# Monotonic and cyclic behaviour of sand-silt mixtures through the equivalent state parameter

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## ABSTRACT

This paper presents the results of a laboratory investigation into the effect of non-plastic fines on the monotonic and cyclic behaviour of sand-silt mixtures. For this purpose, drained and undrained triaxial monotonic and undrained stress-controlled cyclic triaxial tests were performed on clean sand and its mixtures with non-plastic silt. The Critical State theory known as a characteristic state of soil behaviour and the equivalent state concept were used to the interpretation of the laboratory tests results. By estimating parameter b, which recognizes that different percentages of fines contribute differently to the strength of the sand and consequently the equivalent intergranular void ratio, (eg)eq, a single Critical State Line, CSL, is determined in the (eg)eq-log(p΄) plane, and a single liquefaction resistance curve at the CRR15-(eg)eq, independently of fines content, fc, based on the monotonic and cyclic tests results, respectively. It is shown that parameter b depends on the fines content, fc, and on the loading type of the laboratory test conducted, while (eg)eq proves to be a suitable parameter for the estimation of the monotonic behavior and undrained critical state strength as well as the liquefaction resistance of granular mixtures up to the threshold fines content value, fcth, independently of their fines content. The effectiveness of state parameter, ψ, and equivalent state parameter, (ψg)eq, in the estimation of the undrained critical state strength and liquefaction resistance of sand-silt mixtures is confirmed.

**Keywords:** equivalent intergranular void ratio; critical state; liquefaction resistance; sand-silt mixtures.

## 1. Introduction

Understanding how the addition of fines affects the monotonic and cyclic behaviour of granular soils, in conjunction with the effect of stress and density, constitutes a major challenge in Geotechnical Engineering.

The Critical State theory is widely used as a characteristic state of soil behaviour, according to which there is a unique relationship between the void ratio, e, and the mean effective stress, p', which is expressed by the Critical State Line (CSL), in the e-log(p΄) plane. According to the critical state concept, the behaviour of a sand depends not only on density, but also on stress level. The true state of a sand is described by the location of its current state of stress and volume relative to the CSL. When the state of a sand is above the CSL, the sand has the tendency to contract upon shearing, whereas when its state is below the CSL it has the tendency to dilate. Various normalized parameters have been proposed to characterize the difference between the actual state and the CSL. Been and Jefferies (1985) have quantified the distance of the current state from the CSL by means of a state parameter, $\psi = e - e_{cs}$, which is the difference in void ratios between the current state and the CSL at the current mean effective stress, $p'_{cs}$.

It has been proven by various researchers (Zlatovic and Ishihara 1995, Thevanayagam et al. 2002, Naeini and Baziar 2004, Yang et al. 2006, Hsiao and Phan 2016, Porcino et al. 2019) that the addition of non-plastic fines to sands results in a downward shift of the CSL up to a threshold value of the fines content, $f_{cth}$, which suggests that contractive stress increases with increasing fines for a given void ratio. The $f_{cth}$ is an important parameter determining the transition from the sand-dominated to the silt-dominated behaviour of mixtures and is related to their particle packing, mean diameter ratio, and separation distance as well as gradation, mineralogy, and particle shape characteristics (Papadopoulou and Tika 2008).

The Critical State Soil Mechanics framework through the state parameter may be also of great use in the investigation of the cyclic behaviour and liquefaction resistance of sand-silt mixtures. It has been shown by various researchers (Polito 1999, Xenaki and Athanasopoulos 2003, Dash et al. 2010) that liquefaction resistance curves in the $CRR_{15}$-e plane shift downwards up to an $f_{cth}$, value. Both in the case of the CSLs and liquefaction resistance curves, this shift can be explained by the change in the structure of the sand-fines mixtures, which, however, cannot be satisfactorily attributed by the void ratio parameter.

In laboratory studies, sands containing fines are considered as consisting of two matrices, the sand grains matrix and the fines matrix and their behaviour is analysed in terms of the interaction with each other (Vaid 1994, Thevanayagam et al. 2000, Polito and Martin 2001). The nature of the contribution of sand and fines matrices may be expressed in terms of the intergranular and interfine void ratios, respectively. The intergranular void ratio, $e_g$, expresses the relative contribution of sand fraction on the behaviour of the mixture and is given by the following equation (Mitchell 1976):

$$e_g = \frac{V_w+V_f}{V_g} = \frac{w\cdot\left(\frac{G_{sf}}{S_r}\right)+f_c}{(1-f_c)\cdot\left(\frac{G_{sf}}{G_{sg}}\right)} \quad (1)$$

where $V_w$ is the volume of voids, $V_f$ is the volume of the fines, $V_g$ is the volume of sand grains, $f_c$ is the fines content in decimal, w is the water content of the specimen, $G_{sf}$ is the specific gravity of the fines and $G_{sg}$ is the specific gravity of sand grains. For saturated specimens ($S_r=100\%$) and considering that $G_{sf}\approx G_{sg}$, the intergranular void ratio after the consolidation of the specimen is expressed as follows:

$$e_g = \frac{e_c+f_c}{1-f_c} \quad (2)$$

where $e_c$ is the void ratio of the mixture after consolidation. Equations (1) and (2) hold for $f_c \le f_{cth}$.

