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An evaluation of non-linear undrained behaviour in the moderate strain range for fine-grained soils

Mair Beesley[1#], *Erdin* Ibraim[2], and *Paul J.* Vardanega[2]

[1]*Colorado School of Mines, Department of Civil and Environmental Engineering, Golden, CO 80401, USA.*
[2]*University of Bristol, Department of Civil Engineering, Bristol, BS8 1TR, UK.*
[#]*Corresponding author: mbeesley@mines.edu*

ABSTRACT

To select appropriate stress-strain parameters for serviceability limit state calculations, an understanding is needed of the likely variation of stress-strain behaviour within the model displacement mechanism. One approach that may be utilised to investigate variations in stress-strain behaviour is by employing a simple non-linear model (with a small number of physically significant parameters) that simulates experimental measurements of soil stress-strain with reasonable precision. By testing the sensitivity of the model parameters to changes in physical properties that can be expected to be related to them the reliability of different models can be established. Recently, empirical analysis of the published triaxial test database RFG/TXCU-278 identified a significant positive correlation between γ_{50} and *OCR* for four test modes (CIUC, CIUE, CKUC and CKUE). In this paper, a new experimental dataset from a programme of reconstituted soil tests on Kaolin and Bothkennar Clay is used to investigate the validity of a simple non-linear model.

Keywords: stress-strain; variability; experimental; database development.

1. Introduction

Prediction of the stress-strain behaviour of soils is affected by many variables related to the material or method of measurement of stress and strain. Each of these factors has an associated uncertainty and hence there is always a range of possible stress-strain curves from which to select for geotechnical design. To mitigate the risk of settlements causing damage to sensitive buildings, utilities, and infrastructure, the minimum change in soil displacement with the relevant stress path needs to be estimated with sufficiently acceptable accuracy. Simply, overestimating ground movements in a supporting soil can result in excessive design mitigations and, hence, increased financial cost and embodied energy/carbon in final designs.

An approach to the selection of stress-strain parameters that can be incorporated into a risk-based design assessment calls for empirical analysis of large test parameter databases. Regional infrastructure projects such as high-speed railways and urban metro systems may produce the quantity of Ground Investigation (GI) data required per geological deposit/material type to discern meaningful insights from geospatial variability analyses. Consultants who tend to work on smaller-scale construction projects can also adopt data-driven methods of ground characterisation by developing regional GI databases from multiple past projects.

Empirical analysis of geomechanical parameters for ground characterisation is not new to the geotechnical profession. However with increasing digitalisation of project delivery there is a need to consider appropriate methods of data interpretation. Such data-driven methods to characterise ground conditions for settlement predictions are heavily reliant on the appropriate choice of tests and parameters to include in any database for use in this space.

In this paper, the validity of using a simple non-linear soil model for stress-strain variability characterisation is investigated. A methodology is described and tested for the selection of a suitable model and corresponding model parameters (for a summary of the methodology, see Section 2); specifically, the parameters representing mobilised shear strains are the focus of the paper. The paper concludes with a discussion on the expected suitability of a simple non-linear model for variability characterisation of natural soils and earthwork materials in regional infrastructure projects. Further details on the statistical analysis, database building and experimental work can be found in Beesley (2019). A companion paper (Beesley et al. 2023) on comparison of simple stress-strain models has been prepared.

2. Methodology

To select a simple non-linear soil stress-strain model, with the aim to assess the significance of soil variability to ground movement predictions, a six-step methodology is proposed that includes:

a) characterising strains by reference to a mobilised stress range and maximum stress relevant to the design scenario;
b) (2) selecting a simple curve-fitting non-linear modelling procedure by assessment of the least curve-fitting error and corresponding potential candidates for model parameters;

(3) developing a test parameter database, considering the appropriate geotechnical test type and including the selected model parameters and other test

variables that may reasonably be considered to influence a measured stress-strain curve;

(4) database analysis of the parameters, for example using single and multiple linear regression analysis;

(5) testing the output regression models of the database analysis using an independent test dataset;

(6) accounting for model error in parameter estimates when using the selected empirical regression models, by analysing the information provided by the ratio of measured strains divided by predicted strains.

The next sections present further explanation and demonstration of the methodology using a database of reconstituted soil tests, RFG/TXCU-278, and data from an experimental programme conducted at the University of Bristol Geomechanics Laboratories.

2.1. Selection of stress range

Given that fine-grained soils are characteristically non-linear over the full pre-failure range (e.g., Jardine et al. 1986, Atkinson 2000, Brosse et al. 2017), a simplified approach to characterising stress-strain parameter variability is to identify a stress range relevant to the design problem under consideration. Parameters are then utilised for variability analyses corresponding to this stress range. It is useful to consider the applied stresses in relation to a selected maximum stress for the soil type; for example, a measured yield or peak stress that also represents a limiting condition for the design.

Considering the design scenario of predicting initial foundation settlement in shallow fine-grained soils under static load changes, the stresses mobilised in the ground must be less than peak undrained shear strength, c_u, to avoid undrained instability. An upper limit of 80% c_u is equivalent to applying a minimum design safety factor of 1.25 on available soil undrained strength (Vardanega & Bolton 2011). A mobilised stress lower limit of ≈20% may be considered relevant to undrained shallow foundation settlements, for example beneath a footing or embankment. The same lower limit is also relevant to undrained volume loss for open-face tunnelling; for example, field observation data of tunnels in London Clay (e.g., Macklin 1999, Dimmock & Mair 2007, and Klar & Klein 2014), indicate Load Factor values ranging from 0.32 to 0.52.

