Behaviour of saturated sand under cyclic loading: model approach with experimental validation

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ABSTRACT

This study presents a comparison between triaxial test results and constitutive model simulations for saturated sand subjected to a wide range of stress, relative density and both monotonic and cyclic loadings. This is a preliminary part of a project which is focussed on developing models for the behaviour of unsaturated soils under cyclic loading. There are currently limited experimental data of the cyclic behaviour of unsaturated soils due in part to the difficulty of controlling stress state, degree of initial saturation and uniformity of samples in experiments and their interpretation is challenging due to the mathematical complexity in constitutive models. This study attempts to bridge the abovementioned gap by creating an experimentally validated constitutive model for dynamic loading on saturated sand and then to extend this for unsaturated sands. The paper reports results from a series of monotonic triaxial tests and cyclic triaxial tests on saturated sand. Specimens of 100mm diameter, 200mm height and 54mm diameter, 110mm height have been prepared with different void ratios. These samples have been subjected to a wide range of deviatoric stress in both monotonic and cyclic tests. The results from these tests on saturated Sydney sand are presented and used to demonstrate the ability of the constitutive model. As the constitutive model has been developed for variable saturation demonstration of its success with saturated sand for which considerable data is available is the first stage in simulating the cyclic behaviour of the unsaturated sand.

Keywords: Saturated sand, Cyclic behaviour, triaxial tests, Constitutive model.

1. Introduction

The general behaviour of partially saturated soils has been researched and investigated for many years because of its widespread application. However, studies of partially saturated soils under cyclic loading are limited. Experimentally difficulties arise in controlling the stress state, the degree of saturation and specimen uniformity, especially at reasonable rates of cyclic loading. Available numerical models that capture the cyclic deformation mechanism are also extremely limited (Ghayoomi et al. 2013).

The cyclic behaviour of soil is generally studied through soil element tests such as triaxial tests, and these have been used to investigate the cyclic resistance of partially saturated sands (Chaney. 1978; Ishihara et al. 2004; Unno et al. 2008; Kimoto et al. 2011). These studies have demonstrated a relationship between soil stiffness and degrees of saturation, i.e., soil stiffness and cyclic resistance increase as the degree of saturation decreases, and it is found that they are dependent on soil grading and compressibility (Tsukamoto et al. 2014). Cyclic loading studies have been performed for partially saturated silty soils (Whang et al. 2004; Chin et al. 2010) and well graded road-base like materials (Ohiduzzaman et al. 2012; Ishikawa et al. 2014) in which sufficient information to understand the soil behaviour (pore pressures, volume changes and degree of saturation) has been measured. Cyclic tests of partially saturated well graded metallic ores with significant fines contents have also been reported (Wang et al. 2016, Kwa and Airey 2020) as part of studies to assess liquefaction of cargoes in ships. It is observed that the cyclic testing of partially saturated soils has predominantly used triaxial apparatus.

The ultimate purpose of these experiments is to understand the behaviour of the soil and to formulate an experimentally validated model. Most of the numerical approaches are formulated within the finite element framework to describe the continuum equations of the mixture of air, water and solid phases, based on the concept of effective stress, with different assumptions and simplifications (Lewis and Schrefler 1982; Khalili et al. 2008; Shahbodagh-Khan et al. 2015; Ghorbani et al. 2016 and Ghorbani et al. 2018).

For this study a recent constitutive model, developed for cyclic loading of multiphase, partially saturated, granular soil (Ghorbani and Airey, 2021) has been adopted. As a first step in assessing the model's ability to simulate the cyclic behaviour with variable saturation this paper is limited to simulations of fully saturated behaviour. The key assumptions of the model and process of selecting the many model parameters are discussed. Finally, the model predictions are compared with results from selected monotonic and cyclic triaxial tests covering a range of deviatoric stress means and amplitudes.

2. Soil characterization

The tested material is washed Sydney beach sand. It is classified as SP in the Unified System, which is uniform in grain size and poorly graded. The grain size distribution curve is shown in Fig. 1. Sydney sand has been studied by several researchers and there is extensive triaxial data available (Rahman and Lo, 2007 and Mohammadi and Airey, 2019). However, due to the natural material being sourced from different locations with slight variations in particle characteristics, parameters such as those describing the position of the critical state line, are not consistent.