Thevanayagam (2000) suggested the introduction of an additional parameter, b (0<b<1), in the expression of $e_g$ in order to take into account the percentage of fines, which actually contribute to the maintenance of the contacts between the grains and consequently to sustaining the stresses, proposing for $f_c \le f_{cth}$, the equivalent intergranular void ratio, $(e_g)_{eq}$:

$$\left(e_g\right)_{eq} = \frac{V_w+(1-b)\cdot V_f}{V_g+b\cdot V_f} = \frac{e+(1-b)\cdot f_c}{1-(1-b)\cdot f_c} \quad (3)$$

The parameter b recognizes that different percentages of fines contribute differently to the strength of the sand. A value of parameter b equal to 0 indicates that there is no contribution of the fines in supporting the coarse grains skeleton at sustaining forces, while a unity value of b indicates that the total of fines, actively participate in sustaining the imposed to the coarse grain skeleton forces.

The objective of this study is the estimation of b parameter and consequently $(e_g)_{eq}$, in order to determine a single CSL and a single liquefaction resistance curve at the $(e_g)_{eq}$-log(p′) and $CRR_{15}$-$(e_g)_{eq}$, planes, respectively, independently of fines content for $f_c \le f_{cth}$, for the tested sand-silt mixtures. The CSLs were obtained from drained and undrained triaxial monotonic tests and the liquefaction curves from undrained stress-controlled cyclic triaxial tests.

The effect of $f_c$ on the estimated b values is investigated together with the suitability of $(e_g)_{eq}$ to be used as an estimation parameter of the monotonic behaviour and undrained critical state strength as well as the liquefaction resistance of granular mixtures up to their threshold fines content value. Based on the tests' results correlations of b with $f_c$ based on the type of test are derived. The coincidence of the b values estimated from monotonic and cyclic tests is also examined. The effectiveness of state parameter, ψ, and equivalent state parameter, $(\psi_g)_{eq}$, in the estimation of critical state strength and liquefaction resistance of sand-silt mixtures is investigated.

## 2. Tested materials

The materials used in the testing programme were artificial sand-silt mixtures, made from a clean quartz sand (S) with well-rounded grains and a non-plastic silt (F), a ground product of natural quartz deposits. Samples were prepared by mixing the sand (S) with the silt (F) at percentages of 5, 10, 15, 25, 35, 40 and 60% of the total dry mass of the mixture (noted as SF5, SF10, SF15, SF25, SF35, SF40 and SF60, respectively). Detailed description of the sand-silt mixtures is given in Papadopoulou (2008). Tests were also conducted on specimens of clean sand and pure silt.

The physical properties and grain size distributions of the tested materials are presented in Table 1 and Fig. 1, respectively.

**Table 1.** Physical properties of tested materials

| Soils | $G_s$ | $D_{50}$ (mm) | $D_{10}$ (mm) | $C_u$ | $e_{min}$ | $e_{max}$ |
|---|---|---|---|---|---|---|
| Sand (S) | 2.649 | 0.30 | 0.24 | 1.3 | 0.582 | 0.841 |
| Silt (F) | 2.663 | 0.02 | 0.004 | 7.5 | 0.658 | 1.663 |

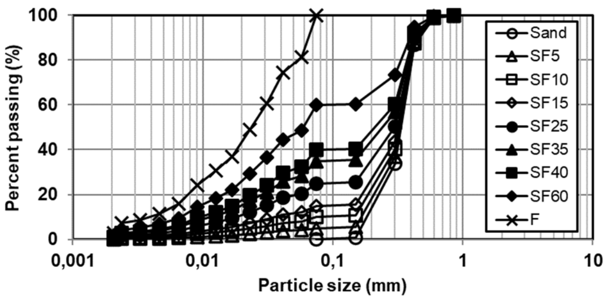



**Figure 1.** Grain size distributions of the tested materials.

## 3. Testing equipment and experimental procedure

The testing programme consisted of monotonic and cyclic triaxial tests for the determination of the critical state line and the liquefaction resistance ratio of the tested materials, respectively. Both types of tests were performed using a closed-loop automatic cyclic triaxial apparatus (MTS Systems Corporation, U.S.A.). Its principles of operation are given in detail in Papadopoulou (2008).

The specimens (height/diameter≈100mm/50mm) were formed by moist tamping at a water content varying between 4% and 12% for all the tested materials and 35% only for the silt specimens, using the undercompaction method, introduced by Ladd (1978). Moist tamping was preferred to other preparation methods, as it produces specimens of varying densities (Verdugo and Ishihara 1996). Saturation was achieved by percolating throughout the specimen, from the bottom to the top drainage line, first carbon dioxide gas ($CO_2$) for 20 minutes and then de-aired water. A suction pressure of 15 kPa was applied while dismantling the specimen, measuring its dimensions and assembling the triaxial cell. In order to ensure full saturation, a series of steps of simultaneous increasing cell pressure and back pressure were performed, while maintaining an effective confining stress of 15 kPa. In all the tests, a final back pressure of 500 kPa was found to be sufficient, as the parameter of pore water pressure, $B=\Delta u/\Delta\sigma$, did not increase by further increasing back pressure. In all the tests the parameter B had values from 0.95 to 1.00. No correction of the results was made for membrane penetration, because of the uncertainties, associated with such a correction and so the raw test results are presented here. After the completion of saturation, the specimens were isotropically consolidated under an effective isotropic stress, $p'_0$, ranging from 50 to 625kPa. A period of time equal to double the consolidation time of the specimens was allowed before shearing. During consolidation the volume change and the axial displacement of the specimens were recorded in order to calculate the post consolidation void ratio.

In the monotonic tests, the specimens were subjected to either undrained (CU), or drained (CD) compression at a constant rate of axial displacement of 0.05 mm/min.

