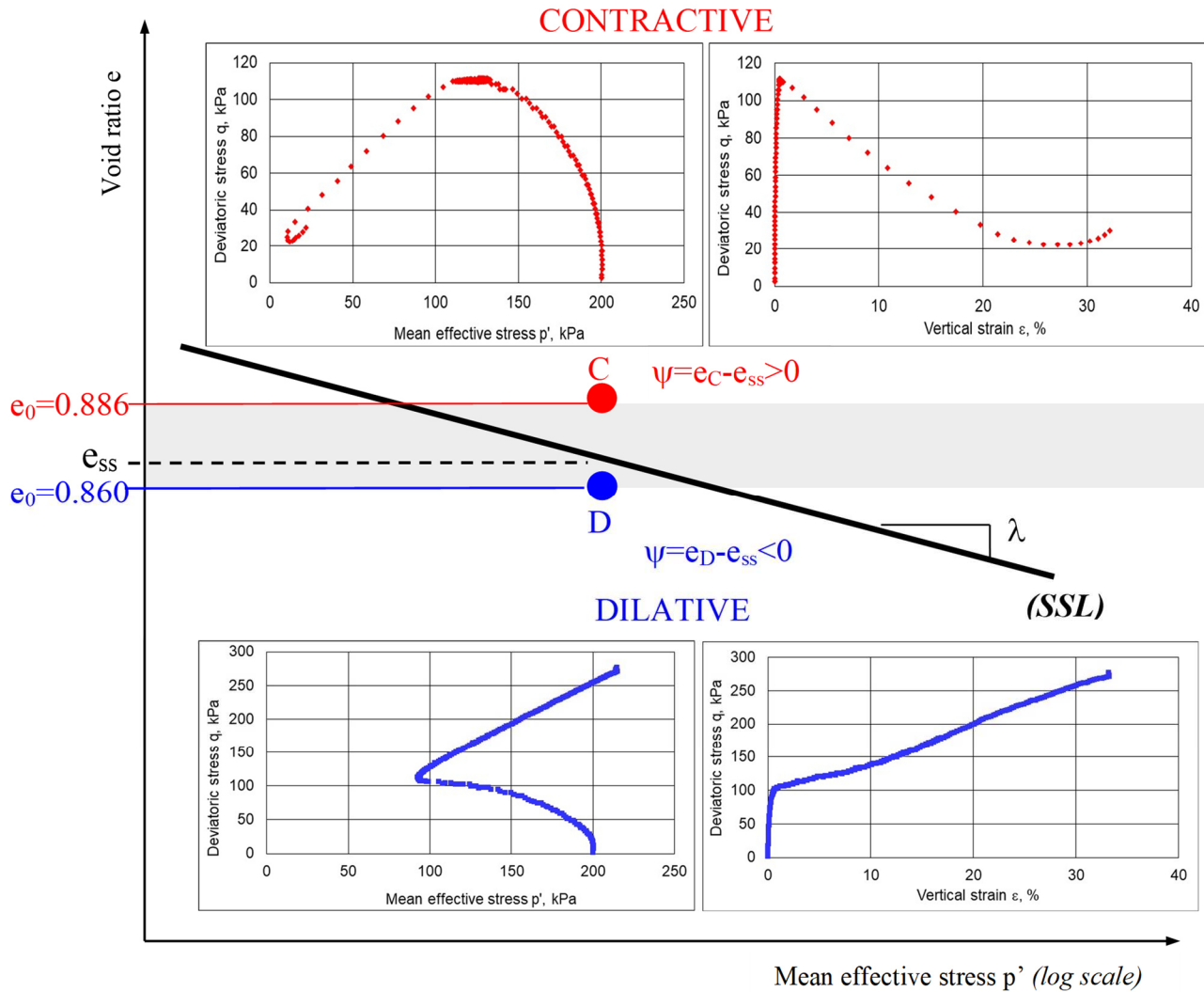


Figure 2. Contractive and dilative response of coarse sand with respect to state parameter.

$$V_s = (m_1 - m_2 e)(p')^{0.25} \quad (1)$$

where:

- e - void ratio
- p' - effective mean normal stress
- m_1, m_2 - material constants (represent soil fabric)

Due to the fact that shear wave velocity is propagated through the contacts in the soil skeleton and depends on void ratio and current stresses, it is conceivable that the actual state of material prior to destructive loading commencement, can be evaluated. It is important to notice that major values that contribute to state of soil i.e. void ratio and effective stress determine value of shear wave velocity.