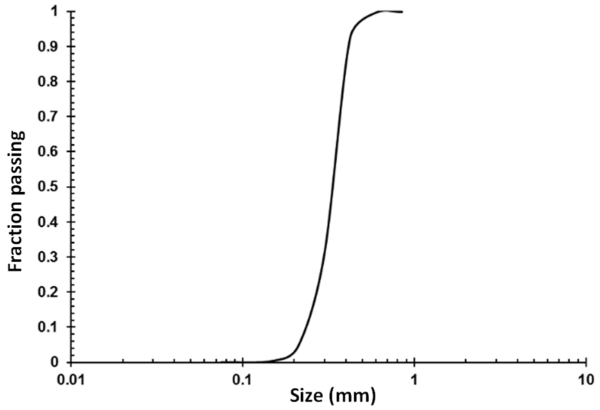

Figure 1: Grain size distribution of Sydney

The basic characteristics of the sand are listed in Table 1. Cylindrical samples have been prepared in two sizes i.e., 54mm diameter and 110mm height, and 100mm diameter and 190mm height. All the samples have been prepared by moist tamping of 10 equal layers for 110mm height samples and 12-15 layers for 190mm height samples. For saturated specimens elevated back pressures were used to ensure B values of greater than 0.95.

Table 1. General sand characteristics

S.no	Description	Value
1	Mean grain size, D_{50}	0.32 mm
2	Coefficient of curvature, C_c	1.1
3	Coefficient of uniformity, C_u	1.3
4	Specific gravity, G_s	2.63
5	Minimum void ratio, e_{min}	0.54
6	Maximum void ratio, e_{max}	0.85

3. Testing Conditions

The tests were conducted using a computer controlled triaxial system developed at the University of Sydney capable of performing tests on cylindrical samples of various sizes. The tests performed include monotonic drained and undrained, and cyclic drained and undrained on both saturated and unsaturated specimens. The soil specimens were prepared by a moist tamping process with relative densities D_r between 0 (Loose) and 0.87 (Dense) based on the minimum and maximum void ratio determined as per guidelines of ASTM D4254-16. The saturated soil specimens were then isotropically consolidated to initial confining stresses of between 50 kPa and 1500 kPa. The monotonic drained and undrained tests were conducted to large deformations to identify the critical state. The cyclic tests were conducted in both drained and undrained conditions with a constant confining stress, and the deviatoric stress was varied using an approximately sinusoidal waveform as shown in Fig. 2 at a frequency of 0.16 Hz. The low frequency was necessary because of the limitations of the motor control for the load frame. Over 200 tests have been conducted which have included standard drained and undrained tests and a variety of cyclic tests with constant cell pressure and cyclic deviator stresses covering a wide range of means and amplitudes. Cyclic tests have also been performed drained and undrained on unsaturated specimens. Because of the limitations of the conference proceedings only a limited number of the saturated tests are reported herein. The list of selected tests is provided in Table 2. The label can be interpreted by type of loading as first letter (M-Monotonic, C-Cyclic), drainage condition as second letter (U-Undrained, D-Drained), and the following number indicates the initial effective mean stress p'.

Table 2. Details of triaxial tests

S. no	Test Label	e_i	D_r	p_i'	Test condition
1	MD50	0.837	0.042	50	M- Drained
2	MD100	0.606	0.787	100	M- Drained
3	MD100i	0.835	0.048	100	M- Drained
4	MD150	0.831	0.061	150	M- Drained
5	MD200	0.603	0.797	200	M- Drained
6	MD300	0.606	0.787	300	M- Drained
7	MD1500	0.801	0.158	1500	M- Undrained
8	MU100	0.584	0.858	100	M- Undrained
9	MU100i	0.792	0.187	100	M- Undrained
10	MU200	0.660	0.613	200	M- Undrained
11	MU200i	0.796	0.174	200	M- Undrained
12	MU300	0.636	0.690	300	M- Undrained
13	MU300i	0.781	0.223	300	M- Undrained
14	MU1500	0.741	0.352	1500	M- Undrained
15	CU100	0.578	0.877	100	C- Undrained Isotropic
16	CU100i	0.764	0.277	100	C- Undrained Anisotropic

*M- Monotonic, C- Cyclic.