In the cyclic triaxial tests, a sinusoidally varying axial stress ($\pm\sigma_d$) was applied at a frequency of f=0.1Hz, under undrained conditions. In this work, cyclic stress ratio, $CSR=\sigma_d/2p'_0$, corresponds to double amplitude axial strain, $\varepsilon_{DA}\approx5\%$ and liquefaction resistance or cyclic resistance ratio, $CRR_{15}$, is defined as the cyclic stress ratio, $CSR=\sigma_d/2p'_0$, required to cause double amplitude axial strain, $\varepsilon_{DA}\approx5\%$ at 15 cycles of loading.

## 4. Test results and discussion

### 4.1. Monotonic behavior

Fig. 2a presents the CSLs of the sand-silt mixtures in the e-$p'_{cs}$ plane. As it can be seen, the CSLs are moving downwards with increasing $f_c$ up to the percentage of 35% and then upwards with further increasing $f_c$, while the CSLs of the mixtures with $f_c$=35% and 40%, practically coincide. Consequently, the threshold fines content is in the range of 35-40%. It is noted that, essentially, $f_{cth}$, is determined as a range of fines content values and not as a strictly defined value (Papadopoulou 2008, Papadopoulou et al. 2010).

In Fig. 2b, the CSLs of the tested materials are presented, for the fines contents up to the 35%, that is for $f_c \leq f_{cth}$, in the $(e_g)_{eq}$-$p'_{cs}$, plane. The solid line depicts the CSL of the sand while the dashed one the CSL of all the mixtures including the sand, to show their coincidence. This was the criterion-the maximum possible coincidence of the CSL of each mixture with that of the sand- based on which the value of parameter b, was determined for each one of fine content values studied in the testing program, to be then possible to calculate the $(e_g)_{eq}$ and finally the depiction of a single CSL, for the sand and the sand-silt mixtures.

Figs. 3a and 3b present the variation of $e_g$ and $(e_g)_{eq}$ with void ratio - within the range of the maximum, $e_{max}$, and minimum, $e_{min}$, void ratio values-of SF mixtures with $f_c$ values lower than or equal to the threshold value. It can be noted that for all sand-silt mixtures with $f_c \leq f_{cth}$ and for a given e value, the corresponding $(e_g)_{eq}$ values are significantly lower than the corresponding $e_g$ values and the differences between various $f_c$ values are notably lower.

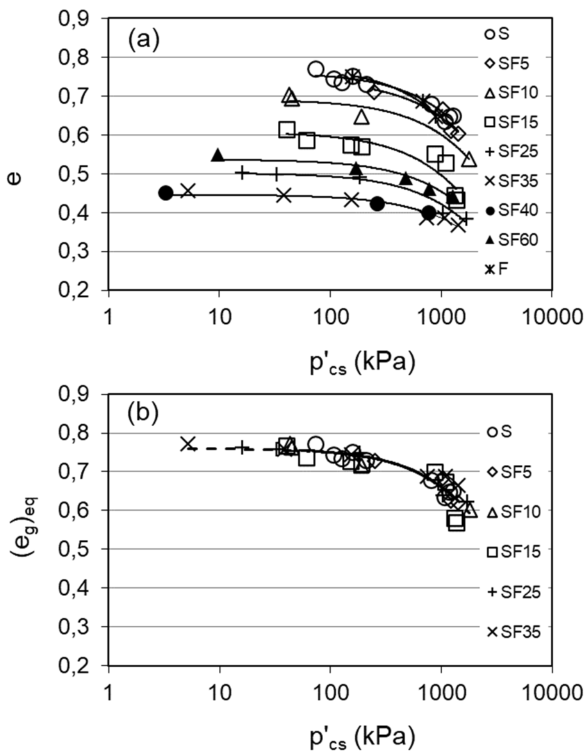



**Figure 2.** Critical State Lines of the sand-silt mixtures on the (a) e-$p'_{cs}$ and (b) $(e_g)_{eq}$-$p'_{cs}$, plane.

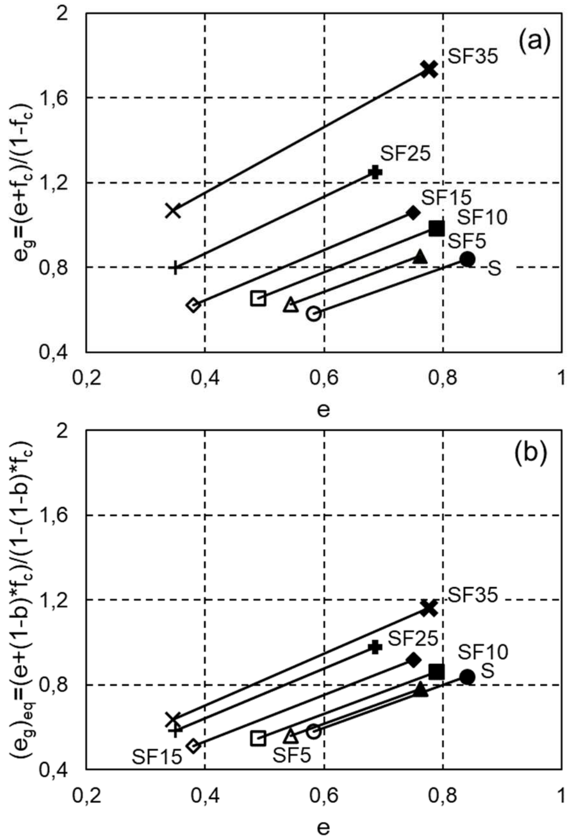



**Figure 3.** Variation of void ratio, e, with a) intergranular void ratio, $e_g$, and b) equivalent intergranular void ratio, $(e_g)_{eq}$, for the tested sand-silt mixtures up to the threshold fines content, $f_{cth}$=35%, based on their $e_{max}$ and $e_{min}$ values.