The parameter termed stress ratio (Casey et al. 2016, Vardanega & Bolton 2016a, Beesley & Vardanega 2020), denoted by S in this paper, is used to define the stress range:

$$S = \frac{\tau_{mob} - \tau_0}{c_u - \tau_0} \qquad 0.2 \leq S \leq 0.8 \qquad (1)$$

where, τ_{mob} = the mobilised shear strength, and τ_0 = initial shear stress.

2.2. Selection of a simple non-linear stress-strain model

2.2.1. Choice of settlement calculation procedure

It is important to consider the appropriate settlement calculation procedure that will be used for design since the procedure will determine the information that needs to be captured by the stress-strain model. Settlement predictions are often compared with a limiting displacement set by the tolerance of nearby constructions to ground movements (cf. Vardanega & Bolton 2016b). Since tolerances can vary considerably by asset type and location, for performance assessments of non-linear soils it is useful to relate the local strain limit to the mobilised strength of the local soil.

By defining c_u as the maximum stress for the soil type, the selected stress-strain parameters can be used to undertake Mobilisable Strength Design (MSD) calculations for various construction scenarios e.g., shallow foundations (Osman & Bolton 2005, McMahon et al. 2014) and tunnels (Klar & Klein 2014). Following the MSD procedure, a serviceability criterion (i.e., a limiting displacement) can be converted into a soil strain limit that in turn corresponds to a maximum stress ratio (or mobilised strength).

However, this procedure relies on a single stress-strain curve to simultaneously represent all compatible strains taking place within the plastic strain mechanism and all shear stresses mobilised along the potential failure surface (Osman & Bolton 2005). Similar routine settlement calculation procedures utilising the assumed self-similarity between the element test stress-strain curve and the full-scale load-settlement curve have been described by Skempton (1951) and Atkinson (2000).

Alternatively, routine settlement calculation procedures are available that adopt a single or multi-layered soil profile where an equivalent linear undrained modulus is selected per soil layer (e.g., Poulos & Davis 1974, described by Atkinson 2000). In this case, not only does the representative stress-strain curve for each soil layer within the mechanism need to be known, but also the stress-strain increment involved per soil layer. For this reason, the MSD calculation procedure is selected for further investigation in this work, noting that the likely variation of stress-strain behaviour within the model displacement mechanism needs to be investigated in more detail.

2.2.2. Choice of test data

Development of large test parameter databases involves a choice of test procedures to include in the database. Routine monotonic undrained triaxial tests were chosen in this study as the tests measure non-linear soil stress-strain behaviour in increments up to peak strength. Although not always representative for the soil conditions around foundations of geotechnical systems, in practice it is the triaxial test that has traditionally been used for the measurement of soil deformation and shear strength (e.g., Bishop & Henkel 1962, Germaine & Ladd 1988).

By categorising tests into test modes, the variety of test procedures involved in characterising the parameter variability distribution is reduced. The term 'test mode' has been used by Ching & Phoon (2013) to differentiate between c_u distributions using undrained triaxial tests with various consolidation and shear modes, i.e., isotropic consolidation triaxial compression (CIUC); isotropic consolidation triaxial extension (CIUE); K_0-consolidation triaxial compression (CKUC); and K_0-consolidation triaxial extension (CKUE). This paper follows the same convention.

2.2.3. Choice of stress-strain function

Multiple published non-linear stress-strain models relevant to the moderate strain range were reviewed in accordance with selected criteria by Beesley (2019), including non-linear elastic moduli at $S = 0.25, 0.5, 0.75$ (Casey et al. 2016), a non-linear elastic hyperbolic model (Duncan & Chang 1970), power-law models (e.g., Matlock 1970, Vardanega & Bolton 2011, Zhang & Andersen 2017 and other sources reviewed in more detail in the companion paper Beesley et al. 2023), an exponential model (Klar & Klein 2014), and a logarithmic model (Puzrin & Burland 1996).

Three mathematical functions were selected to investigate the simple modelling of non-linear stress-strain behaviour in fine-grained soil; exponential (Equations 2a and 2b, Klar & Klein 2014), power (Equations 3a and 3b, Vardanega & Bolton 2011), and logarithmic (Equations 4a and 4b, Beesley 2019);

$$S = \left(1 - e^{-0.693\frac{\gamma}{\gamma_{50\,CIU}}}\right) \qquad 0.2 \le S \le 0.8 \quad (2a)$$

$$S = \left(1 - e^{-0.693\frac{\gamma}{\gamma_{50\,CKU}}}\right) \qquad 0.2 \le S \le 0.8 \quad (2b)$$

$$S = 0.5\left(\frac{\gamma}{\gamma_{50\,CIU}}\right)^{b_{CIU}} \qquad 0.2 \le S \le 0.8 \quad (3a)$$

$$S = 0.5\left(\frac{\gamma}{\gamma_{50\,CKU}}\right)^{b_{CKU}} \qquad 0.2 \le S \le 0.8 \quad (3b)$$

$$S = \beta_{CIU} \cdot log_{10}\left(\frac{\gamma_{50\,CIU}}{\gamma}\right) + 0.5 \qquad 0.2 \le S \le 0.8 \quad (4a)$$

$$S = \beta_{CKU} \cdot log_{10}\left(\frac{\gamma_{50\,CKU}}{\gamma}\right) + 0.5 \qquad 0.2 \le S \le 0.8 \quad (4b)$$

where, γ = shear strain (= $1.5\varepsilon_a$); ε_a = axial strain; CIU = isotropically-consolidated undrained conditions; CKU = K_0-consolidated undrained conditions; γ_{50} = a reference shear strain to mobilise $0.5(c_u - \tau_0)$ according to each function; b = an exponent to describe non-linearity (power-law function); β = a factor to describe non-linearity (logarithmic function); the exponential function is inflexible in shape.

