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Linking laboratory quasi-steady state strengths to field scale performance of tailings

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ABSTRACT

Current state of practice for the assessment of liquefied strengths of tailings relies primarily on empirical or semi-empirical correlations based on penetration testing. That is, despite liquefied strength being arguably the most important strength parameter for the design of brittle tailings storage facilities, there is much less success or acceptance of the use of laboratory element tests to support strength selection compared to other forms of strengths inferred in geotechnical engineering. This is particularly the case for tailings at a state near or slightly dense of the critical state line (CSL) for which there is ample evidence of field-scale flow liquefaction but where laboratory element tests often behave in a manner inconsistent with such field-scale response – at least at large strains. The current paper examines the quasi steady state (QSS) strength of sands and tailings for which the CSL has been measured, linking the observed strengths to inferred in situ behaviour through the state parameter. Particular focus is placed on QSS strengths obtained from simple shear tests carried out within a hollow cylinder torsional shear system where the stress state in the test is a better representation of in situ below-slope conditions that the triaxial compression test. In particular, the marked effect of intermediate principal stress on the QSS in sands is highlighted. Alternatively, the negligible anisotropy seen in a sandy silt gold tailings, and the potential implications in the context of QSS strengths and field-scale behaviour, are examined and emphasised.

Keywords: Tailings; quasi-steady state; liquefied strength

1. Introduction

The significant risks posed by loose deposits of hydraulically placed fills have been made clear through a series of recent major tailings storage facility (TSF) failures. Partially as a result of these failures, engineering practice is moving towards mandating that loose deposits be stable under the assumption that “triggering” will happen, and the mobilisation of a minimum post-peak strength occurs. Triggering in this context refers to in situ elements of soil relevant to slope stability reaching or exceeding their instability stress ratio by means of various plausible in situ stress paths, leading to a loss of strength.

While ensuring stability should triggering occur is undoubtedly likely to make such structures safer, it brings additional focus on the selection of an appropriate minimum post peak strength – with the term “liquefied shear strength” being used in this paper. Current state of practice focuses largely on empirical techniques involving penetration testing, with negligible input provided by laboratory tests. This limited use of laboratory testing likely being a result of the significant discrepancies seen between full scale (via back analyses of flow liquefaction case histories) and laboratory tests – the most common laboratory tests being triaxial compression (TXC) that often show dilation at a range of state parameters Ψ (Been and Jefferies 1985) relevant to many flow liquefaction case histories.

An important subtlety in the potential selection of an appropriate liquefied shear strength from element tests is shear localisation. For example, refer to the idealised test data shown in a schematic mean effective stress p' – deviator stress q plot in Figure 1. Three idealised responses are shown: Path A shows unambiguous post-peak strength loss to a minimum strength. Path C shows strain hardening and dilation consistent with a dense soil, and thus strength loss considerations are obviated for that particular soil under those particular conditions. Finally, and the focus of the current work, Path B exhibits a post-peak strength loss followed by subsequent strain hardening, where the “local minimum” values observed is referred to as the quasi-steady state (QSS) strength. This behaviour has received much attention, with it forming a promising candidate for the relevant strength in an element test that would be most representative of what would occur in the field for an element sheared under such conditions (Alarcon-Guzman et al. 1988, Konrad 1990, Sadrekarimi 2014, Jefferies and Been 2015). Broadly, this response is seen for many types of sands and sandy silts when sheared from Ψ of about -0.05 to +0.05. While the QSS behaviour is in general well recognised, in the authors’ perspective, it has not been analysed sufficiently such that QSS strengths are regularly obtained and used to refine in situ estimated values in design and engineering practice.

