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# Determine the yield curve for unsaturated soils using a novel test approach

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## ABSTRACT

An important step to developing a good understanding of the behaviour of unsaturated soils is to adopt a formal and easy constitutive framework, and to conduct high quality and short-time experiments. One of the most accepted constitutive modelling of unsaturated soils is the Barcelona basic model (BBM) which explained the elasto-plastic behaviour and many other features of unsaturated soils. During the development of the elastoplastic models for unsaturated soils, it is essential to determine the shape of the yield curve and its evolution during plastic hardening. The BBM and other important constitutive models for unsaturated soils were developed upon results from the Suction-controlled triaxial (SCTX) tests. However, previous discussions have proven that the analyses of the SCTX tests results are not theoretically correct for the characterization behaviour of unsaturated soils. At present, there is no available method to correctly determine of the evolution the loading/collapse yield curve during yielding. This paper presents a capable, effective, simple, and theoretically solid approach for the characterization of unsaturated soils and determining the shape and evolution of LC yield curves during yielding. This research proposed applying a photogrammetry-based method during undrained triaxial tests with the Modified State Surface Approach (MSSA). The results from UTT were analysed using the MSSA approach and the model parameter were calibrated with a correlation coefficient of 93.4%. Then the yield curves were determined and plotted in the s-p space.

**Keywords:** Unsaturated Soils, Yield curve, Photogrammetry.

## 1. Introduction

An important step to developing a good understanding of the behaviour of unsaturated soils is to adopt a formal and easy constitutive framework and to conduct high quality and short-time experiments. The elastoplastic behaviour of unsaturated soils was paid significant effort by many researchers as it can be better linked to the irrecoverable behaviour of unsaturated soils.

The first elastoplastic model for unsaturated soils was proposed by Alonso et al. (1987) and then was developed as a full mathematical model by Alonso et al. (1990). This model, with few slight modifications, came to be known as the Barcelona Basic Model (BBM) (Gens et al. 2006). One of the main aspects that were investigated by many researchers was the occurrence of yielding which is an important feature of the stress-strain behaviour of soils (Zakaria 1994). Alonso et al. (1990) developed the loading/collapse (LC) yield curve in the p-s plane on which the yield stress increases with the increase in the suction. Later, the model was extended to triaxial stress states, and the yield curves were represented by an elliptical shape at constant suctions in the q-p-s stress space. The LC yield curve can be determined using soil specimens with identical stress histories from loading tests under constant suction or from wetting tests under constant applied stress (Alonso et al. 1990; Alonso and Gens 1992; Alonso et al. 1994; 1999; Delage and Graham 1996). Although these methods are theoretically correct, their practical implementation is hard to be achieved as discussed by Zhang and Li (2010) and Zhang (2016). the SCTX tests are actually stress-path controlled consolidated drained tests which are based on measuring the soil volume change relative to the initial soil volume to obtain the final volume change at different stages of testing. However, the test results might be misleading in case the initial or any intermediate volume measurements are not accurate. As discussed by Fayek et al. (2020), the relative volume measurement method might cause misleading determination of soil strength and deformation. It can be concluded that the use of the SCTX tests for the constitutive modelling of unsaturated soils is problematic as well as laborious and time consuming. So, the existing analysis methods based on the SCTX tests to develop the BBM and another constitutive modelling for unsaturated soils may not be reasonable.

It is necessary to develop another method to overcome the limitations of the SCTX tests for the constitutive modelling of unsaturated soils and to correctly determine the shape and evolution of the LC yield curves during yielding. Based on an overview of existing constitutive models for unsaturated soils and their associated potential barriers, a modified state surface approach (MSSA), proposed by Zhang and Lytton 2009a; Zhang and Lytton 2009b, is considered in this research to determine the shapes of yield curves for unsaturated soils. This is due to the ability of the MSSA to explain the elastoplastic behaviour of unsaturated soils. Undrained triaxial tests (UTT) can be used an alternative testing method to characterize unsaturated soils. The UTT has many advantages: (1) the ability to catch the soil coupled behaviour of soil in case the soil behaviour is measured during the undrained stress path; (2) a higher reliability as the soil specimen is expected to have fewer disturbances in a short testing time; (3) the ability for water content measurement of the soil specimen before and after the test; (4) Ability to measure soil specimens at different loading conditions and moisture contents; (5) Ability to get accurate results for the plastic loading process; (6) Short testing time (few hours) and cost In addition, the methods proposed by Zhang et al. (2015) and Fayek et al. (2020) were applied with minor modifications to determine the absolute volume of soil specimens at different stages of UTT.