Since 1963 considerable progress has been done in the field of body wave application in solving geotechnical engineering problems. Large number of data that have been accumulated during the next years concerned technique of measurement, e.g. (Shirley 1978, Roesler 1979) improvement of data interpretation, e.g. (Brignoli et al. 1996, Viggiani and Atkinson 1995, Viana di Fonseca et al. 2009), and taking into account the effect of anisotropy (Stokoe et al. 1985, 1991, Hardin and

Blandford 1989, Lo Presti and O'Neill 1991, Bellotti et al. 1996, Jamiolkowski et al. 1995, Lo Presti 1989).

4. The method and material tested

The data for evaluation of applicability of shear wave velocity to quantify static liquefaction problem were obtained on the basis of laboratory and field tests. The majority of experimental work was done in the laboratory. Triaxial tests were carried out on reconstituted specimens 70 mm diameter and HD ratio of around 2. In order to get a precise value of initial void ratio, the specimens were prepared by moist tamping undercompaction method as described by Ladd (1978). When a triaxial cell was assembled the specimen was flushed with CO₂, and then saturated with deaired water. It is important to note that during first flushing with water the volume of specimen was controlled by proximity transducers as described by Lipinski et al. (2020). Next the specimen was saturated by back pressure until Skempton's parameter B was in the range value 0.96-1.00. After saturation the specimens were consolidated anisotropically with zero lateral strain, thus the relation between principal effective stresses at the end of consolidation stage depended on the initial density of soil. When consolidation was terminated, shear wave

velocity was measured. Piezoelectric transducer of bender type was used for sending and receiving the signals. The lab set up for shear wave velocity measurement is shown in Fig. 3a. In order to project data from the laboratory to a field profile, it was necessary to measure shear wave velocity in situ. In Figure 3b scheme for shear wave velocity measurement in the vertical direction are shown. Besides down hole tests which are carried out in a specially prepared borehole, the shear wave velocity might be routinely measured during static penetration SCPTU and SDMT. These measurements are taken every meter of depth. Seismic dilatometer is equipped with two geophones while the CPTU probe usually has one geophone. The data obtained for the purpose of the described test programme came from SCPTU probe.

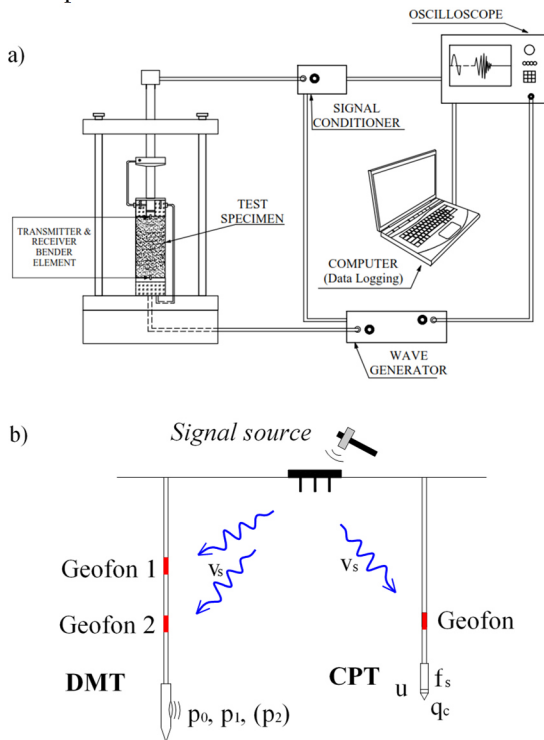


Figure 3. Laboratory and field equipment for shear wave velocity measurement.

Laboratory triaxial tests were carried out on two cohesionless materials with various amount of fines (understood as a material passing ASTM sieve # 200 having diameter 0.075 mm). More sandy material had 10% of fines while the finer one 36%. Both materials were predominantly silicious. Their grain size distributions are shown in Fig. 4.