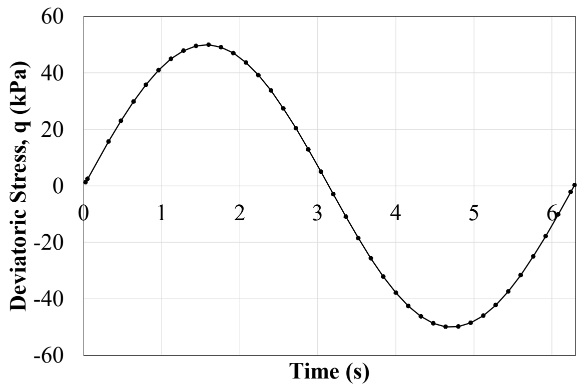

Figure 2: Sinusoidal waveform for cyclic loading

4. Model outline

The constitutive model developed by Ghorbani and Airey (2021) that can capture the cyclic behaviour of multiphase (saturated and unsaturated) granular soil has been adopted for this study. The model can simulate monotonic and cyclic triaxial tests for a wide range of stress states and degrees of saturation. It can also describe the wetting and drying processes, cyclic anisotropy and changing compressibility using parameters of isotropic and kinematic hardening. The smooth transitions from unsaturated to fully saturated states and vice versa are achieved by basic concepts of effective stress of partially saturated soils. The hysteretic nature of the wetting and drying process is addressed by Soil Water Characteristic Curve (SWCC) model. The model is capable of smooth transitions between kinematic and isotropic hardening using several smoothing techniques. It also has an automatic error controlling system to have a better control over speed and accuracy of the analysis.

4.1 General approach and assumptions

The constitutive model is based on the Sanisand family of models (Manzari and Dafalias 1997) with modifications being made to facilitate its ability to handle unsaturated soil behaviour (Ghorbani and Airey 2021).

These models incorporate the important influence of stress-ratio on the stress-dilatancy behaviour as recognised by Taylor (1948) and others (Rowe 1962; Nova 1982; Bolton 1986). By neglecting particle crushing for low to moderate stress, a simplified plasticity model, Sanisand, was developed (Manzari and Dafalias, 1997; Dafalias and Manzari, 2004; Dafalias and Taiebat, 2016). The shearing process is represented by a kinematic hardening law. When the crushing of particles is significant the kinematic hardening can be combined with an isotropic volumetric hardening law.

Implementation of these models is difficult where saturation/desaturation occurs. This problem is addressed by adopting an integration scheme with automatic error control outlined by Sloan et al. (2001) and furthermore developed efficiently in Ghorbani et al. (2018).

Classical elasto-plastic methods are adopted in many granular soil models. These models comprise a purely elastic zone and a hardening yield surface and require complex sub-algorithms to calculate the stress in the plastic zone in each analysis step. This problem is avoided by adopting bounding surface plasticity (Dafalias and Popov. 1975; Kreig. 1975) which removes the need for a yield surface. The assumption of plastic strains accompanying all deformations agrees with the experimental data, as the truly elastic region for soils, if it even exists, is very small. Even the loading with simple stress paths exhibits plastic deformations at very low strain level (Pestana and Whittle. 1999; Bellotti et al. 1989).

The current model is formulated by the evolution of the isotropic hardening parameter and equation of critical state line. The model also uses the influence of suction and degree of saturation on the isotropic hardening parameter and critical state line (CSL). The softening and hardening mechanisms are dealt with by a kinematic hardening law. This is achieved by modelling the difference between peak and ultimate stress ratios using a state parameter.

4.2 Model Parameters

The model parameters can be categorised into elasticity, plasticity, isotropic hardening, critical state, dilatancy and bounding surface as in Table 3. As only saturated tests are simulated in this paper the SWCC and unsaturated parameters are not included. The elastic parameters can be determined from early-stage data of triaxial tests or some trial-and-error methods (Taiebat and Dafalias. 2008).