## 2. Modified State Surface Approach for Unsaturated Soils Characterization

The Modified State Surface Approach (MSSA) was first proposed by Zhang and Lytton (2009a, 2009b) to investigate the elastoplastic mechanical behaviour of unsaturated soils under isotropic conditions. MSSA was developed to study the unsaturated soils' volume change using independent stress state variables of mean net stress and suction. Later, MSSA was extended to study the hydro-mechanical behaviour of unsaturated soils (Zhang and Lytton 2011). The coupled MSSA considered the compatibility among the elasto-plastic relations for air-water-soil solids. In addition, the coupled MSSA overcomes the limitations of most existing models by its ability to predict the unsaturated soil behaviour under undrained conditions (Zhang 2016; Zhang et al. 2009).

Under isotropic conditions, the unsaturated soil behaviour can be represented in the e-p-s as represented in Fig. 1. The void ratio is divided into two surfaces: (A) a movable unloading/reloading elastic surface; (B) a fixed plastic surface. Based on a qualitative study of the unsaturated soil behaviour, the soil volumetric behaviour is divided into elastic and plastic surfaces based on many criteria as follows:

1) The shape and position of the plastic surface ABCDEF in the e-p-s space are fixed for the same soil. Plastic loading only changes the range of the plastic range.

2) During an elastic loading or unloading process, the shape and position of the unloading–reloading elastic surface MABC remain unmovable in the e-p-s space. The volume change of any isotropic elastic loading or unloading stress path must fall on the elastic surface in the e-p-s space.

3) During a plastic loading process, the shape of the unloading–reloading elastic surface stays fixed. However, the position of the plastic loading surface changes by moving downward in parallel with the original unloading–reloading elastic surface. For example, from MABC to M'DEF under loading paths of either AD, BE, or CF. The volume change of any isotropic plastic loading stress path must fall on the plastic surface in the e-p-s space.

4) The yield curves are the interception of the unloading–reloading elastic surface and the plastic surface.

Based on the previous criteria, Zhang and Lytton (2009a) derived the mathematical expressions of the elastic surfaces for the BBM. MSSA was used successfully to represent many unsaturated soil behaviours including the stress path independence under isotropic conditions.

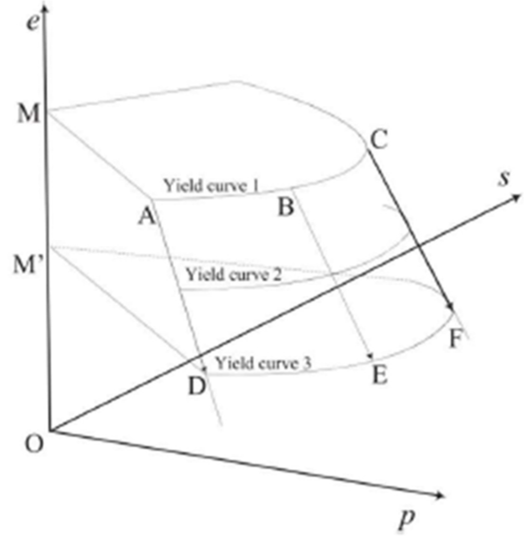


**Figure 1.** Schematic plots of the state boundary surface in e-p-s space (Modified from Delage and Graham 1995).

## 3. Experimental Investigation

This section presents the specimen preparation procedures, then a description of the development of the high-suction tensiometer, and the application of the photogrammetric method to determine the absolute soil volume during triaxial testing. Further details of the experimental investigation are also presented.

### 3.1. Development of High Suction Tensiometer (HST)

Direct measurement of matric suction is a great advantage to characterize the behaviour of unsaturated soils in laboratory testing. A proper suction sensor is required to measure suction variations during the undrained triaxial testing. A review of suction measurement methods (Fredlund and Rahardjo (1993)) shows that conventional methods have two serious disadvantages: (1) limited measurement range lower than 0.1 MPa and (2) slow response time. The low measurement ability is caused mainly by water cavitation.