2.3. Database development

Beesley (2019) and Beesley & Vardanega (2020) described the compilation of a large database of 278 published triaxial experiments named RFG/TXCU-278. The database includes tests on reconstituted fine-grained soils using CIUC, CIUE, CKUC and CKUE procedures. Curve-fitted model parameters corresponding to Eq. 2, Eq. 3, and Eq. 4 were calibrated for 271 out of 278 tests.

In Fig. 1, the error data (in total n = 2069 data points) demonstrate that all three functions naturally do not fit the data exactly for the CIUC tests of RFG/TXCU-278. (Fitting error is similarly distributed in the other three test modes – see Beesley et al. 2023.) The 10th and 90th percentiles of the factor error quantify the model fitting error; 80% of shear strain measurements in CIUC are modelled within a factor error of ×/1.09 to 1.17 using Eq. 3 or Eq. 4, whereas Eq. 2 is considerably more biased – overestimating γ by a factor of up to 1.85 at the 10th percentile (noting that greater factor errors are observed in other test modes – see Beesley et al. 2023).

The data scatter of Fig. 1 shows that fitting error varies with S and γ for the three models: Eq. 2 tends to

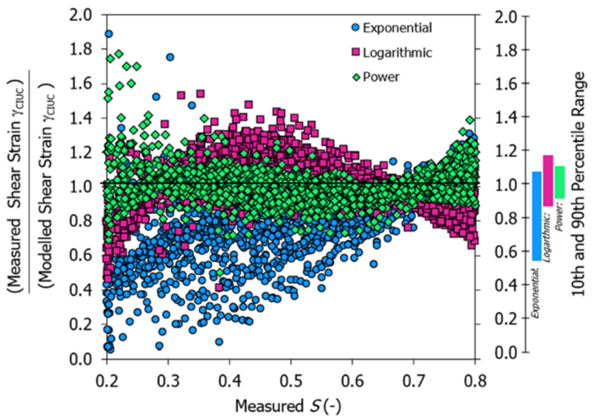

Figure 1. (Measured shear strain)/(Modelled shear strain) of all CIUC stress-strain data between 20%≤S≤80% of database RFG/TXCU-278 using the exponential, power-law, and logarithmic models: 114 tests, n = 2069 data points

overestimate γ where $S < 0.65$; most measurements of γ have values between the two approximations by power (Eq. 3) and logarithmic (Eq. 4) functions. Eq. 4 estimates γ about as well as Eq. 3 if only the bulk of the error data (the middle 80%) are considered without evaluating the error variation with S, but Eq. 3 is closer to the shape of the stress-strain curve over a larger range of S than Eq. 4.

Given the evidence of smaller errors of modelled strain (see also Beesley et al. 2023), the power-law function (Eq. 3) was selected for the subsequent phase of the parameter variability study. In addition to γ_{50}, curve-fitted reference shear strains γ_{30} and γ_{70} mobilised respectively at $0.3(c_u - \tau_0)$ and $0.7(c_u - \tau_0)$ were included based on least error identified in Fig. 1. In the database these parameters were related to other reported experimental variables per test including OCR, vertical and horizontal effective stress after consolidation, σ'_{v0} and σ'_{h0}, undrained strain rate, $\dot{\varepsilon_a}$, liquid limit, w_L, plastic limit, w_P, and specific gravity, G_s (assuming $G_s = 2.7$ if unreported, according to G_s ranges reported by Bell 1992). Void ratio after consolidation, e_0, was not always available in the reported test data and where available has been estimated here for this paper using reported water content, w_0, and assuming saturation = 100%. Table 1 summarises the parameter ranges of RFG/TXCU-278.

2.4. Experimental programme

To evaluate the results of the CU parameter database analysis presented in Section 3.1, an independent set of new experimental data was obtained from a laboratory programme of isotropically-consolidated undrained triaxial compression and extension tests undertaken by the first author at the University of Bristol. In total, 11 tests on reconstituted Kaolin and one test on reconstituted Bothkennar clay are presented in this paper.

Powdered Speswhite Kaolin was sourced from a UK supplier. The Bothkennar clay was available from a resealed intact block sample that had been used in a previous experimental investigation by Sukolrat (2007). The composition of Kaolin is 66% to 73% silt and the remainder clay; the Bothkennar is composed of 96% silt, 2% clay, and 2% sand. Deaired deionized water was added to each material to form a slurry at a water content

Table 1. Parameter ranges of RFG/TXCU-278 (curve-fitted tests only)