It has been recommended that the slope of the NCL be determined by conducting isotropic compression tests or oedometer tests to high stresses (Pestana and Whittle. 2002). In the current paper the stress levels have been limited to those in conventional triaxial apparatus, 1500 kPa, and thus the NCL has had to be estimated by trial and error to best match the data. The slope of critical state line, M_c can be simply derived from the p'- q plot. N_c can be determined from $ln\ e$-$ln\ p'$ plot. Likewise, the other material parameters can be determined using appropriate methods.

The dilatancy and kinematic hardening parameters namely h_0, c_h, n^b, n^d, A_0, c_z and z_{max} play prominent roles in the model as explained briefly in Dafalias and Manzari, (2004).

To capture the unsaturated behaviour of the soil including moisture variations caused by either suction or void ratio changes additional parameters to quantify the SWCC, and also the hysteretic behaviour that occurs when the soil is subjected to repeated wetting and drying, are required.

Table 3. The Model parameters for Sydney sand.

Type	Symbol	Values
Elasticity	G_0	186
	v	0.3
Plasticity	N_l	22
	h_0	7.05
	c_h	0.968
	n^b	1.05
	n^d	0.5
	A_0	1.06
	c_z	600
	z_{max}	6
Critical State	λ	0.38
	α_{csl}	4400
	N_c	19.5
	M_c	1.2
	c	0.97

4.3 Simulations of triaxial tests

Over 200 triaxial tests have been conducted on Sydney sand to explore its behaviour and selected tests have been used here to explore the ability of the model to simulate the triaxial test results. As mentioned in the previous sections, both monotonic and cyclic tests for saturated sand were conducted under drained and undrained conditions. For the simulation, the material parameters, CSL and other parameters have been obtained from experimental data. The experiments and simulations have covered a wide variety of tests with material behaviour varying from highly dilatant to highly contractant. This was achieved by varying the combinations of relative density, void ratio, and initial stress state. The results of the experiments and the results from the model simulations are compared in Figures 3 to 9.

Fig. 3 shows the critical state line determined from the experimental data, which is an input to the model, together with the *e, p′* responses from drained triaxial test paths. The simulated responses agree reasonably with the experimental data, however in all cases the drained experiments have terminated before reaching the CSL, a common observation during testing associated with the development of non-homogenous straining. For dense samples, the simulations also appear to terminate before reaching the critical state, which is a consequence of limiting the axial strain to 20% in the simulations. The deviator stress, axial strain responses for drained triaxial experiments on loose and dense specimens isotropically compressed to p' = 100 kPa, and their simulations, are shown in Fig. 4. The volume strains for these tests and simulations are shown in Fig. 5. The general trends in the data are well predicted across the range of consolidation stresses from 50 kPa to 1500 kPa. But, as shown in Fig. 4 and Fig. 5 the simulations show more dilation and yet lower peak deviator stresses than the experiments at p' = 100 kPa. The agreement in these figures can be improved by adjusting the fabric and dilatancy parameters, however if these predictions are improved other aspects of the behaviour are less well predicted. Fig. 6 shows the effective stress paths for undrained tests on specimens isotropically compressed to 100 kPa. The general trends from the simulations again agree well with experiments as shown in Fig. 6 and for a range of consolidation stresses.

Several cyclic triaxial tests have been conducted comprising of drained, undrained and multistage cyclic tests starting from isotropic and anisotropic stress states. Currently the model can simulate cyclic undrained tests for saturated and unsaturated conditions. For this paper two saturated cyclic undrained tests are reported; one with isotropic and one with anisotropic loading and these are compared with the model simulations.

The effective stress path from the simulation of a cyclic undrained test of an initially dense specimen with deviator stress amplitude of 50 kPa and mean of zero is shown in Fig. 7 together with the experimental response. The simulation shows the correct pattern with the strain per cycle increasing as the critical state stress ratio is approached and with cyclic mobility behaviour developing, which concurs with the experiment. The effective stress path for a cyclic undrained test with deviator stress amplitude of 15 kPa and mean of 10 kPa is shown in Fig. 8 for an initially loose specimen. The deviator stress, axial strain responses for this test are shown in Fig. 9. Again, the general pattern in experiment and simulation is similar with both leading to failure and liquefaction of the specimen after a few cycles. Similarly reasonable patterns of response are observed in the simulations of other cyclic tests. The loading reversal and decrement of the load is predominantly controlled by the fabric dilatancy parameters z_{max} *and* c_z and with trial and error a set of parameters which gave reasonable simulations across many of the cyclic tests was obtained.