The response time of suction measurement is influenced by the time required for the soil specimen to reach suction equilibrium and the reaction time of the suction sensor itself (Zhang et al. 2016).

In this study, a miniature HST built in-house was developed according to the principle proposed by Ridley and Burland (1993) and similar to the design used by Li and Zhang (2014). The miniature HST has a diameter of 6.5 mm and a height of 11.6 mm. As shown in Fig. 2, the miniature HST consists of (1) a miniature pressure transducer; (2) a metal housing; (3) a 15-bar high airentry disc to prevent the miniature HST from cavitating at high suction; (4) a water reservoir to generate negative water pressure to be detected by the pressure transducer. The miniature HST was saturated in a closed cell filled with water by applying water pressure cycles up to 500 kPa for around one week (Li and Zhang 2014). Then, the miniature HST was calibrated by comparing the value of the applied pressure to the miniature HST readings to get the linear calibration parameters. The maximum attainable suction of the miniature HST was determined to be around 550 kPa by applying the free evaporation test reported by Guan and Fredlund (1997). More information about the tensiometer fabrication can be found in Li and Zhang (2014).

### 3.2. Photogrammetry-based Method for Absolute Volume Measurement

Measuring the soil volume during triaxial testing is an essential part of the characterization of stress-strain behaviour for unsaturated soils. In this study, a non-contact method is applied to measure the volume of the soil specimen using a conventional triaxial test apparatus. The photogrammetric method proposed by Zhang et al. (2015), Fayek et al. (2020), and Xia et al. (2022) is used to reconstruct the three-dimensional (3D) model of the soil specimen from which the absolute volume is calculated. This method is cost-effective, simple, and highly accurate.

Photogrammetry is based on the ideal pinhole camera model where the light beam from an object point passes through the pinhole and forms an image point (Mikhail et al. 2001). By taking images from multiple positions for an object, the different pixel positions for the same points on different images can be used to calculate the original camera orientation and positions. Then, the camera orientations can be used with the images to reconstruct the 3D object shape. However, the application of the photogrammetry principle during triaxial testing can be challenging. Due to the refraction effect at the air – acrylic cell - water interfaces, the light ray is bent twice, and the collinearity condition cannot be met. This disadvantage was overcome by Zhang et al. (2015) by extending the conventional photogrammetry from one optical medium to multiple optical media for triaxial soil testing. Only one commercial camera was used to capture images from outside the conventional triaxial cell apparatus as shown in Fig. 2. This photogrammetric method was utilized to determine the orientations of the camera where the images are taken and the shape and location of the acrylic cell. In addition, multiple optical ray tracings were employed for refraction correction at the air-acrylic cell and acrylic cell–water interfaces, and a least square optimization technique is applied to estimate the coordinates of any point on the specimen surface. So, Zhang et al. (2015) were able to measure both the global and local deformations for both saturated and unsaturated soil. Validation on the cylinder and soil specimen verified the high level of accuracy of 0.05% volume measurement. However, this method was not able to locate the top and bottom boundaries between the soil sample, the specimen cap, and the specimen base respectively since the top and bottom boundaries were covered by a membrane during testing. Assuming the soil sample's boundaries makes the accuracy of the soil volume measurements questionable.

To overcome this limitation, Fayek et al. (2020) proposed a simple and rigorous photogrammetry-based technique to determine the top and bottom boundaries of the soil specimens mathematically. The principle is based on the fact that the distances from the peripheral coded targets to the surface plane of each pedestal will remain unchanged, regardless of their locations and orientations during the triaxial tests. The only additional requirement for the proposed method is to post some coded targets (high contrast dots that can be automatically identified by photogrammetry software) to the peripheral of the top and bottom pedestals. The photogrammetric analysis is performed for each of the top caps and bottom pedestal (as shown on the left side of Fig. 2) to determine the coordinates of all coded targets from which the distances of each code target on the peripheral to the surface plane are calculated (Fig. 2). So, by applying the method proposed by Fayek et al. (2020), the absolute soil volume is measured with high accuracy without making any assumptions about the specimen's shape or boundary. Following a similar procedure, the tilting and eccentricity of the triaxial specimen can be calculated. More discussions regarding vertical displacement, tilting, and eccentricity can be found in Fayek et al. (2023a; 2023b).