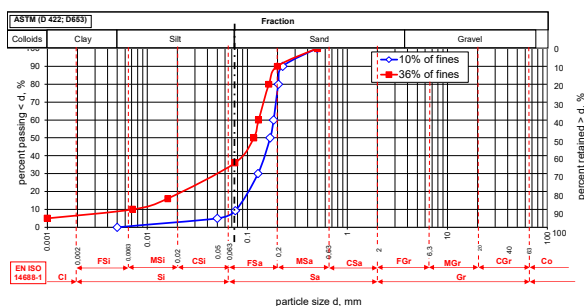


Figure 4. Grain size distribution curves

5. Laboratory test results

Data obtained in the laboratory concerned two important stages. The first one concerned a state prior to shearing from which the value of void ratio and corresponding shear wave velocity was obtained. The second set of data concerned the end of shearing stage on the basis of which the steady state lines for each material were obtained.

Regarding shear wave velocity measurement as indicated in Section 3, V_s value depends on void ratio, fabric component and stress level. However, since void ratio and state of effective stress contribute to a state of soil, it is necessary to eliminate influence of state of stress on measured value of shear wave velocity. This can be done by normalisation of shear wave velocity with respect to stress. One of the possible approaches was proposed by Robertson et al. (1992).

$$V_{s1} = V_s \left(\frac{Pa}{\sigma'_v} \right)^{0.25} \quad (2)$$

where:

V_{s1} - normalized shear wave velocity (m/s)

V_s - shear wave velocity (m/s)

σ'_v - vertical effective stress (kPa)

Pa - reference pressure (usually 100 kPa)

In order to check to what extent shear wave velocity is capable to reflect state of soil with various fines content the data obtained for two tested materials were presented in Fig. 5. For each material tested the data were shown on two charts. In charts (a), raw results of measured V_s were shown against void ratio, while in charts (b) a measured shear wave velocity was normalized with respect to the vertical effective stress, as indicated above. The data for each material are different significantly. For sandy material with 10% of fines the raw results represent large scatter of points and normalization procedure makes a nice relationship between void ratio and normalized shear wave velocity. Such relationship enables easy determination of void ratio when the values of shear wave velocity and vertical component of effective stress are known. Quite different situation is in the case of material with 36% of fines. In this case, although the trend of regression line is clear, the normalization procedure does not seem to work as efficient as in case of sandy material. This difference in behaviour of two materials results from various compressibility characteristics in both cases.

The above conclusion seems to be confirmed by steady state lines for both materials which are shown in Fig. 6. The slope of both steady state lines is quite similar but the major difference is in a position of SSL. For soil which contains 10% of fines the steady state line plots much higher than for sand with 36% of fines. This clearly reflects different compressibility characteristics. Void ratio of material having fines in the range 25-40% reveals minimum values of void ratio (Lipinski et al. 2017a, 2017b).