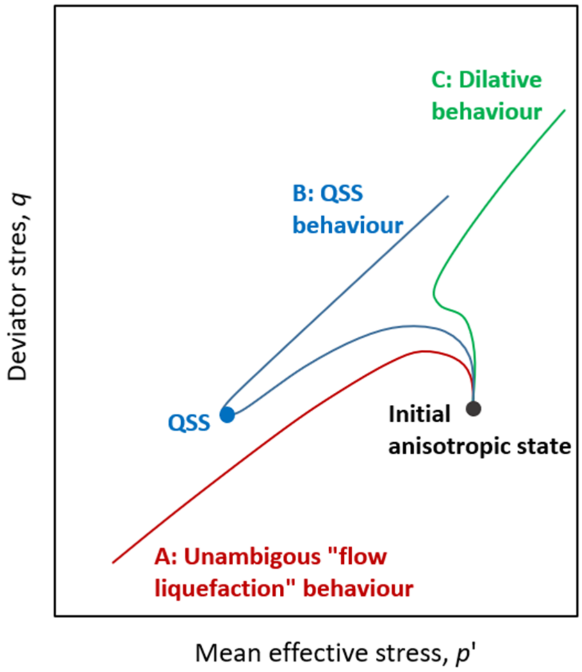

Figure 1. Schematic of different forms of undrained shearing behaviour, with an emphasis on the QSS

The purpose of the current work is to further examine the relevance of QSS strengths measured in the laboratory to field scale behaviour seen in flow liquefaction case histories. The linkage between these two conditions is made based on the state parameter. To enable the comparisons, relevant QSS data and Ψ values from element tests are obtained from the literature and through additional testing carried out to support this study.

2. Background and current state of practice

The current state of practice for the assessment of flow liquefaction susceptibility of TSFs and other loose sandy or silty fills, while varied and lacking standardisation, does feature some common elements that are relevant to the scope of the current paper:

- Increasingly, the significant risks of flow liquefaction and likely lack of obvious indicators before such brittle failures has led to the view that stability of earth structures comprising loose saturated material must be ensured assuming "triggering" occurs, and thus that liquefied strengths are mobilised (Robertson 2010, Been 2016, Jefferies 2018, Morgenstern 2018).
- Based on the work of Shuttle and Cunning (2007) and Jefferies and Been (2015), a boundary of flow liquefaction susceptibility of Ψ = -0.05 sees widespread adoption across soils ranging from sands to clayey tailings.
- To estimate liquefied strengths for zones with Ψ>-0.05, screening-level techniques based on penetration testing are often applied (Olson and Stark 2002, Robertson 2021) owing to their simplicity to employ, widespread adoption, and their ability to capture, in an approximate empirical sense, field-scale processes that may not be reproducible in laboratory element tests.
- The "best practice approach" proposed by Jefferies and Been where liquefied strength is estimated directly from Ψ is finding increasing application, including strong recommendation in recent tailings guidelines (e.g. ICOLD 2022).

The adoption of -0.05 as the boundary of flow liquefaction susceptibility requires further considerations prior to proceeding. This criterion is largely based on interrogation of case histories: for example, Jefferies and Been (2015) by examining case histories such as the Nerlerk berm infer Ψ values in the range of -0.05 to 0.0. As a more recent example, the characteristic state for the Fundão TSF was +0.02 (Morgenstern et al. 2016) and clearly underwent a brittle response resulting in flow liquefaction, while the coarse tailings at Feijão appear to have a Ψ range from -0.05 to 0.0 and were inferred to have a liquefied strength ratio of 0.05 to 0.10 (Robertson 2021, Reid et al. 2022c). While these case histories therefore appear to suggest the clear ability of in situ soil that is dilative, or at most only slightly contractive (at high strains) to undergo flow liquefaction, TXC element tests on specimens prepared to similar states generally produce quite contradictory behaviour.

The field vs element scale behaviour differences for soils with Ψ of -0.05 to 0, for example, can perhaps best be examined through three figures prepared by Jefferies and Been (2015) comparing (i) back analysis of flow liquefaction case histories, (ii) the "steady state" calculated values, if assuming undrained behaviour, contraction to the CSL, and a critical state friction ratio, and (iii) the best practice trends proposed by Jefferies and Been. These figures are reproduced here in Figure 2, with the soils grouped in terms of approximate stiffness as: "stiff" (e.g. sandy), "intermediate", and "compressible", and where the CSL slope λ_{10} is used to define intrinsic compressibility in this context. For the sandy/stiff soils presented in Figure 2a, the significant discrepancy between the steady state approach and the back analysed values from case histories is evident. However, as compressibility increases, the discrepancy between the steady state approach and case history behaviour appears to decrease. It is the overall behaviour shown in Figure 2, with special considerations towards the different behaviour of sands and more compressible materials, that forms the focus of the remainder of this work.