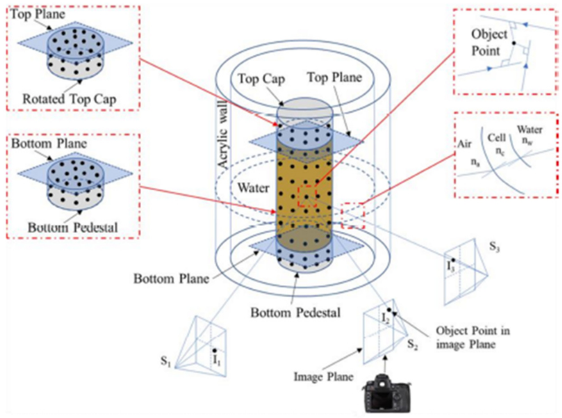


**Figure 2.** Schematic plot showing the principal of the photogrammetry-based method including the methodology to calculate the specimen ends.

### 3.3. Tested Materials and Specimen Preparation

The soil was collected from the Missouri riverbed and classified per USCS as silty sand (SM). Then the soils were oven-dried and pulverized. The retained soil passing sieved No. 200, which is non-plastic silty soil, was used to conduct the laboratory tests. The silty soils have a specific gravity of 2.7, an optimum moisture content ($w_{opt}$) of 15%, and a maximum dry density of 18 kN/m$^3$. The oven-dried silt was mixed with distilled water at the optimum moisture content. Then, the wet soil was sealed in plastic impervious bags in a humidity and temperature-controlled room for two days to equalize the water content in the whole soil mass (Li and Zhang 2015). The soil was then compacted in 6 layers in the split mold of 2.8 inches in diameter and 5.6 inches in height until the specimen reaches the desired height. The under-compaction method was applied to prevent excessive densification of the lowest layers (Ladd 1978). Four soil specimens were compacted using the same preparation steps (compaction energy and procedure) and the moisture content to ensure approximately the same stress history. After compaction, the specimens were stored in sealed plastic bags for at least one month. To reach the desired moisture levels, the soil specimens were exposed to air for different periods by controlling the number of exposures to around 15 min/day. More information about the condition of the soil specimens used in this study is summarized in Table 1.

**Table 1.** Soil Specimen Used in Testing Program

| Test # | Water content | Void ratio | Matric Suction | Stress Paths |
|---|---|---|---|---|
| 1 | 7.1 | 0.873 | 219.7 | Isotropic loading or unloading path. Loading to 300 kPa, Unloading to 200 kPa, Reloading till 600 kPa |
| 2 | 8.9 | 0.835 | 102.2 | Isotropic loading or unloading path. Loading to 250 kPa, Unloading to 150 kPa, Reloading till 400 kPa |
| 3 | 11.9 | 0.817 | 67.84 | Isotropic loading or unloading path. Loading to 300 kPa, Unloading to 150 kPa, Reloading till 500 kPa |

### 3.4. Undrained Triaxial Tests for Unsaturated Soils

As discussed earlier, the SCTX tests have many limitations which make using its derived results in constitutive modelling questionable. On the other hand, in undrained triaxial tests, the suction is the soil specimen can reach equilibrium instantaneously everywhere (Bishop AW, Donald1961; Thu et al. 2007). Thus, the soil specimen can be considered as a representative elementary volume (REV) as stress and suction are in equilibrium in every location. Consequently, the test results derived from undrained triaxial tests can be used for the constitutive modelling purpose. UTT is not just technically sound but also easy and fast to perform. The setup of the undrained triaxial test for unsaturated soils is shown in Fig. 3. The specimen installation process involved moving out the specimen of the container and installed in the triaxial testing system. One circular opening was cut in the membrane to facilitate the tensiometer installation. Then, the specimen was quickly covered with a latex membrane to maintain the desired water content. One miniature HST was installed at the mid-height of the soil specimen to measure the suction changes during testing. Coded targets were previously printed on the membrane following the design in Xia et al. (2022). The number of targets is 816 which allows accurate measurement of the specimen shape and the change in the lateral and horizontal directions at different locations. The conventional triaxial acrylic cell was filled with water to apply the all-around pressure. The confining pressure was applied with an interval of 50 kPa, and the suction and absolute volume were monitored.