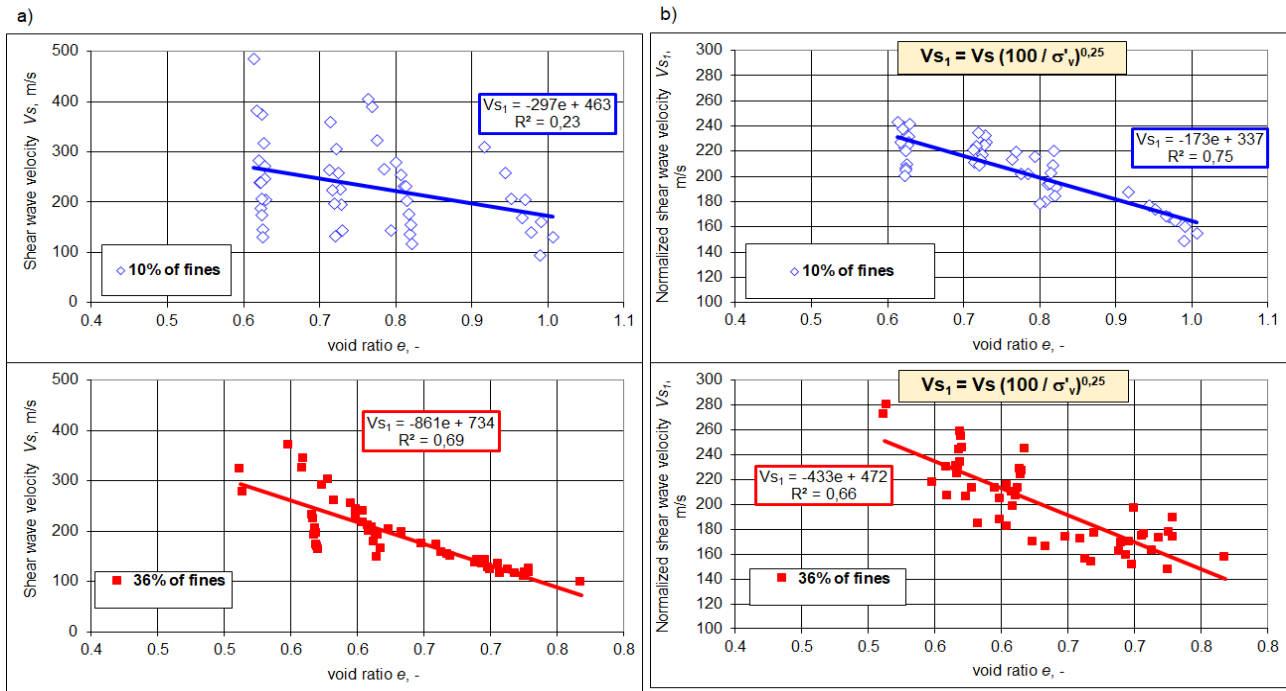


Figure 5. Raw (a) and normalized (b) shear wave velocity measured in the laboratory shown against void ratio for materials containing 10 and 36% of fines.

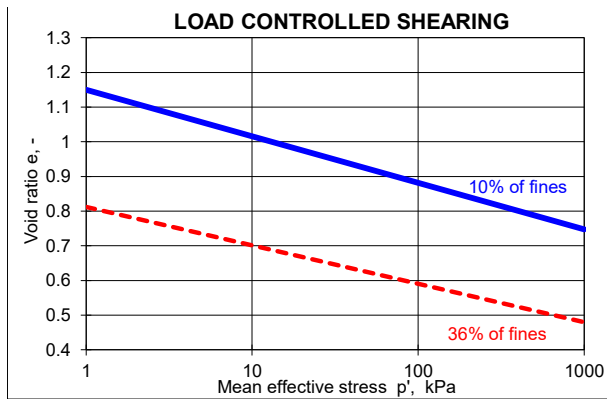


Figure 6. Steady state lines for materials containing 10 and 36% of fines.

6. Hybrid approach to determine state parameters *in situ*

The term "hybrid approach" is understood here as a correlation of laboratory and *in situ* test results, consisting in projection of void ratio profile determined from shear wave velocity on the basis of the formula obtained during laboratory tests. The idea of this method in application to evaluation liquefaction hazard in tailings dam constructed by upstream method were described by Lipinski et al. (1997). The scheme of this method is shown in Fig. 7. A combination of the formulae for void ratio calculation obtained on the basis of regressions of normalized shear wave velocity shown in Fig. 5b with field measurement of V_s makes possible projection of void ratio on a soil profile. It should be pointed out that relations between shear wave velocity and state of stress can be described by various formulae. The method resting on normalized shear wave velocity with respect to vertical effective stress, although most popular and often used, is only one of at least a few methods conceivable to

use for reduction of the data (Lipinski and Wdowska 2019, 2020).

The results of application of the above described hybrid approach of evaluation of void ratio *in situ* are shown in Fig. 8. The profiles of void ratio are presented for both granulations. The major observations can be summarized as follows:

- Experience acquired with void ratio determined on the basis of conventional sampling indicates that distribution at this parameter determined on the basis of shear wave velocity is less scattered.
- Void ratio values for each tailings batch are approximately in the range observed during compressibility tests.
- Results for upper part of the beach profile (from surface to ca 5 m) indicate that tailings in this zone reveals lower value of void ratio resulting from compaction equipment densified by compaction equipment.
- From approximately 11 m down, an apparent decrease in void ratio is observed. Probably at this depth vertical stress resulting from gravitational forces start to prevail.
- For the same depth and measured shear wave velocity values the following rule holds true: higher fines content is associated with smaller value of void ratio.
- In general, presented void ratio values in a profile show less scatter than obtain on the basis of static penetration (CPTU).

As indicated in the block diagram in Fig. 7, if the steady state line is determined for a given soil kind it is possible to evaluate whether material would reveal contractive or dilative response during shearing. To project steady state void ratio e_{ss} on tailings profile, the knowledge of mean effective stress p'_{ss} and SSL parameters are necessary.

