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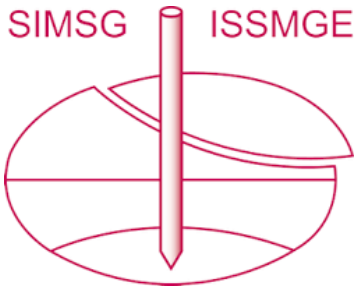

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Small-strain shear stiffness of sand-gravel mixtures

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ABSTRACT

Small-strain shear stiffness (G_0) is a key parameter for seismic ground response analysis and performance evaluation of various earth structures and foundations. To account for its pressure and density dependency, G_0 has been conveniently correlated to mean effective stress (σ_m') and void ratio (e). Such an approach is suitable for conventional uniform materials. However as found in this study, it is not always applicable to estimate the G_0 of sand-gravel mixtures (SGMs), because it essentially fails to account for the combined effects of density and gravel content (G_C). In this study, aimed at addressing this issue and developing a theoretical framework and empirical correlations suitable for estimating the G_0 of SGMs, a series of bender element laboratory tests were carried out on selected SGMs. Specifically, SGMs were obtained by mixing two clean sands – namely New Brighton Sand (mean diameter, D_{50} = 0.2 mm) and Dalton River Washed Sand (D_{50} = 0.75 mm) – and rounded pea gravel (D_{50} = 5.5 mm). Shear wave velocity of specimens having G_C = 0, 10, 25, 40 and 60% and prepared at a relative density (D_r) of 20, 30, 45 and 60% was measured at σ_m' = 50, 100, 150 and 200 kPa. The laboratory results indicated that G_0 of SGMs increases with increasing both the D_r and σ_m', whereas the effect of G_C would be marginal to significant depending on the limiting and threshold sand contents. To correlate G_0 simultaneously with both G_C and D_r, the equivalent void ratio ($e_{f(eq)}$) was adopted. It is shown that the use of $e_{f(eq)}$ makes it possible to uniquely describe the G_0 of SGMs for any combination of G_C and D_r over the full range of σ_m' level applied in this study.

Keywords: Small-strain shear stiffness; Sand-Gravel Mixtures; Equivalent void ratio

1. Introduction

Natural alluvial deposits are often composed of sand and gravel mixtures (SGMs). Such mixtures are also widely used as structural fills for land and port reclamations, earthen embankments for rail and roadways, and backfill materials for retaining walls. The presence of coarse gravel aggregates in a sand matrix may influence the engineering properties (i.e., strength, compressibility, and dynamic behaviour) of SGMs due to the different nature (e.g., mineralogical composition, grain size and shape, roughness etc.) of the host and guest materials.

Small-strain shear stiffness (G_0) is a key parameter for seismic ground response analysis and performance evaluation of various earth structures and foundations. To account for its pressure and density dependency, G_0 has been conveniently correlated to mean effective stress (σ_m') and void ratio (e) (Hardin and Richart 1963; Hardin and Drnevich 1972) as described by Eq. (1):

$$G_0 = A\, f(e) \left(\frac{\sigma_m'}{p_a}\right)^n \quad (1)$$

where σ_m' is the effective mean stress; p_a is the atmospheric pressure (100 kPa); A is material constant; n is a stress-dependent fitting soil parameter and $f(e)$ is a function of void ratio (e) commonly expressed by Eq. (2):

$$f(e) = \frac{(B - e)^2}{1 + e} \quad (2)$$

The value of B can be set equal to 2.17 for rounded grain soils and 2.97 for angular grain soils.

Past studies on sand–silt mixtures (Iwasaki and Tatsuoka 1977; Wichtmann et al. 2015) concluded that the G_0 for sand–silt mixtures decreases with increasing fines content. In addition to the mean effective stress (σ_m') and void ratio (e), grading characteristics (e.g., fines content) are recognized to be key factors influencing G_0 for mixed soils. Compared to the large body of studies on sand and sand–silt mixtures, systematic studies on the G_0 characteristics of gravel and SGMs are relatively limited. Based on shear wave velocity measurements for SGMs reported by Toyota and Takada (2019) and Hubler (2017), it is understood that the G_0 of SGMs is significantly affected by the amount of gravel content (G_C) in the sand matrix at a similar global void ratio (e) or relative density (D_r) and σ_m'.

The above-mentioned studies on mixed soils suggest the inapplicability of sand–based $G_0 - e$ correlations for soils other than clean sand. This is mainly because binary sand mixtures may have an unusual type of void ratio characteristics based on the amount of fines or coarse particles present in the sand matrix. Therefore, it is very difficult to estimate G_0 for mixed soils by traditional indirect methods based on a unique function of void ratio (Kokusho et al. 1995; Thevanayagam and Liang 2001).

The intergrain state framework concept (such as skeleton void ratio, equivalent void ratio, and equivalent state parameter) has been used in past studies (e.g., Rahman et al. 2012) to better understand the combined effect of density and fines content on the mechanical behaviour of sand–silt mixtures, and established a unique function between G_0 and equivalent void ratio $e_{f(eq)}$ for sand–silt mixtures irrespective of the fines content.