The variation of parameter b with $f_c$ is given in Fig. 4a for SF mixtures and as it can be noted, it can be expressed satisfactorily by a quadratic polynomial equation, while its minimum value corresponds to an $f_c$ of 25%, lower than the threshold, $f_{cth}$=35%. Fig. 4b presents the variation of parameter b with the $f_c$ normalized with $f_{cth}$. With parameter b it is recognized that different fine contents contribute differently to sand's strength. The content of fines that practically contribute to sustain the imposed stresses, in sand-fines mixtures, depends on the particles' arrangement and changes as their percentage on the mixture increases, making the use of a constant value for b parameter for all fine contents not suitable.

A crucial question is whether the mixtures of granular soils will have similar monotonic behaviour with that of the sand. In Figs. 5a-5c are presented the stress-strain diagrams, the pore water pressure to strain diagrams and the deviatoric-mean effective stresses diagrams of the sand and the mixtures specimens S, SF10 and SF25, with similar $(e_g)_{eq}$ values, where a contractive type of behaviour can be distinguished. For similar values of $(e_g)_{eq}$, the developed monotonic behaviours display a satisfactory approximation. If the comparison was made based on void ratio, e, or intergranular void ratio, $e_g$, or even relative density, $D_r$, the variations would be significant. Therefore, parameter $(e_g)_{eq}$, may be a sufficient basis for the estimation of the monotonic behaviour of granular soil mixtures with fines contents lower than the $f_{cth}$, based on the behaviour of their coarse grain fraction, given the correct determination of parameter b values.

Figs. 6a and 6b, present the variation of the normalized to the effective stress, $p'_0$, undrained critical state shear strength, $S_{cs}=q_{cs}/2$, defined at the ultimate state, with e and $(e_g)_{eq}$, respectively, for the tested sand-silt mixtures. A decrease of strength with increasing e and $(e_g)_{eq}$, is observed for all tested soils. For the range of e values tested the lowest values of $S_{cs}/p'_0$ are of the order of 0.097, 0.026, 0.022 and 0.079 for $f_c$=10%, 35%, 40% and 60%, respectively. At the same e, the results indicate that the presence of non-plastic fines may induce a decrease in $S_{cs}$ of the mixtures. This may be attributed to the fact that at a constant e value, as $f_c$ increases, $e_g$, and $(e_g)_{eq}$, increase resulting in a looser structure of the sand grains into the mixture. Similarly, at the same $e_g$ value, the presence of non-plastic fines results in the increase of $S_{cs}$ owing to the decrease of e, as $f_c$ increases. In Fig. 6b, the solid line in the $S_{cs}/p'_0$ - $(e_g)_{eq}$, variation represents the sand while the dashed one all the mixtures including the sand, to show their coincidence. It is shown that given the density of a mixture through the void ratio, e, its fines content and by estimating b value, $(e_g)_{eq}$, may be calculated and by using the $S_{cs}/p'_0$ - $(e_g)_{eq}$, variation for the clean sand the critical state strength of the mixture may be estimated with a good approximation.

Figs. 7a and 7b show the variation of $S_{cs}/p'_0$ with $\psi$ and $(\psi_g)_{eq}$, for the tested mixtures, respectively. A decrease of $S_{cs}$ with increasing both $\psi$ and $(\psi_g)_{eq}$, is observed, due to increasing contractiveness of the soils. The solid lines represent the sand while the dashed ones all the mixtures including the sand, to show their

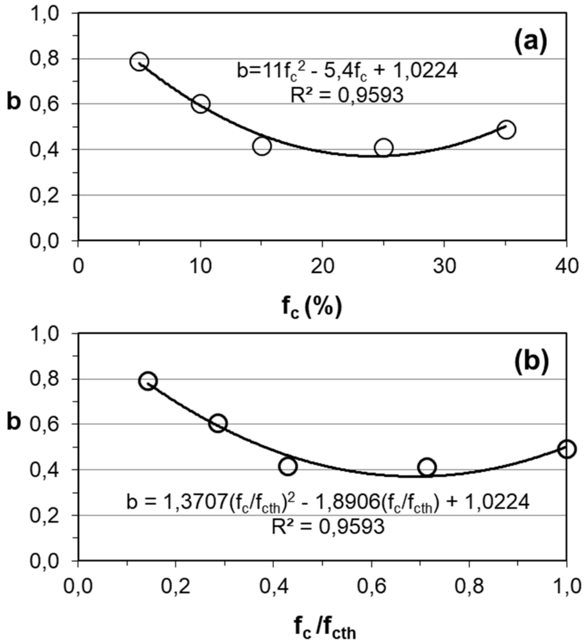
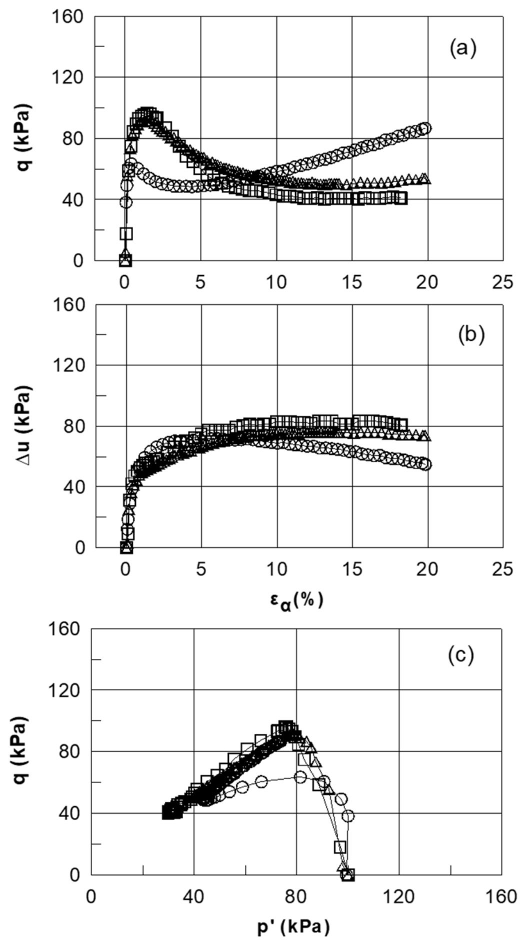