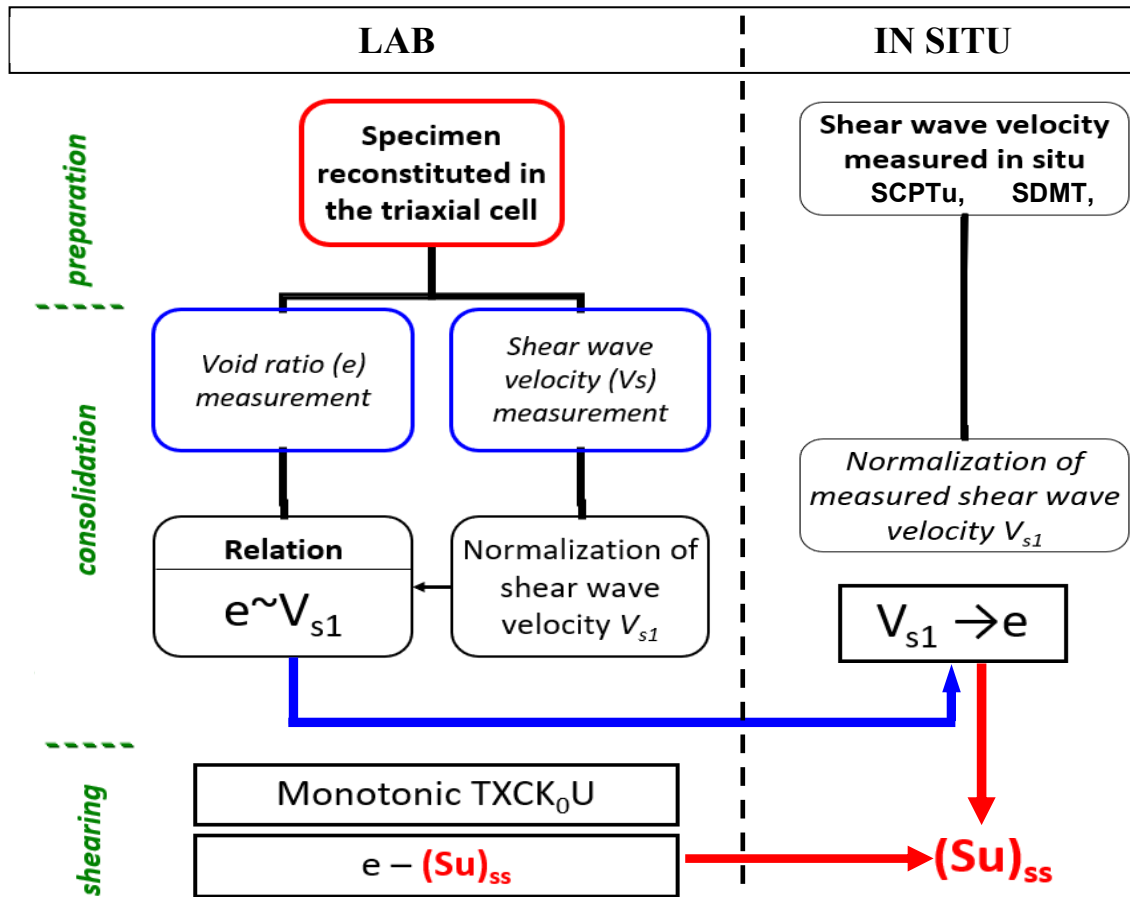


Figure 7. Approach to evaluate in situ state of cohesionless materials and undrained shear strength $S_u(ss)$ during liquefaction on the basis of shear wave velocity measurement.

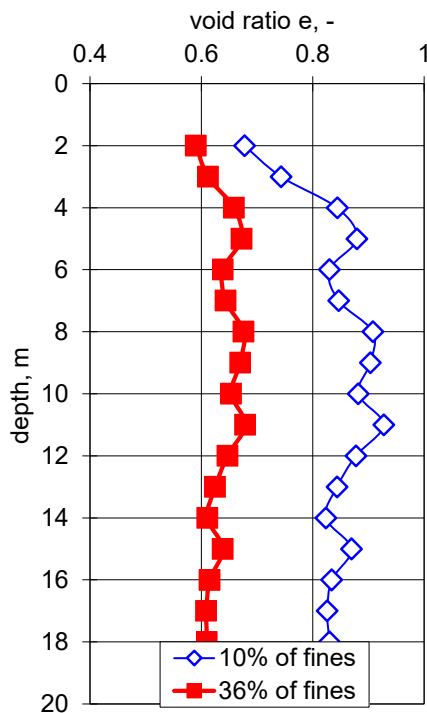


Figure 8. Void ratio distribution projected on soil profile on the basis of laboratory and field measurement of shear wave velocity.

$$e_{ss} = -\lambda_{ss} \ln(p'_{ss}) + \Gamma_{ss} \quad (3)$$

Comparison of initial and steady state void ratio in tailings profile for material containing 10 and 36% of fines is shown in Fig. 9.