	CIUC $n = 114$	**CIUE** $n = 55$	**CKUC** $n = 67$	**CKUE** $n = 30$
w_L	0.25-0.74	0.25-0.72	0.25-0.75	0.25-0.75
w_P	0.13-0.42	0.13-0.40	0.13-0.40	0.13-0.40
I_P	0.12-0.46	0.12-0.32	0.12-0.47	0.12-0.47
G_S	2.46-2.79	2.60-2.74	2.65-2.81	2.60-2.81
σ'_{v0}	9.0-2900	20.7-2900	35.0-9743	40.8-1961
K_0	1.00	1.00	0.44-2.00	0.44-1.67
OCR	1-32	1-12	1-10	1-9.6
e_0	0.38-1.50 (n = 98)	0.43-1.31 (n = 43)	0.42-1.30 (n = 58)	0.42-1.26 (n = 20)

of 1.95w_L and the slurry was compressed in a tall floating ring 1D-consolidometer to create a single 50mm x 100mm triaxial specimen. Each specimen was consolidated to a target *OCR* of 1, 2 or 8 in the triaxial cell at 5-8kPa/hour (Kaolin) or 1kPa/hour (Bothkennar) to minimise excess pore pressures (Δu). Once Δu had dissipated, with σ'_{v0} equal to 49-403kPa, all samples were sheared undrained using a displacement-control loading frame at a rate of 0.002mm/minute (Kaolin) or 0.0013mm/minute (Bothkennar). For further details on the experimental methodology, see Beesley (2019).

3. Results

3.1. Database analysis

The selected strain parameters, γ_{30}, γ_{50}, and γ_{70}, of database RFG/TXCU-278 are plotted against *OCR* in Fig. 2 and Fig. 3 by test mode. All three reference strains increase with *OCR* regardless of stress ratio and test mode. Using single linear regression analysis, each regression equation of γ_{50} and *OCR* is significant whether describing the relationship using linear axes or transformed axes, i.e., $\log_{10}(OCR)$ and $\log_{10}(\gamma)$.

To test the validity of a relationship between $\log_{10}(OCR)$ and $\log_{10}(\gamma)$, multiple linear regression analysis of each reference strain and *OCR*, water content ratio (w_0/w_L), initial void ratio (e_0), and void ratio at w_L, (e_L), was completed using the logarithmic transformations of the parameters.

Table 2 includes the results of the multiple linear regression analysis (Eq. 13-16 and 21-24) next to the best-fit single linear regressions (Eq. 5-8) previously reported by Beesley & Vardanega (2020) for γ_{50}. Single linear regressions between $\log_{10}(OCR)$ and $\log_{10}(\gamma_{70})$ are included (Eq. 9-12) for comparison with the multiple linear regression equations (Eq. 17-20 and 25-28). A visualisation of the same data is shown in Fig. 4-5 with the corresponding 10th and 90th percentile factor errors of the predictions; the shaded area indicates a factor error range of ·/1.5. Results for γ_{30} demonstrated the highest factor errors about the 1 to 1 line when compared with the regression equations for γ_{50} and γ_{70} and therefore have been excluded here for the sake of brevity.

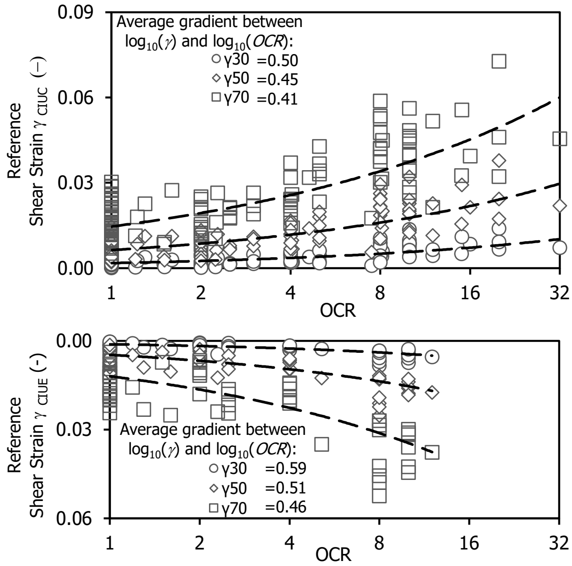

Figure 2. Variation of CIU reference shear strains with *OCR*

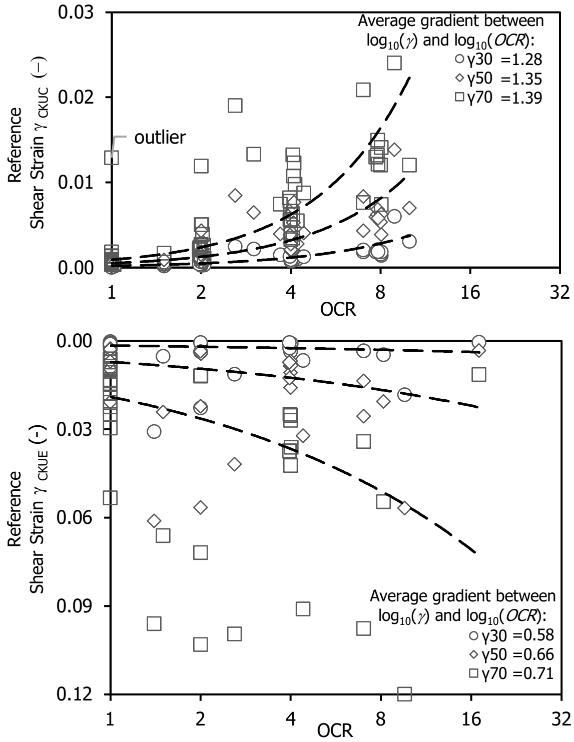

Figure 3. Variation of CKU reference shear strains with *OCR*

The results of the database analysis provide evidence that having values of e_0 and e_L per triaxial test reduces the prediction error of γ_{50} and γ_{70} respectively by 20-60% and 12-71% compared with using *OCR* as the only independent variable. If only the parameter w_0/w_L (equal to e_0/e_L for saturated remoulded clays and silts) is used with *OCR* to estimate γ_{50} and γ_{70}, prediction error is equal to or (up to 63%) greater than the error of using *OCR*, e_0 and e_L to estimate the same strain parameters. Hence, the database analysis demonstrates that the variability of γ_{50} and γ_{70} can be explained in part by the variation of e_0 and e_L. Holding *OCR* constant, the regression coefficients suggest that γ_{50} and γ_{70} are more sensitive to e_L than to e_0 and that, regardless of test mode, at S = 0.5 or 0.7 shear strain increases as e_L increases and e_0 reduces.