The model is coded to work with total strain and number of steps, so it is not possible to specify the desired number of cycles for the simulation. This limits the ability to compare tests and their simulations, especially when large numbers of cycles are involved. The performance of the model for the saturated sand is reasonably consistent at different void ratios and initial stress states and other testing conditions. The same model parameters can be used with the addition of the SWCC parameters to simulate the unsaturated sand tests. Preliminary results suggest that similarly reasonable simulations are obtained to those shown here for the saturated tests.

Considerable trial and error were required to obtain the fits to the data shown in these Figures. Although only limited results are shown the objective was to obtain reasonable fits to all the available test data with a single set of parameters. Even though the material is same throughout the tests a few parameters were varied slightly to achieve the best fit curves. For the monotonic tests, the alterations were mostly on the dilatancy parameters and plastic modulus and for cyclic tests the alterations were made to the fabric-dilatancy tensor. To keep the model consistent, the maximum alteration of any parameter was limited to 5% of the original value, but the critical state and elastic parameters were unaltered in all the simulations.

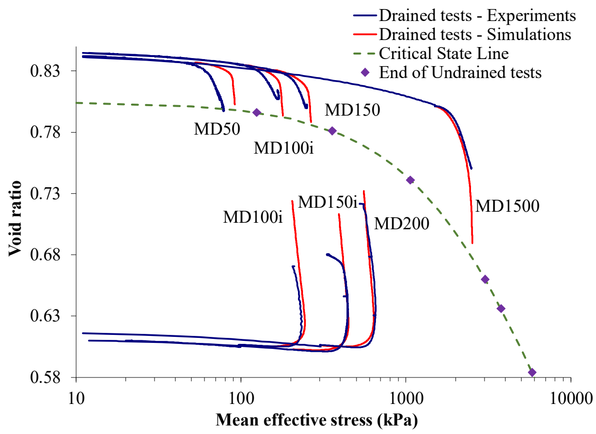

Figure 3. Summary of drained tests in e and log p' space.

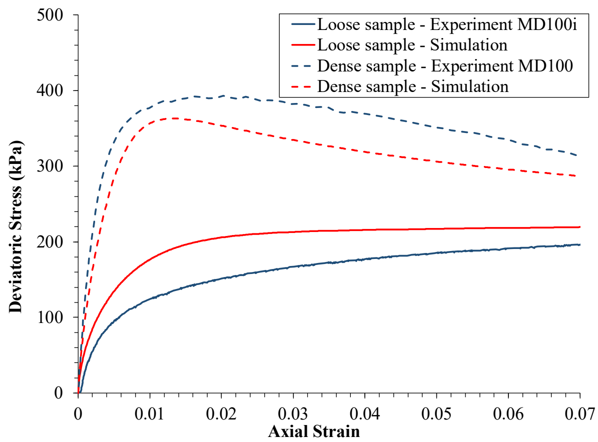

Figure 4. Comparison of simulation and experiment on deviatoric stress and axial strain space.

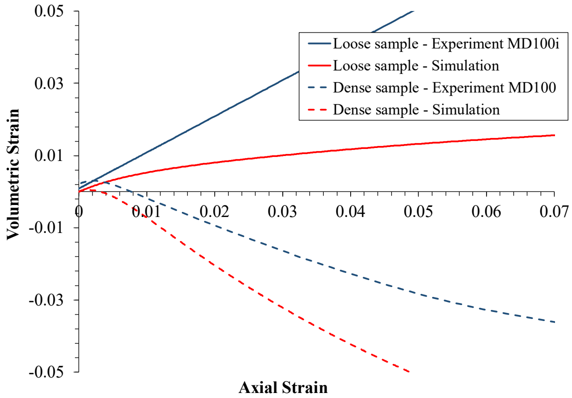

Figure 5. Volumetric strain against axial strain.