The data seem to confirm a hypothesis that fines content might considerably change the state of soil. The results show that for sandy material with 10% of fines content in profiles negative values of state parameter ψ prevail, which means that the response during shearing would be dilative. On the contrary for material containing 36% of fines the state parameter would be positive implying contractive response. This means that such soil might be susceptible to liquefaction. In order to quantify this statement, the data from Fig. 9 were converted to steady state undrained shear strength $(S_u)_{ss}$ profile shown in Fig. 10. The obtained profiles for considered granulation are significantly different. Shear strength profile for soil with 36% of fines is extremely low and pretty uniform. $(S_u)_{ss}$ distribution for coarser material (10% of fines) is nonuniform and changes in a very wide range (30-300kPa). It should be emphasized that the obtained profiles bear consequences of inherent simplified assumptions. Shear wave velocity measured in the laboratory can be assigned to tested material while V_s measured in situ is for real tailings with very nonuniform granulation.

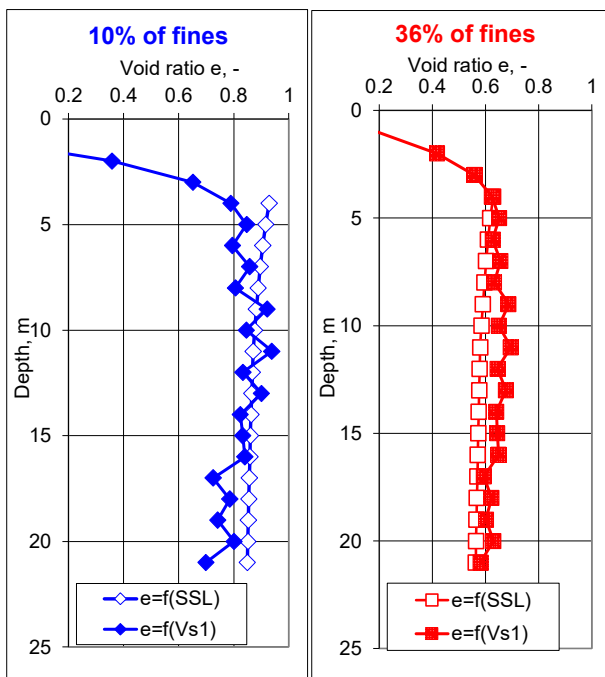


Figure 9. Relation between void ratio calculated on the basis of normalized shear wave velocity and steady state void ratio determined for considered granulation.

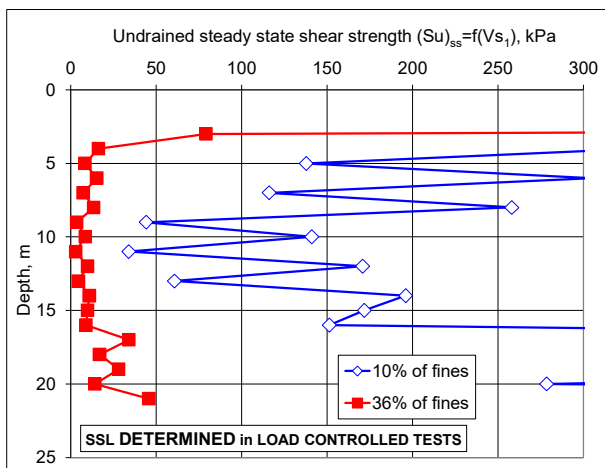


Figure 10. Distribution of undrained steady state shear strength in a profile for considered granulation on the basis of hybrid shear wave velocity measurements.

7. Conclusions

In evaluation of state of tailings dams constructed by upstream method there is necessity to look for solution which meets severe requirement concerning accuracy of void ratio determination in deposited slurry. The paper presents approach based on parallel measurement of shear wave velocity in the laboratory and in the field. The experimental work has proved that correlation between void ratio and normalized shear wave velocity obtained in the laboratory on two kinds of reconstituted material containing 10 and 36% of fines can be successfully projected on the soil profile in the field. However, the conversion of the data to undrained shear strength profiles through steady state line does not seem realistic. In case of $(S_U)_{ss}$ values, the correction accounting for grain size distribution in a real profile is necessary to obtain more consistent results.

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