Table 2. Tested regression equations for power-law reference shear strains γ_{50} and γ_{70} using RFG/TXCU-278. Regression Factor Error calculated as Measured/Predicted reference strain using the 10th, 50th, and 90th percentiles (/factor or ·factor).

Equation			n	$p<$	R^2	Factor Error 10th	50th	90th	% data points within ·/1.5	·/1.75
5		$\gamma_{50\ CIUC} = 0.0010(OCR) + 0.0074$	114	0.001	0.51	/2.50	/1.05	·1.55	62%	78%
6		$\gamma_{50\ CIUE} = 0.0013(OCR) + 0.0042$	55	0.001	0.65	/1.97	/1.02	·1.60	65%	84%
7	§	$\gamma_{50\ CKUC} = 0.00049(OCR)^{1.35}$	67	0.001	0.79	/2.01	·1.00	·2.19	57%	73%
8		$\gamma_{50\ CKUE} = 0.0038(OCR)$	30	0.001	0.45	/2.07	·1.03	·2.85	45%	55%
9	§	$\gamma_{70\ CIUC} = 0.0015(OCR)^{0.41}$	114	0.001	0.55	/1.66	·1.04	·1.58	68%	86%
10	§	$\gamma_{70\ CIUE} = 0.0012(OCR)^{0.46}$	55	0.001	0.50	/1.74	/1.01	·1.67	64%	87%
11	§	$\gamma_{70\ CKUC} = 0.00090(OCR)^{1.39}$	67	0.001	0.73	/1.97	/1.08	·2.09	58%	66%
12	§	$\gamma_{70\ CKUE} = 0.015(OCR)^{0.71}$	30	0.001	0.47	/2.06	·1.08	·2.88	37%	70%
13	§	$\gamma_{50\ CIUC} = 0.0035(OCR)^{0.48}(w_0/w_L)^{-1.03}$	98	0.001	0.58	/1.83	·1.06	·1.68	63%	82%
14	§	$\gamma_{50\ CIUE} = 0.0041(OCR)^{0.47}(w_0/w_L)^{-0.64}$	43	0.001	0.60	/1.62	/1.03	·1.58	72%	91%
15	§	$\gamma_{50\ CKUC} = 0.0002(OCR)^{1.35}(w_0/w_L)^{-1.56}$	58	0.001	0.84	/1.66	·1.03	·1.44	76%	88%
16	§	$\gamma_{50\ CKUE} = 0.0040(OCR)^{0.58}(w_0/w_L)^{-0.28}$	20	0.001	0.54	/1.73	/1.05	·1.56	65%	75%
17	§	$\gamma_{70\ CIUC} = 0.0099(OCR)^{0.42}(w_0/w_L)^{-0.73}$	98	0.001	0.59	/1.55	·1.04	·1.57	76%	93%
18	§	$\gamma_{70\ CIUE} = 0.0116(OCR)^{0.43}(w_0/w_L)^{-0.34}$	43	0.001	0.54	/1.69	·1.06	·1.54	74%	93%
19	§	$\gamma_{70\ CKUC} = 0.0005(OCR)^{1.38}(w_0/w_L)^{-1.52}$	58	0.001	0.76	/1.77	/1.05	·1.39	74%	84%
20	§	$\gamma_{70\ CKUE} = 0.0087(OCR)^{0.72}(w_0/w_L)^{-0.66}$	20	0.001	0.70	/1.61	·1.03	·1.36	80%	90%
21	§	$\gamma_{50\ CIUC} = 0.0032(OCR)^{0.49}(e_L)^{1.29}(e_0)^{-1.09}$	98	0.001	0.58	/1.90	·1.07	·1.64	69%	83%
22	§	$\gamma_{50\ CIUE} = 0.0030(OCR)^{0.48}(e_L)^{1.60}(e_0)^{-0.92}$	43	0.001	0.76	/1.44	/1.03	·1.36	86%	95%
23	§	$\gamma_{50\ CKUC} = 0.00018(OCR)^{1.42}(e_L)^{2.91}(e_0)^{-1.5}$	58	0.001	0.92	/1.47	·1.01	·1.41	86%	93%
24	§	$\gamma_{50\ CKUE} = 0.0028(OCR)^{0.68}(e_L)^{1.62}(e_0)^{-0.8}$	20	0.001	0.67	/1.62	/1.03	·1.46	75%	80%
25	§	$\gamma_{70\ CIUC} = 0.0082(OCR)^{0.43}(e_L)^{1.19}(e_0)^{-0.83}$	98	0.001	0.62	/1.51	·1.05	·1.53	78%	95%
26	§	$\gamma_{70\ CIUE} = 0.0076(OCR)^{0.45}(e_L)^{1.65}(e_0)^{-0.72}$	43	0.001	0.89	/1.29	·1.01	·1.23	98%	100%
27	§	$\gamma_{70\ CKUC} = 0.00029(OCR)^{1.49}(e_L)^{3.40}(e_0)^{-1.45}$	58	0.001	0.89	/1.55	/1.09	·1.60	79%	95%
28	§	$\gamma_{70\ CKUE} = 0.0055(OCR)^{0.84}(e_L)^{2.39}(e_0)^{-1.36}$	20	0.001	0.87	/1.26	/1.03	·1.38	90%	100%

NB. e_L is the void ratio when water content is equal to the liquid limit of the soil. § indicates the R^2 and p values relate to the logarithmic transformed version of the equation. Adjusted R^2 values are reported for MLR equations.