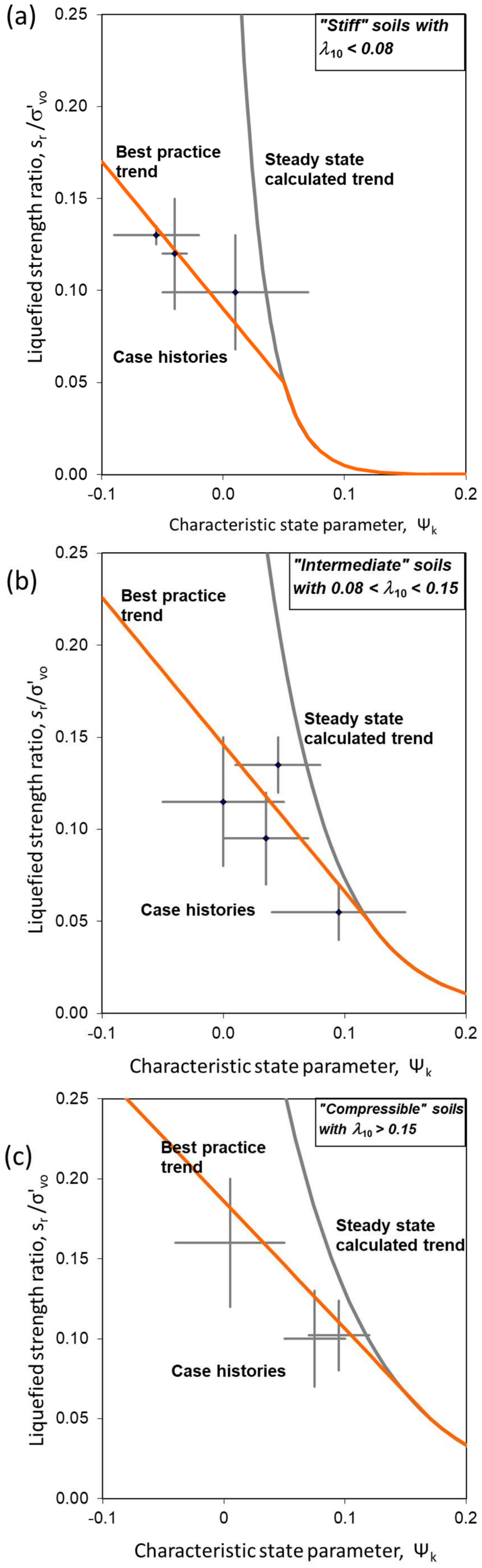

Figure 2. "Best practice approach" developed by Jefferies and Been (2015): (a) "stiff" soils, sands and silty sands, (b) intermediate soils, (c) more compressible soils. Used with permission from CRC Press, Boca Raton, FL.

3. Sand behaviour

3.1. Criteria for selection

It has long been established that hydraulically-deposited sands feature fabric anisotropy (Arthur and Menzies 1972), which means that the angle between the major principal stress and the vertical axis (herein defined as principal stress angleα), during shearing can have a significant effect on the undrained strength and shearing response of sands depending on the particle morphology and arrangement – including at the QSS (Sivathayalan and Vaid 2002). Similarly, given the effect of intermediate principal stress ratio *b* on critical state friction ratio, this can also result in differing shear behaviour in situ (below a slope) than in a TXC test. Therefore, while the QSS represents a strong candidate for the laboratory-derived strength most relevant to field scale post-peak behaviour, measuring the QSS under loading conditions relevant to below-slopes is important.