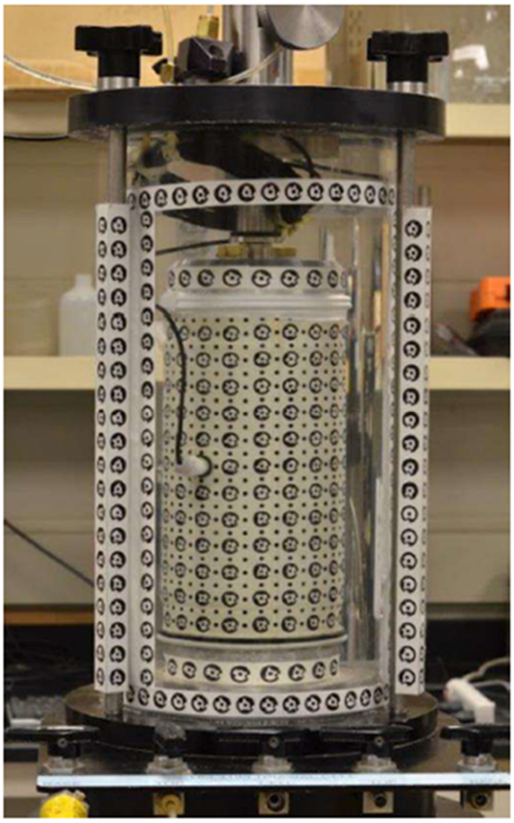

**Figure 3.** Undrained triaxial test setup with a miniature high-suction tensiometer.

## 4. Test Results and Interpretation

Fig. 4 shows the stress paths followed in the v:p:s space and test results for three isotropic compression tests with different moisture contents. The specimens were loaded following the stress paths described in Table 1. The lower the moisture content of the specimen, the higher the suction is. During isotropic undrained loadings, all three specimens underwent a reduction in the matric suction. This is caused by the increase in the radius of the meniscus of the water-air interface of macro-pores in the soils when the soil is compressed; consequently, the matric suction would reduce during loading (Zhang 2016). In terms of volume change, the soil specimens have yield stresses between 200 and 400 kPa. During undrained unloading and reloading, all soil specimens demonstrated irrecoverable volume changes.

MSSA is used to analyse the results from undrained triaxial tests. As discussed previously, the yield curves are the interception of elastic and plastic surfaces where the evolution of the yield curves forms the plastic surface. The first step to determine the yield curve is to select the model parameters that accurately predict the soil behaviour under isotropic conditions. Then, the selection of correct parameters will automatically result in the correct estimate of the form and shape of the LC yield curves.

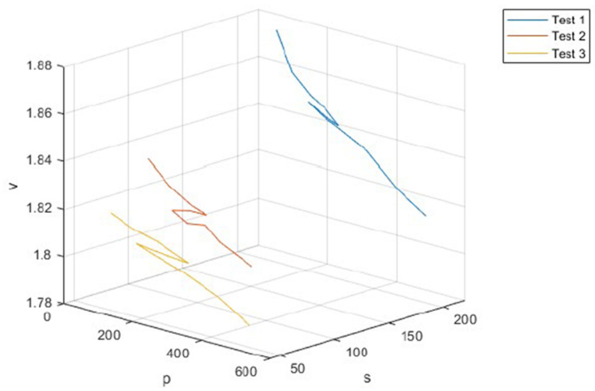


**Figure 4.** Stress paths followed in the v:p:s space for Undrained Triaxial Tests.

The calibrated model parameters are presented in Table 2. The parameter calibration process according to MSSA can be simplified in the following steps:

(1) Determine the parameter κ and κs from the undrained unloading stress paths. The data of the undrained unloading stress paths are used to best fit a plane in MATLAB. Then the values of κ and κs were derived from the plane equation. The averaged κ and κs for all undrained loading paths were found to be 0.01 and 0.15, respectively.