**Figure 4.** Variation of b parameter estimated from CSLs of SF mixtures, with (a) $f_c$ (%) and (b) $f_c/f_{cth}$.




| Symbol | Tested material | e | $e_g$ | $D_r$(%) | $(e_g)_{eq}$ |
|---|---|---|---|---|---|
| ○ | S | 0.771 | 0.771 | 27 | 0.771 |
| △ | SF10 | 0.703 | 0.892 | 29 | 0.773 |
| □ | SF25 | 0.499 | 0.999 | 56 | 0.758 |

**Figure 5.** Monotonic behavior of sand S and of mixtures SF10 and SF25 samples, with similar $(e_g)_{eq}$ values, in the a) q-$\varepsilon_\alpha$, b) $\Delta u$-$\varepsilon_\alpha$ and c) q-p′, planes.

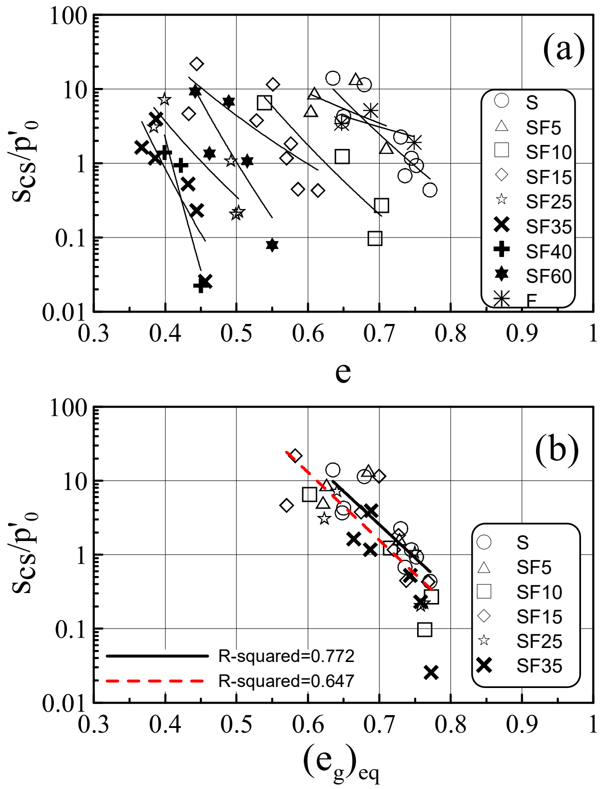



**Figure 6.** Variation of the normalized undrained critical state strength, $S_{CS}/p'_0$, with (a) void ratio, e, (b) equivalent intergranular void ratio, $(e_g)_{eq}$, for the SF mixtures.

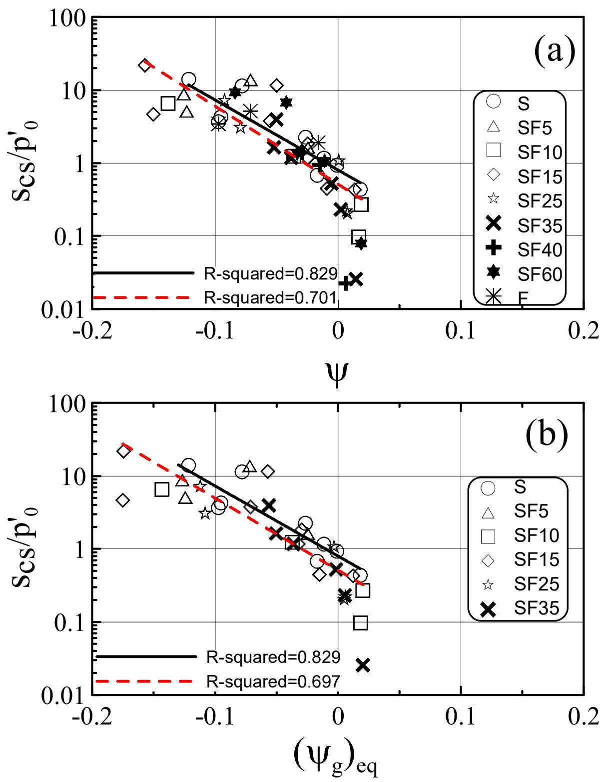



**Figure 7.** Variation of the normalized undrained critical state strength, $S_{CS}/p'_0$, with (a) state parameter, ψ, (b) equivalent state parameter, $(\psi_g)_{eq}$, for the SF mixtures.

coincidence. Equally, the state and the equivalent state parameters, ψ and $(\psi_g)_{eq}$, prove to be suitable and effective in the estimation of the undrained critical state strength, as they combine the effect of density and effective confining stress. The $S_{cs}/p'_0$-ψ correlation covers the whole range of $f_c$ values. These correlations should be used with engineering judgement in the cases where ψ≥0.