The narrower bandwidths of prediction error (a reduction of 19-26% for CIU and CKUE tests) also indicate that γ_{70} correlates better with OCR, e_0, and e_L compared with γ_{50}.

An advantage of using the proposed multiple linear regression models is that, when OCR is constant, e.g., OCR=1, they can be used to explain the variation in γ_{50} and γ_{70}. For comparison, Fig. 6 and Fig. 7 present the reference strains of normally consolidated samples in relation to w_0/w_L. Since w_L is constant per reported soil in the database, these plots show that $\gamma_{50\ NC}$ and $\gamma_{70\ NC}$ in reconstituted soils increase as water content or void ratio reduces, i.e., as consolidation stresses increase. The average gradient and strain magnitudes are about the same for CIU and CKUE tests, whereas CKUC tests have lower strains and lower rates of change with w_0/w_L. These relationships between reference shear strains, void ratio, and w_L, for normally consolidated soils can be related to Burland's Intrinsic Compression Line (ICL) framework for reconstituted clays which makes use of the same material properties (e and w_L) to predict one-dimensional compression strains (Burland 1990).

3.2. Experimental results

To test the proposed regression models, the observed and predicted values of γ_{50} and γ_{70} from the experimental programme described in Section 2.4 are presented in Fig. 4, Fig. 5, Fig. 6, and Fig. 7. A full description of the experimental results is given in Beesley (2019). Only values predicted using Eq.'s 21, 22, 25 and 26 are included in Fig. 4 and Fig. 5. Observed values of $\gamma_{50\ NC}$ and $\gamma_{70\ NC}$ from the CIUC and CIUE experiments on Kaolin are presented in Fig. 6 and Fig. 7.

The strain parameters observed by the new CIUC tests tend to be underpredicted by Eq. 21 and 22 and overpredicted by Eq. 25. Of the tested regression models, the least error is observed between Eq. 25 and the measured values of γ_{70}, although 2 out of 8 CIUC test values plot outside the 10th and 90th percentile prediction error bounds reported in Table 2. The normally consolidated Kaolin samples deformed in compression to shear strains close in value to the average $\gamma_{50\ NC}$ and $\gamma_{70\ NC}$ values identified by the trends in Fig. 6 and Fig. 7. It is perhaps unsurprising that the highest (over)prediction error in compression using Eq. 25 is observed in the Bothkennar test data, since the w_L of the soil (77.3%) is higher than the range included in RFG/TXCU-278 (see Table 1).

Beesley (2019) described the measurement of initial strain caused by the vacuum connection between the load cell and specimen top cap at the end of consolidation of the CIUE test specimens. This initial compliance strain caused larger values of γ_{50} and γ_{70} than would be expected from Eq. 22 (with exception of one test) and Eq. 26. The

Measured/Predicted: —— Line of Equality ◆ γ_{50} = f(OCR) ◈ γ_{50} = f(OCR, w_0/w_L) ◇ γ_{50} = f(OCR, e_0, e_L) × New CIUC tests