To enable an examination of relevant sand and silty sand data in a critical state framework, a review was carried out to identify materials for which the following data was available: (i) the CSL, to enable calculation of Ψ, (ii) triaxial compression (TXC) test data that included tests with Ψ less than about +0.05 to examine the behaviour of soils likely to exhibit a QSS, and (iii) where possible, hollow cylinder torsional shear (HCTS) test data either carried out as a simple shear test (HCTS SS) or as undrained shearing at constant α and b relevant to below slope conditions, under the same range of Ψ as sought for the TXC tests. Sands and silty sands identified that meet these criteria are summarised in Table 1. Toyoura sand was previously investigated in this context by Reid et al. (2022), with Silica Fine Sand (SFS) testing by Fanni et al. (2022) and Ottawa sand and silty sands tested by Murthy et al. (2007) and Ottawa 20-30 tests as HCTS SS by Alarcon-Guzman et al. (1998) with reference CSL provided by Santamarina and Cho (2001).

Table 1. Summary of sands and silty sands considered

Sand	References
Toyoura	(1996) Yoshimine et al. (1998) Nakata et al. (1998) Chiaro et al. (2013) Umar et al. (2021)
SFS	Fanni et al. (2022b)
Ottawa **(0, 5, 10, 15% FC)**	Murthy et al. (2007)
Ottawa 20-30	Alarcon-Guzman et al. (1988) Santamarina and Cho (2001)

As an example of the QSS behaviour seen in the HCTS tests reviewed, Figure 3 presents two tests on SFS sand (Fanni et al. 2022b) sheared undrained with α=45°, b=0.5, prepared using the dry pluviation technique to achieve a Ψ < -0.02, while avoiding tamping-induced overconsolidation that is likely if attempting to prepare to such a density using compaction. The dramatic post-peak strength loss to a low QSS strength is seen, with a resulting liquefied strength ratio of $s_r/p'_c = 0.06$. This behaviour is entirely consistent with field-scale observations of flow liquefaction for deposits at similar Ψ. Alternatively, a TXC from such conditions shows strong dilation through undrained shearing, and no post-peak drop to a QSS strength.

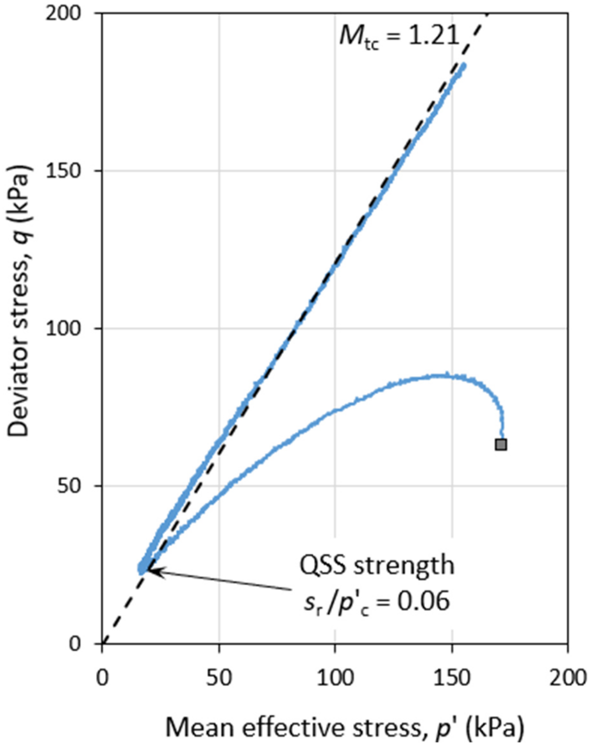

Figure 3. Example HCTS undrained test, dry pluviated specimen of SFS sand. $\Psi = -0.02$, $\alpha=45°$, $b=0.5$. After Fanni et al. (2022b).

3.2. Synthesis of results

The results of the tests on the sands summarised in Table 1 are presented in Figure 4 in the same general form as Figure 2. State parameter is based on the void ratio of each test and the published CSLs for the sands, while the relevant liquefied shear strength is taken as the QSS strength, consistent with the framework and hypothesis of the current paper. Included in the plot for reference are the steady state-calculated trend lines and best practice approach lines based on an M_{tc} of 1.25 (average for the sands considered) and λ_{10} values of 0.018 and 0.051, which represent the range of values for the sands examined. It is reiterated that these best practice trend lines were developed by Jefferies and Been (2015) based on back analysed case histories, and thus are the most reliable data available for what liquefied strengths would manifest at the field-scale from those Ψ values.