(2) Calibration of the model parameters by finding a combination of the model parameters of N(0), λ(0), r, β, and pc to best fit the experimental (undrained test) results at virgin states. The model calibration is done by considering the following:

a. Some model parameters have physical meanings (Zhang and Xiao 2013). So it is important to set some constraints for example N(0) > 0, λ(0) > 0, and pc > 0

b. Minimizing the difference between the experimental data at virgin states and the theoretical results by finding a combination of parameters N(0), λ (0), β, r, and $p^c$.

**Table 2.** Soil Specimen Used in Testing Program

| Parameter | Unit | Best Fit Value |
|---|---|---|
| κs | - | 0.15 |
| κ | - | 0.01 |
| N(0) | - | 1.717 |
| λ(0) | - | 0.02 |
| r | - | 15.088 |
| β | - | 1.218 |
| $p_c$ | - | 15.591 |
| $p_{at}$ | MPa | 0.1 |
| Standard Deviation | | 0.016 |
| Correlation coefficient | | 93.4% |

Curves can be determined using the MSSA. MSSA does not necessitate the knowledge of the initial yield stress. In addition, the soil specimens with nonidentical stress histories can be used in the constitutive modelling of unsaturated soils using the MSSA approach. It is necessary to test soil specimens beyond their yield stress to ensure that enough data can be obtained in the virgin states. Then the LC yield curve can be obtained which summarizes the criterion stated in MSSA: (1) the LC yield curve is the interception of the unloading–reloading elastic surface and the plastic surface. (2) the LC yield curve can be obtained by varying the value of C1. The yield curves in the s-p plane from the undrained triaxial tests are presented in Fig 5. It can be noticed that on the yield curves, the suction increases with an increase of the mean net stress, which conforms with the typical collapsible behaviour for compacted soils, as shown in Alonso et al. (1990) and Zhang and Lytton (2009a, 2009b).

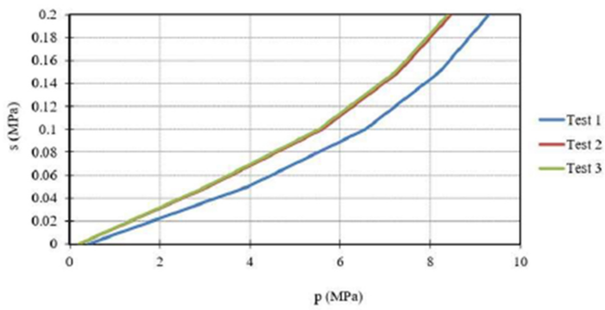


**Figure 5.** Evolution of yield curves in the tests stress range.

## 5. Conclusions

The suction-controlled triaxial testing method has been extensively used as a standard approach to characterize constitutive modelling for unsaturated soils. The SCTX tests are based on the divide-and-conquer approach which implies that the soil behaviour is stress-path independent. However, this approach opposes the behaviour of unsaturated soils which is elasto-plastic and stress-path dependent. The previous discussion of SCTX tests has proven that the analyses of their results are not theoretically correct for the characterization behaviour of unsaturated soils. In addition, the SCTX tests have many disadvantages including the long, costly, and laborious tests required for the characterization behaviour of unsaturated soils. A new approach is needed to be used in the constitutive modelling of unsaturated soils and to correctly determine the shape and evolution of yield curves. This research proposed using the UTT with the MSSA as an alternative approach that is simple, fast, and theoretically correct. The photogrammetry-based method was applied to determine the absolute soil volume of soil specimens during UTT. A miniature HST built in-house was developed and installed at the mid-height of the soil specimen to measure matric suction during testing. The results from UTT were analysed using the MSSA approach and the model parameter were calibrated with a correlation coefficient of 93.4%. Then the yield curves were determined and plotted in the s-p space. It can be concluded that the proposed new approach is theoretically sound, and simple in comparison with the

SCTX tests. The UTT requires only a few hours to be performed in contrast with 2 to 3 months for SCTX tests. In addition, this approach can increase the implementation of unsaturated soil mechanics in the routine engineering project at low cost.

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