### 4.2. Cyclic behavior

Figs. 8a, 8c, 8e and 8g show the variation of $CRR_{15}$ with e, at $p'_0$=50, 100, 200 and 300 kPa, for the sand and the sand–silt mixtures, at values of $D_r$ ranging from 7% to 100%. At a given $p'_0$ and density, $CRR_{15}$ decreases with increasing $f_c$ up to a threshold fines content value, $f_{c,th}$, and increases thereafter with further increasing $f_c$. For the tested sand–silt mixtures, $f_{c,th}$ is 35% and 25% at $p'_0$=50–200kPa and 300kPa, respectively. The behaviour of the mixtures at $f_{c,th}$ is characterised by instability and flow liquefaction. Moreover, it is shown that at a given density, $CRR_{15}$ decreases with increasing $p'_0$ and that the effect of $p'_0$ on $CRR_{15}$ diminishes with increasing $f_c$.

In Figs. 8b, 8d, 8f and 8h the variation of $CRR_{15}$ with $(e_g)_{eq}$, at $p'_0$=50, 100, 200, and 300 kPa, for the sand and the sand–silt mixtures, is presented. For the calculation of $(e_g)_{eq}$, the values of parameter b were estimated with the criterion of the best coincidence of the liquefaction curve of each one of the tested mixtures with that of the sand. In the above Figs., the solid curves represents the sand while the dashed ones all the mixtures including the sand, to show their coincidence.

Figs. 9a and 9b show the variation of parameter b with $f_c$ and $f_c/f_{cth}$, respectively, as estimated from the cyclic triaxial tests (CTX), for the tested sand-silt mixtures at the $p'_0$=50, 100, 200 and 300kPa. Together, the values of b as estimated from the triaxial tests (TX) results are plotted, for comparison. Parameter b estimated from CTX tests depends on the fines content and the effective stress. Again, its variation with $f_c$ may be expressed satisfactorily by a quadratic polynomial equation, while its minimum values correspond to an $f_c$ of 25% lower than the threshold, $f_{cth}$=35% in the cases of $p'_0$=50, 100 and 200kPa, and an $f_c$ of 15% lower than the threshold, $f_{cth}$=25% in the case of $p'_0$=300kPa. With increasing $p'_0$, the values of b tend to decrease. It is worth noting that the b-$f_c$ and b-$f_c/f_{cth}$ curves, as derived from the TX tests results, form a lower band since the b values estimated from the CTX tests results are higher with the exception of the mixture of the sand with 5% fines at the $p'_0$=100kPa. It can be concluded that parameter b besides $f_c$ depends also on the loading type of the laboratory test conducted.

The variation of liquefaction resistance, $CRR_{15}$, with the state parameter, ψ, is presented in Figs. 10a, 10b, 10c and 10d, for the clean sand, the pure silt and their mixtures with 5, 15, 25, 35, 40 and 60% fines, at the effective stresses of 50, 100, 200 and 300kPa, respectively. The solid curves correspond to the sand and the dashed ones to all the mixtures including the sand and the silt. It is shown that the state parameter can serve as a suitable parameter for the estimation of the monotonic behaviour in relation to the liquefaction resistance in mixtures with fines, irrespectively of their $f_c$, for the values of $p'_0$=100, 200 and 300kPa.

Figs. 11a, 11b, 11c and 11d show the variation of $CRR_{15}$, with the equivalent state parameter, $(\psi_g)_{eq}$, for the sand and the tested mixtures up the threshold value, $f_{cth}$,