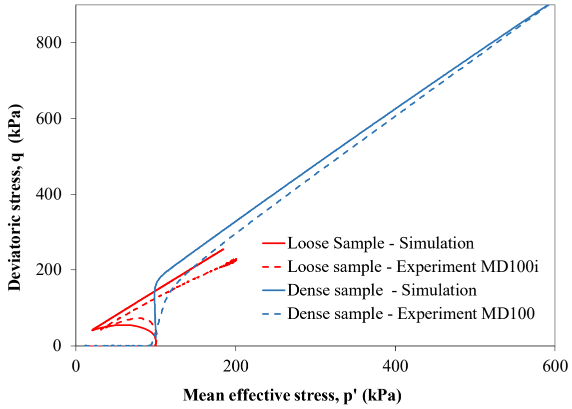

Figure 6. Comparison on simulation of loose and dense sample in q and p' space under monotonic undrained loading.

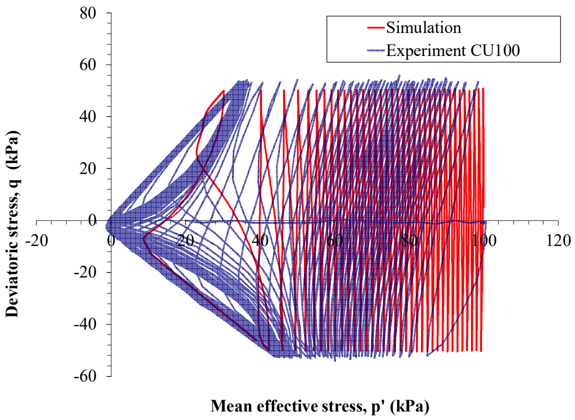

Figure 7. q against p' on isotropic cyclic loading.

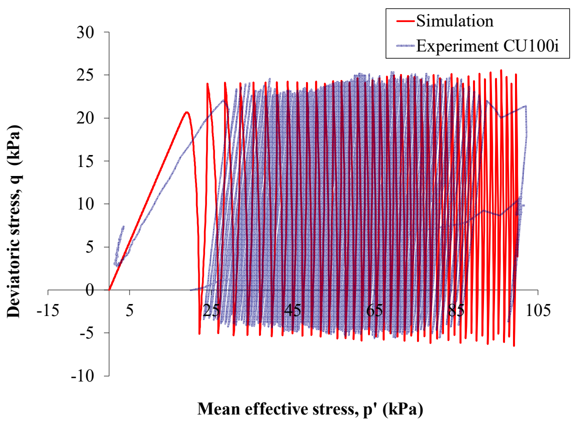

Figure 8. q against p' on anisotropic cyclic loading.

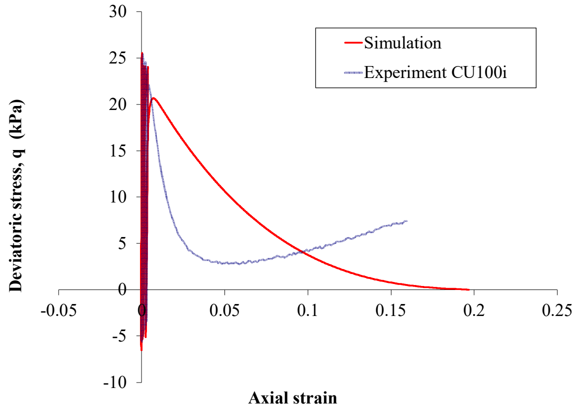

Figure 9. Deviatoric stress against axial strain under anisotropic cyclic loading.

5. Conclusion

The simulation of the triaxial tests in the model initially done with monotonic tests (both drained and undrained). Having a greater number of parameters in the model makes us to determine the exact numbers for the material and the testing conditions. Determining the precise values of kinematic hardening is more challenging which involves complex concepts. The model can simulate precise results only when all the parameters are precise and that too involves more trial-and-error approach. The results of monotonic simulations are well correlated except the dilatancy behaviour which needs rigorous iterative corrections. Similar approach is adopted in cyclic drained tests yield similar results. The number cycles are indirectly defined by number of increments and increment intervals. The capacity of the model in yielding and dilatancy behaviour must be improved. Moreover, number of effective parameters should be identified and others to be eliminated to make the model more effective.

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