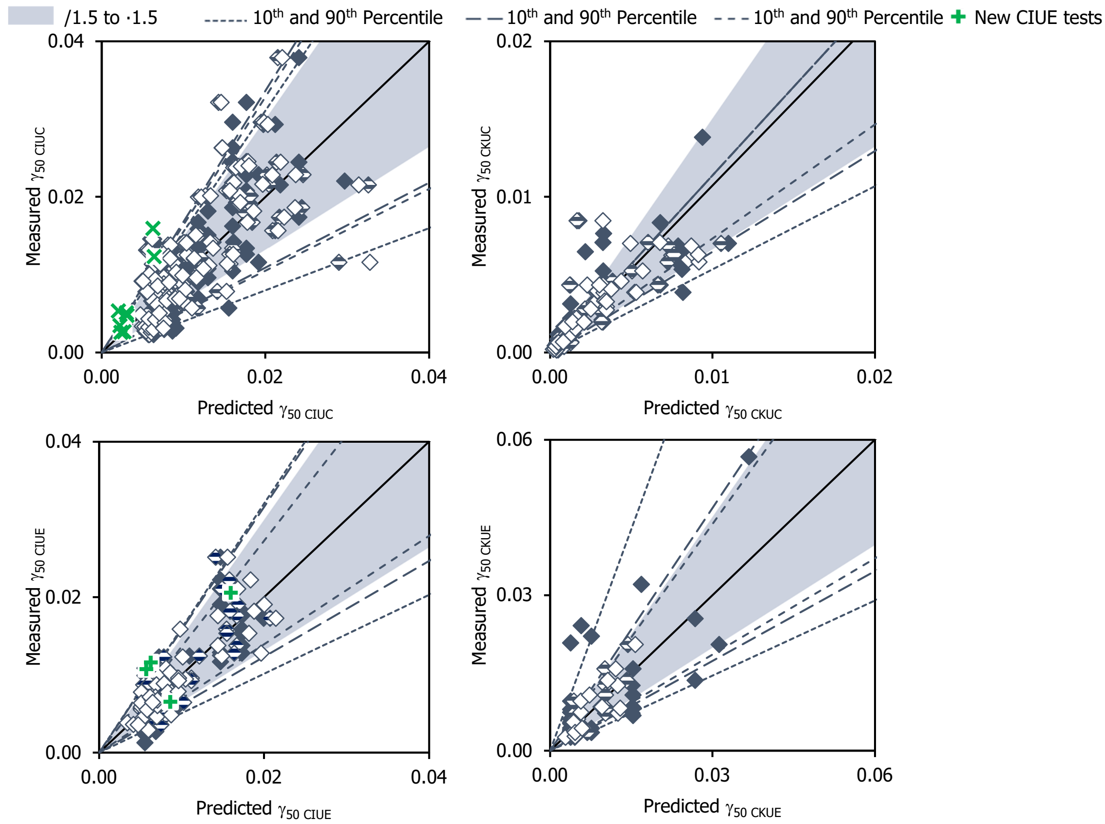

Figure 4. Comparison of observed and predicted values of reference shear strain γ_{50} using Eq. 5-8, 13-16 and 21-24. Values of γ_{50} of the new CIUC and CIUE tests are predicted respectively by Eq. 21 and 22.

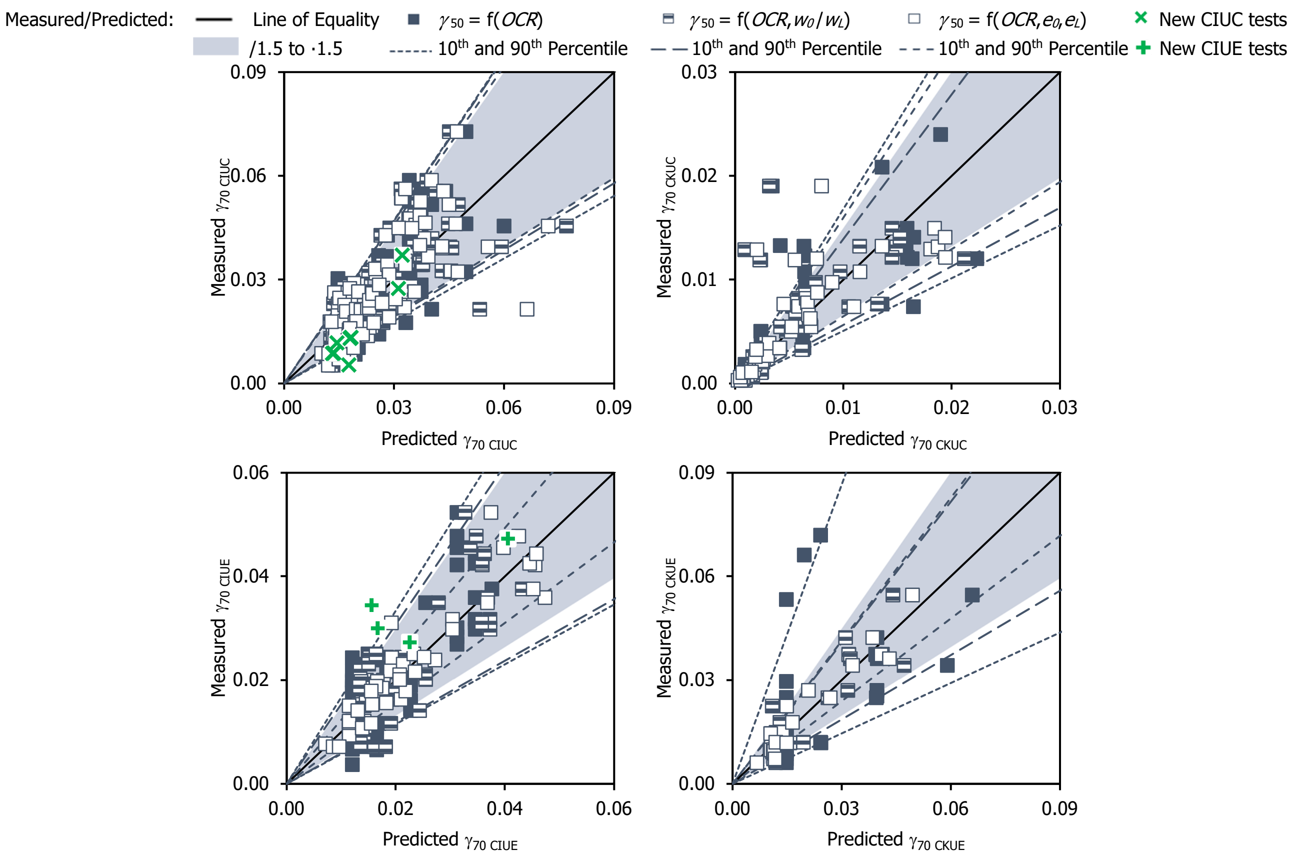

Figure 5. Comparison of observed and predicted values of reference shear strain γ_{70} using Eq. 9-12, 17-20 and 25-28. Values of γ_{70} of the new CIUC and CIUE tests are predicted respectively by Eq. 25 and 26.

extension reference strains, measured by experiment at OCR=1, are 2.8 to 4.0 times higher than the average strain values of RFG/TXCU-278 in Fig. 6 and Fig. 7