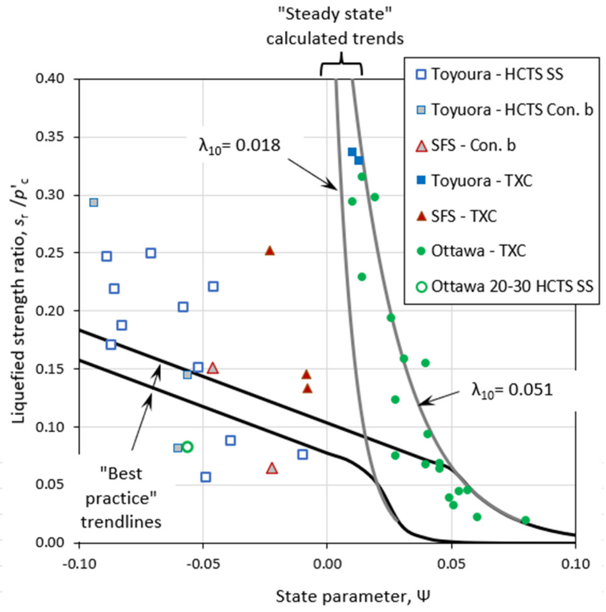

Figure 4. Synthesis of sand and silty sand TXC and HCTS-SS tests in framework developed by Jefferies and Been (2015). Experimental data and reference trends from Steady State calculations and best practice trend included for reference.

In the framework of Figure 4, it is immediately clear that the TXC tests, even when taking the value from the QSS, far exceed those suggested by the best practice trends (which are derived from field-scale observations). Alternatively, HCTS tests sheared undrained in modes relevant to below-slope conditions (HCTS SS or constant α/*b*) agree far better with the expected range based on flow liquefaction case histories. This is a promising result, and is logical given much historic argument as to the importance of anisotropy in sands in the context of their field scale behaviour. However, it is particularly clarifying to view the data in the framework of state parameters that enables comparison to case histories.

4. Sandy silt gold tailings behaviour

4.1. Materials and methods

While evidence supporting the relevance of the QSS strength obtained from shearing modes consistent with below-slope conditions in relatively stiff sands and silty sands appears clear, the question then turns to the behaviour of more compressible materials such as predominately silt tailings. As noted previously (e.g. Figure 2), there is evidence that increasing compressibility leads to less discrepancy between the steady state-calculated strengths and those inferred from back analyses – and thus potentially a decreasing relevance/prominence of QSS behaviour. Further investigation of this trend could serve to refine the limits of flow liquefaction susceptibility for more compressible soils, something of significant practical benefit to engineering practice.

To the author's knowledge, the literature does not include any predominately silty soil or tailings that includes the requisite data (CSL, tests with QSS, HCTS tests) to develop comparisons such as Figure 4. Therefore, testing specifically to support the current work was carried out on sandy silt gold tailings. The tailings is a sandy silt with 59% <75µm, is non-plastic, and has a Specific Gravity G_s of 2.78. It has been thoroughly characterised including determination of the CSL and investigating sample preparation effects (Reid et al. 2022a), DSS and HCTS-SS tests on loose specimens (Fanni et al. 2022a), various investigations of sample preparation effects on the limiting compression curve (Reid et al. 2023), and drained plane strain HCTS testing (Fanni et al. 2023). Importantly, with a CSL slope of $\lambda_{10} = 0.083$, it falls within the "intermediate" compressibility range in the context of the Jefferies and Been (2015) framework and is therefore well suited to extend the investigation beyond sands.