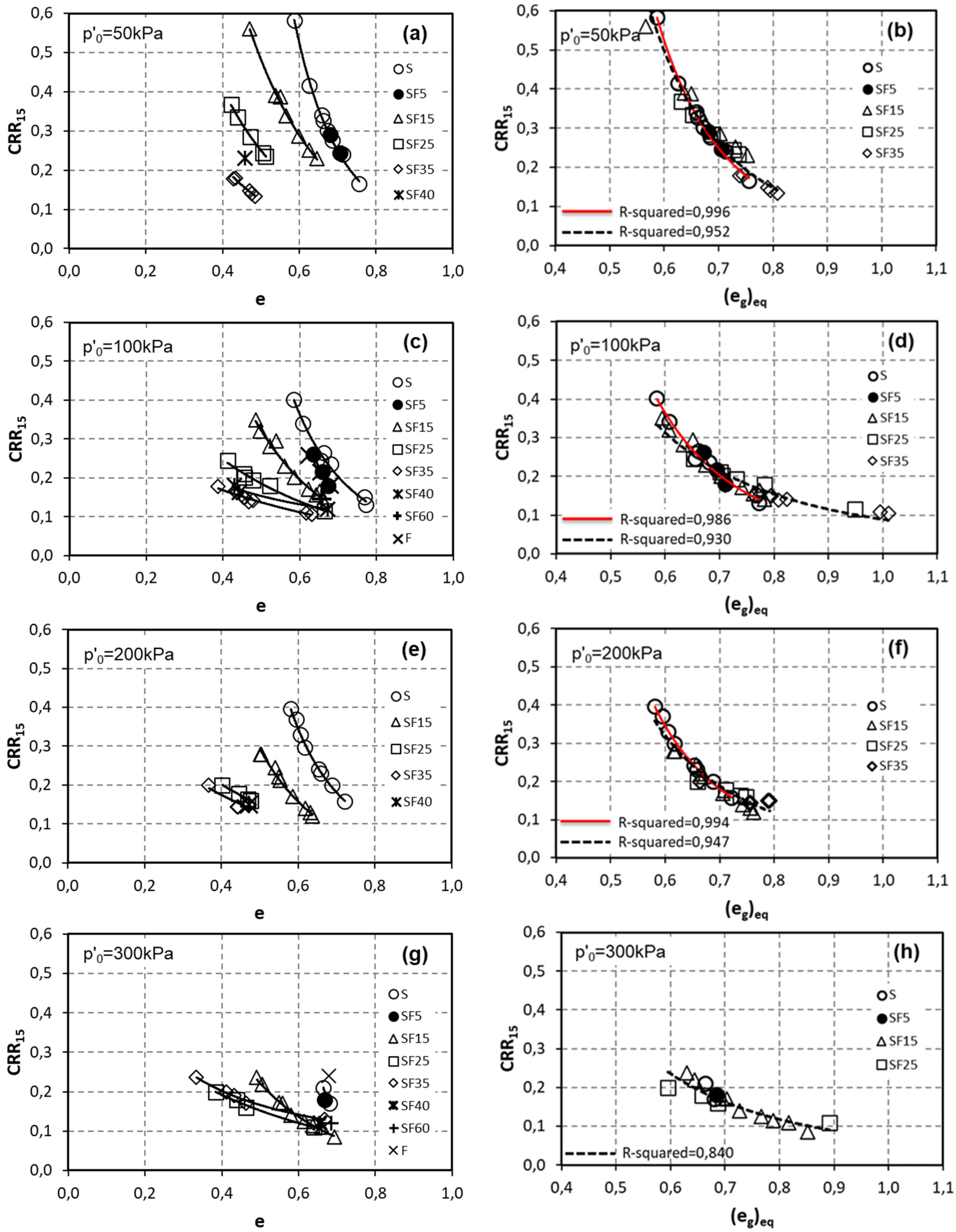



**Figure 8.** Liquefaction Resistance curves of the sand-silt mixtures on the (a, c, e, g) $CRR_{15}$-e and (b, d, f, h) $CRR_{15}$-$(e_g)_{eq}$, plane, at the effective stresses of p'$_0$=50, 100, 200 and 300kPa.

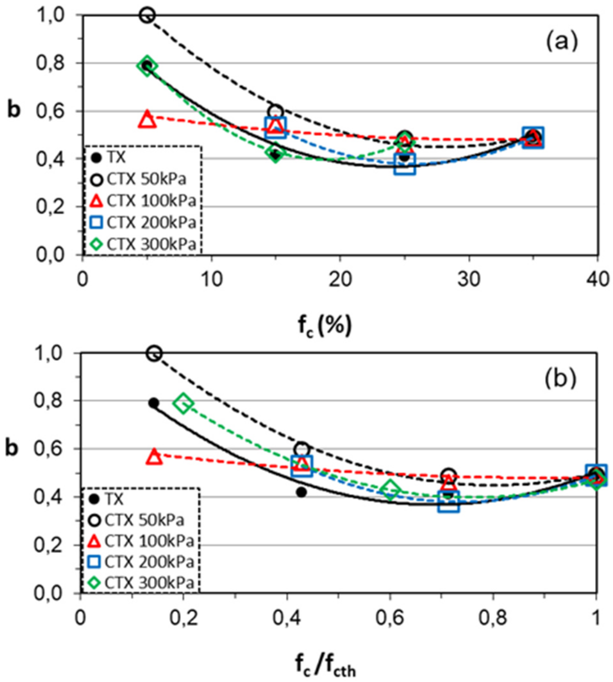


**Figure 9.** Variation of b parameter estimated from triaxial (TX) and cyclic triaxial (CTX) tests results, at $p'_0$=50, 100, 200 and 300kPa with (a) $f_c$ (%) and (b) $f_c/f_{cth}$.

of 35% for $p'_0$=50, 100 and 200kPa and 25% for $p'_0$=300kPa. There is a decrease of $CRR_{15}$, with increasing $(\psi_g)_{eq}$, due to increased contractiveness. The solid curves represent the sand while the dashed ones all the mixtures up to the $f_{cth}$, including the sand, to show their coincidence, which is very good. The equivalent state parameter proves to be efficient in estimating the liquefaction resistance of the tested sand-silt mixtures, much more than the state parameter, but it encounters fine content values up to the $f_{cth}$, while $\psi$ covers all fine content values, including the pure silt.

These correlations are very useful as they connect the monotonic behaviour, contractive or dilative, with the liquefaction resistance for soils such as the tested sands with non-plastic fines, which are commonly encountered in geotechnical engineering works.