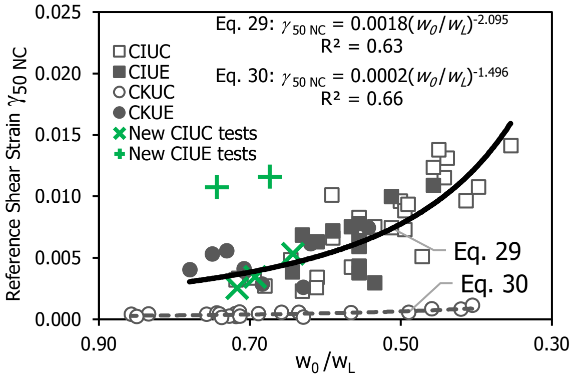

Figure 6. Variation of reference shear strain γ_{50} at OCR=1 with w_0/w_L (Kaolin data reported by Parry & Nadarajah 1974 and Valls-Marquez 2009 are excluded due to high scatter).

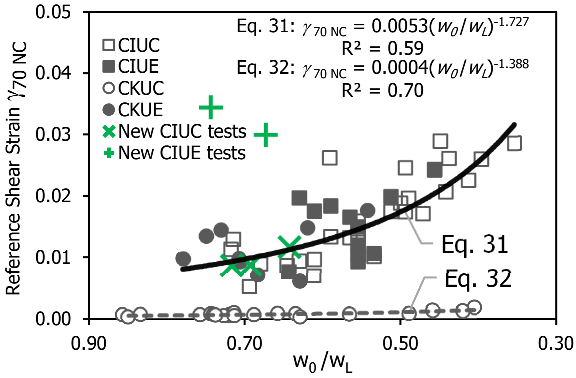

Figure 7. Variation of reference shear strain γ_{70} at OCR=1 with w_0/w_L (Kaolin data reported by Parry & Nadarajah 1974 and Valls-Marquez 2009 are excluded due to high scatter).

4. Discussion

Development of a reconstituted soils test database of stress-strain increments allows an assessment to be made of the significance of stress history, composition, testing procedure, measurement uncertainty, and model error to parameter variability. The results described in this paper present new evidence that the variability of shear strains of reconstituted fine-grained soils, in the moderate strain range (as defined by S of 0.2 to 0.8), is associated strongly with the soil's composition (represented by w_L), changes in density (captured by changes in e_0), and stress history (OCR).

The experimental programme gives new evidence to support the observation of the database analysis that γ_{70} better correlates with OCR, e_0, and e_L than γ_{50}. It may be postulated that perhaps not all the soils have fully yielded at S = 50% whereas at S = 70% arguably more samples have fully yielded; therefore, it may be that there is less scatter in the data post-yield and hence γ_{70} is slightly easier to predict than γ_{50}.

The experimental programme also identified similar variation to the 10th and 90th percentile prediction error bounds of Eq. 25 and 26. The factor errors of Eq. 25-28 (varying from ·1.23 to ·1.6 and /1.26 to /1.55, see Table 2) reflect multiple sources of parameter variability: the model error presented in Fig. 1; the variance of shear strain associated with test procedure; measurement uncertainty of deviator stress pre-failure and at peak failure, shear strain, OCR, e_0 or w_0, e_L or w_L, and G_s; and, inherent material variability that is not explained by w_L (noting that a single value of w_L was reported per tested soil).

5. Concluding remarks

While this paper has produced several new empirical relationships for triaxial shear strains using the published database RFG/TXCU-278, which may be useful for estimating undrained shear strains in other reconstituted soils and soft, unstructured natural soils, the primary purpose of the paper is to present a reliability-based methodology for the selection of stress-strain model parameters for variability analyses on geotechnical projects.

The proposed parameter variability characterisation methodology described in this paper was demonstrated by characterising parameters for short-term undrained settlement in fine-grained soils, with a series of decisions summarised as follows:

(1) shear strains were characterised by reference to stress ratio, S, between a value of 0.2 and 0.8 (Eq. 1);

(2) the MSD method was chosen for settlement calculations, and the suitability of three different simple functions (exponential, logarithmic, and power-law) as approximations of non-linear behaviour was evaluated by least curve-fitting error (Fig. 1);

(3) a test parameter database was developed using CU tests on reconstituted soils; γ_{30}, γ_{50} and γ_{70} (i.e., reference values of mobilised shear strain curve-fitted by the power-law, Eq. 3) were selected as dependent variables for analysis in this paper; OCR, e_0, and w_L were selected as variables that could reasonably be expected to explain variance of shear strain to some degree;

(4) multiple linear regression analyses and Measured/Predicted γ_{50} and γ_{70} were used to assess the significance of OCR, e_0, and w_L to strain variability;

(5) the CIU output regression models and their prediction error bounds were supported by the results of a new experimental programme of CIUC and CIUE tests;

(6) empirical estimates of γ_{70} using the factor errors of Eq. 25-28 (varying from 1.23 to 1.6 and /1.26 to /1.55) can be recommended using the test data in this paper.

Since γ_{70} was shown to correlate with OCR, e_0 and w_L, then γ_{70} may be a useful parameter for variability analyses of natural ground. The methodology may be applied to geotechnical test databases comprising samples of natural soils and earthwork materials to investigate alternative parameters, with careful consideration of appropriate variables to include in step (3). This paper has not considered the variability characterisation of the non-linearity parameter – denoted as b for the power-law function (Eq. 3) – as this was the subject of an earlier paper (Beesley & Vardanega 2021).

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