To investigate the behaviour of this material at a range of ψs that produce a QSS-type response, the comparison focuses on two sets of tests prepared using the "air dried" (AD) sample preparation method (Reid et al. 2022e) that is suitable for achieving states near to the CSL while avoiding tamping-induced overconsolidation, as follows:

- Triaxial tests on AD specimens carried out as part of the development of the AD preparation technique, with an additional triaxial test carried out as part of this study to supplement the available data.
- HCTS-SS tests, also on AD specimens, carried out as part of the current study. Tests were consolidated anisotropically, with a drained static bias α_c ranging from 0 to 0.18, to better simulate below-slope conditions.

A summary of the gold tailings tests considered in this study is provided in Table 2.

Table 2. Gold tailings TXC and HCTS SS tests

Test	p'_c (kPa)	e_c	α_c	QSS s_r/p'_c	ψ
TXC-1	81	0.595	n/a	0.33	-0.037
TXC-2	506	0.536	n/a	0.44	-0.026
TXC-3	80	0.607	n/a	0.33	-0.025
TXC-4	534	0.521	n/a	0.43	-0.038
HCTS SS-1	99	0.659	0	0.25	-0.032
HCTS SS-2	250	0.580	0.14	0.31	-0.012
HCTS SS-3	249	0.582	0.18	0.32	-0.015

4.2. Results

The results of the TXC and HCTS tests are presented in Figure 5 as $p' - q$ plots. All the tests indicated initially contractive behaviour despite being slightly dense of the CSL, consistent with general expectations for the shearing behaviour of specimens near the CSL. All tests show a QSS, although the drop from the initial peak values is very modest compared to the SFS sand behaviour seen in Figure 4. The TXC tests were sheared to relatively high axial strains of ~20%, and were tending to the CSL at the end of shearing, with some clear localisation and shear banding present in the samples. Alternatively, the HCTS SS tests showed significant localisation at 5-10% shear strain, which appears to inhibit the measurement of post-QSS dilation and strain hardening from the global specimen measurements available in the apparatus used. Perhaps most importantly, despite the HCTS SS shearing involving a rotation of principal stress angle under plane strain conditions, the general behaviour of these tests appears very similar to the TXC tests – in sharp contrast to much of the sand comparisons made previously. It is noted that a reduction of fabric anisotropy in silts, compared to sands, has been previously observed in reconstituted specimens (Bahadori et al. 2008), which may be generally consistent with the observations of the reconstituted gold tailings samples presented here.