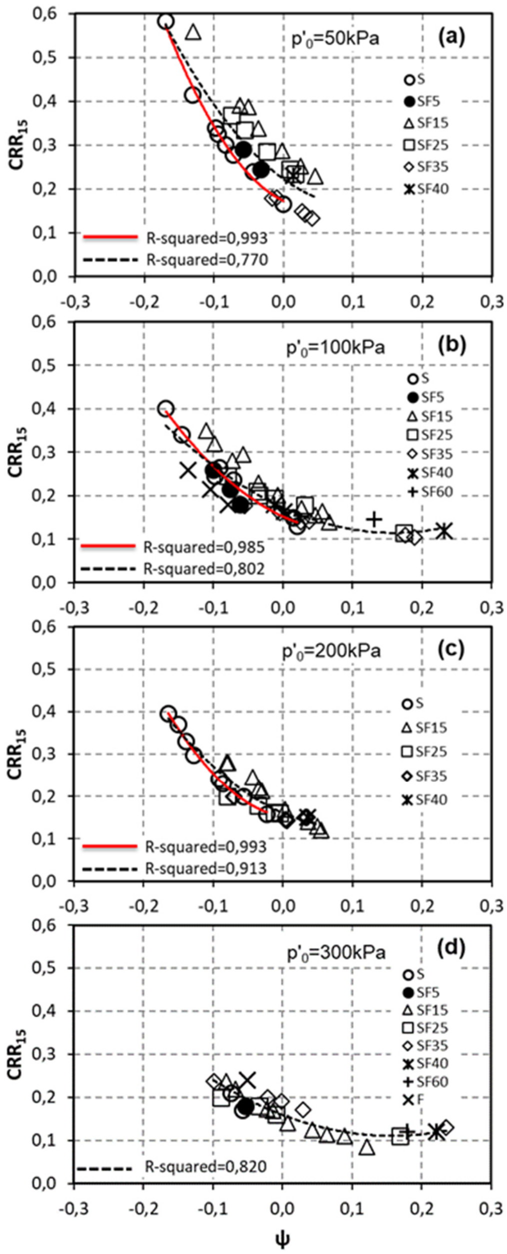


**Figure 10.** Variation of $CRR_{15}$ with $\psi$, for the tested sand-silt mixtures, at the effective stresses, $p'_0$, of (a) 50kPa, (b) 100kPa, (c) 200kPa and (d) 300kPa.

## 5. Conclusions

From the results of the present study the following conclusions can be drawn:

Parameter b depends on the fines content, $f_c$, and can be expressed by a quadratic polynomial equation, with b receiving its minimum value at an $f_c$ value lower than the threshold, $f_{cth}$. In the case where it is estimated from cyclic triaxial tests results, it also depends on the effective stress, $p'_0$.

Parameter b depends on the loading type of the laboratory tests conducted with its minimum values corresponding to triaxial tests results data.

The equivalent intergranular void ratio, $(e_g)_{eq}$, is an appropriate density parameter for the expression of the monotonic behaviour through a single CSL, and a single liquefaction resistance curve for a given $p'_0$, of granular mixtures up to the threshold value of fine grains, $f_{cth}$, irrespective of $f_c$.

The state parameter, $\psi$, proves to be a sufficient parameter for the estimation of the monotonic behaviour and undrained critical state strength, $S_{cs}$, for granular mixtures. The correlations should be used with engineering judgement in the cases of contractive behaviour, $\psi$>0. It can also provide sufficient estimations for the liquefaction resistance of these soils, especially for values of $p'_0 \geq$100kPa.

The equivalent state parameter, $(\psi_g)_{eq}$, can be sufficiently used for the estimation of the monotonic behaviour and the $S_{cs}$, of granular mixtures, but for fine

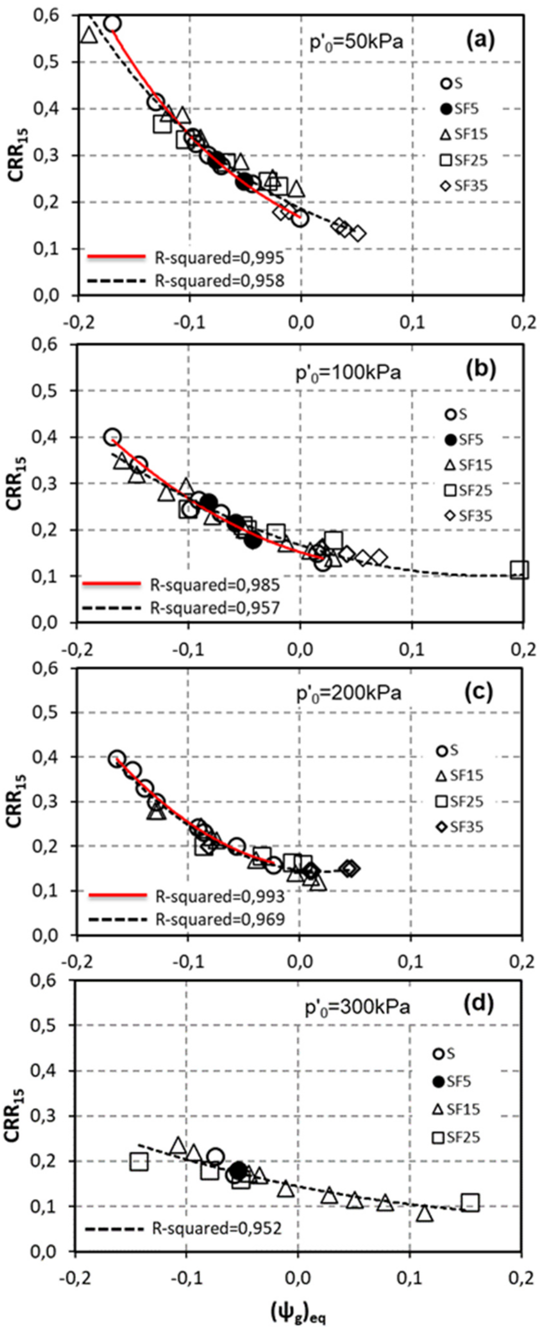



**Figure 11.** Variation of $CRR_{15}$ with $(\psi_g)_{eq}$, for the tested sand-silt mixtures, at the effective stresses, $p'_0$, of (a) 50kPa, (b) 100kPa, (c) 200kPa and (d) 300kPa.

content values up to the threshold, $f_{cth}$. The correlations of liquefaction resistance, $CRR_{15}$, with $(\psi_g)_{eq}$, are very good and should be preferred to the ones with $\psi$, but again they refer to values of $f_c \leq f_{cth}$.

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