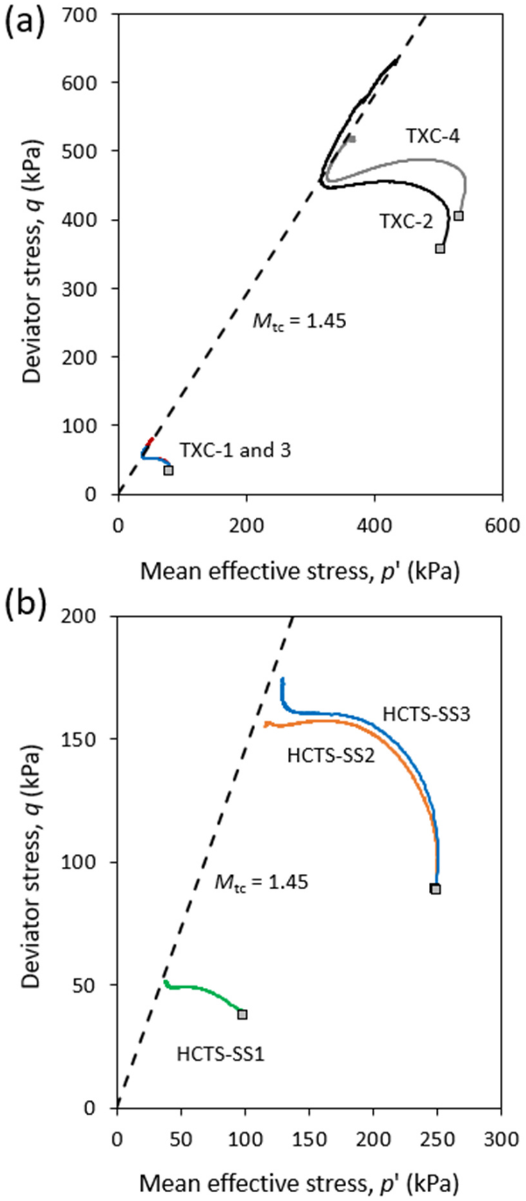

Figure 5. Gold tailings test results, $p' - q$ format: (a) TXC, and (b) HCTS-SS

While the qualitative response seen in Figure 5 did not indicate particularly brittle drops in strength to the QSS following the initial peak, as might be expected in a flow liquefaction scenario, the strengths at the QSS observed nonetheless warrant further investigation. Figure 6 presents the data from the gold tailings in the usual framework adopted here, with state parameter on the x-axis and the Jefferies and Been (2015) derived best practice trend and steady state approach included. Two aspects that are immediately apparent are the significantly higher QSS strengths seen in the gold tailings tests compared to the best practice trend, and the similar apparent strength- Ψ trend for the TXC and HCTS SS results. This is inconsistent with what was seen for sands and implies that the HCTS SS on the AD-prepared specimens, despite achieving desired Ψ values in the -0.05 to +0.05 range for comparison to flow liquefaction case histories with similar conditions, does not appear to reproduce the in situ behaviour seen in field scale behaviour.

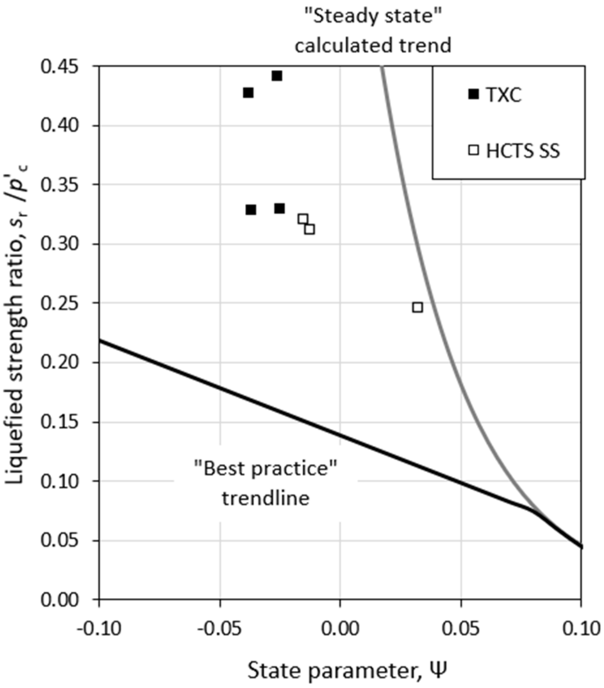

Figure 6. Synthesis of gold tailings TXC and HCTS-SS tests in framework developed by Jefferies and Been (2015). Experimental data and reference trends from Steady State calculations and best practice trend included for reference.

5. Unanswered questions and ways forward

Looking at the HCTS SS tests on gold tailings in isolation, the results would appear to suggest that once one moves to more compressible soils than sands, the importance of the QSS to field-scale behaviour diminishes. For example, the QSS strengths seen in HCTS and TXC tests on the gold tailings were indistinguishable from one another, and the overall behaviour of the AD specimens was such that flow liquefaction would appear unlikely for the Ψ values less than about +0.03 achieved using the AD technique for these tests. However, some significant caveats must be acknowledged:

- The evidence from field scale case histories is inconsistent with the element tests for the gold tailings. This contrasts with the sand testing where the HCTS and field scale behaviour appeared to agree. Given the implications of the classification of tailings as being susceptible (or not) to flow liquefaction, clearly the limited results obtained here are insufficient to challenge the current assumption of Ψ = -0.05 for intermediate compressibility tailings
- An important question then becomes: why are the HCTS SS results showing higher QSS strengths than field scale behaviour? Accepting for the moment the relevance of the QSS, and the premise that laboratory tests could be carried out in a such a way o reproduce to some degree in situ stress conditions, an important consideration is the fabric anisotropy, and in particular how this anisotropy can be reproduced in an element test. It may be that in situ samples that exhibit layering – a ubiquitous feature of hydraulically-deposited tailings (e.g. Reid et al. 2022d) could exhibit greater anisotropy, and thus produce more relevant QSS strengths when sheared under appropriate loading conditions.
- Fortuitously, HCTS and TXC tests were previously carried out on intact specimens trimmed from high quality blocks of the same gold tailings tested in this study (Reid et al. 2022b). The HCTS test relevant to the current work was sheared with constant α of 45 degrees and constant *b* of 0.3, and thus is relevant to below-slope conditions. The behaviour of the HCTS and TXC specimens are compared in Figure 7. Far greater anisotropy is observed in the layered in situ sample compared to the uniform AD specimens reconstituted in the laboratory. This points strongly to the potential for in situ, layered deposits to - logically we would argue, and consistent with other works (Nishimura et al. 2007) - exhibit greater anisotropy than a reconstituted AD specimen prepared from a thick non-segregating slurry. While the techniques that may be required in the future to successfully examine such layered specimens rationally within a critical state context are beyond the scope of the current work, we would argue this likely represents a productive way forward.

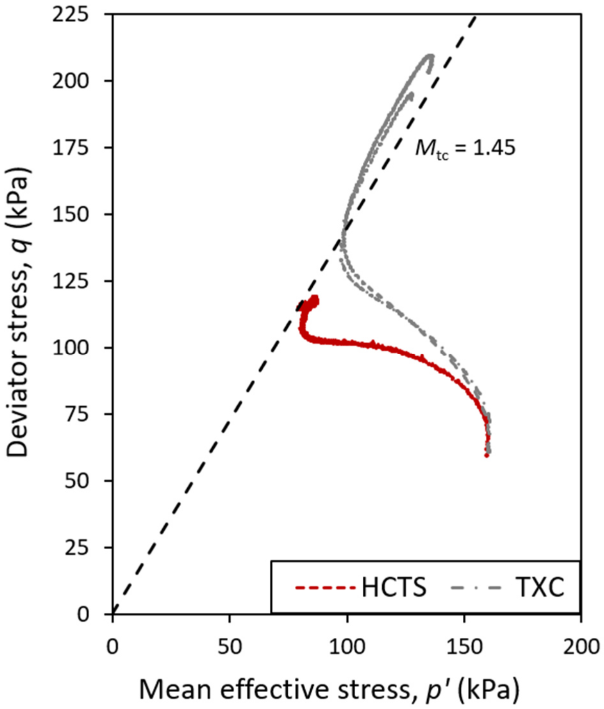

Figure 7. Comparison of intact HCTS and TXC tests on gold tailings. HCTS test sheared with α=45° and *b*=0.3. After Reid et al. (2022b)

6. Conclusions

Liquefied shear strengths of loose saturated tailings and soils are of great importance to the current state of practice for analysis of the stability of earth structures. While several empirical tools exist to estimate liquefied strengths, given the importance of this parameter, the significant implications of over- or under-estimating liquefied strength, and the fundamental disagreement between element and field-scale behaviour of soils with Ψ = -0.05 to +0.05, further investigation is important. The current study compared the QSS strengths of element tests to the back analysed liquefied strengths of flow liquefaction case histories, with particular focus on undrained shearing modes relevant to below-slope conditions. This was accomplished by a literature review of relevant HCTS testing programs on sands, and additional testing of a sandy silty gold tailings to expand the available data to more compressible soils.

The results indicated that sands sheared in modes relevant to below-slope conditions exhibited QSS strengths that aligned well with the back analysed strength values seen for flow liquefaction case histories. In contrast, a more compressible sandy silt gold tailings exhibited QSS strengths from HCTS SS and TXC tests that were similar to one another, and significantly overestimated the expected strengths of such a material at the Ψ values tested. It was speculated that the AD sample preparation method adopted herein may lead to specimens with minimal fabric anisotropy, which seemed to be supported by comparison to other HCTS testing on intact layered specimens of the same gold tailings.

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