However, although suitable for mixed soils, this concept has rarely been applied to SGMs. Therefore, the applicability of such a concept to SGMs needs to be investigated in more detail.

Based on the above background, the main objective of this study is to examine the combined effects of G_C and D_r on the G_0 of SGMs, and assess the applicability of the equivalent void ratio ($e_{f(eq)}$) concept to develop a framework to uniquely describe the G_0 of SGMs for any combination of G_C and D_r over a range of σ_m' values of interest for most geotechnical applications.

2. Equivalent void ratio concept for sand-gravel mixtures

2.1. Threshold sand content

SGMs can be separated into microstructures that are dominated by sand and gravel, depending on the sand content (S_C) in the mixture. The value of S_C at the boundary between sand– and gravel–dominated zones is known as the threshold sand content ($S_{C(th)}$) (Chang and Phantachang 2016).

The threshold fines concentration of sand-silt mixes can be determined using various models, which have been proposed in earlier studies (Lade et al. 1998; Rahman et al. 2009; Zuo and Baudet 2015), and which may also apply to SGMs. Thus, in this current study, to calculate the $S_{C(th)}$ for SGMs, the semi-empirical equation described by Eq. (3) proposed by Rahman et al. (2009) is employed:

$$S_{C(th)} = 0.4\left(\frac{1}{1+exp^{0.5-\frac{0.13}{r}}} + r\right) \quad (3)$$

where $r = d_{50} / D_{10}$, d_{50} = mean diameter of sand grains, D_{10} = effective diameter of gravel grains.

2.2. Equivalent void ratio

It is anticipated that some of the sand particles would actively participate in the strong force chain network even in the gravel–dominated zone, where S_C is lower than $S_{C(th)}$. The equivalent intergranular void ratio ($e_{c(eq)}$) concept proposed by Thevanayagam et al. (2002) can be then used to account for the contribution of sand grains in the load transfer mechanism within the gravel matrix, as described by Eq. (4):

$$e_{c(eq)} = \frac{e + (1-b)S_c}{1-(1-b)S_c} \quad (4)$$

where, b is the finer fraction that contributes to the coarse grain force chain network, and is influenced by the particle size disparity ratio (R_d).

For the sand–dominated zone, where S_C is higher than $S_{C(th)}$, the contribution of coarse particles to the fine–grain strong force chain network cannot be entirely disregarded because gravel particles act as embedded reinforcement elements within the sand matrix up until the limiting fine content $S_{C(lim)}$ (Thevanayagam 2007). Beyond $S_{C(lim)}$, gravel particles are adequately separated without changing the behaviour of the sand matrix. The $S_{C(lim)}$ can be calculated using Eq. (5). Instead, Eq. (6) gives the equivalent void ratio ($e_{f(eq)}$) for the mix, when S_C is higher than $S_{C(th)}$ but less than $S_{C(lim)}$.

$$S_{C(lim)} = \left[1 - \frac{\pi(1+e)}{6\,S^3}\right] \quad (5)$$

$$e_{f(eq)} = \frac{e}{s_C + \frac{(1-s_C)}{R_d^m}} \quad (6)$$

where R_d is the particle disparity ratio ($= D_{50}/d_{50}$), D_{50} is the mean diameter of gravel particles, d_{50} is the mean diameter of sand particles, m is a fitting parameter that depends on the particle gradation and packing condition, and $S = 1 + (10/R_d)$.

However, as per the hypothesis, when S_C is higher than $S_{C(lim)}$, gravel particles are sufficiently separated by sand particles and play no role in the sand matrix force chain network. Therefore, instead of using the equivalent void ratio ($e_{f(eq)}$) concept, the sand skeleton void ratio (e_f^*) should be used. The e_f^* is given by Eq. (7) as follows:

$$e_f^* = \frac{e}{S_C} \quad (7)$$

Equations (3) to (7) were originally developed for sand–silt mixtures based on theoretical studies on uniform spherical arrays; however, in this paper, they have been rewritten and adapted for (SGMs) because the binary packing conditions for SGMs in this study are comparable to those for silt and sand mixtures in earlier studies. This study primarily focuses on the sand–dominated zone to evaluate the effect of G_C and D_r on the G_0 of SGMs.

3. Test materials and procedure

3.1. Test materials

Three types of materials namely New Brighton (NB sand), Dalton River Washed sand (DRW sand) and rounded pea gravel (Gravel) were used to undertake this study. The NB sand and DRW sand were firstly mixed in equal proportion by mass (50:50) to create a well–graded host sand. Then, the pea gravel was added to generate the desired SGMs. Fig. 1 shows the particle size distribution curves of the parent materials and selected SGMs with G_C = 0, 10, 25, 40 and 60% by mass used in this study to investigate the G_0 characteristics of SGMs.

Basic index properties (i.e., specific gravity, G_s; maximum void ratio, e_{max} , and minimum void ratio, e_{min}) and G_0 characteristics of the primary materials are listed in Table 1. G_0 is expressed in the form indicated in Eq. (1). In this study, the values of A (in MPa) and n are 69.2 MPa and 0.5 for NB sand, and 67.8 MPa and 0.58 for DRW sand, respectively. Such coefficients are comparable to those of round–grained Ottowa sand, i.e., A = 70 MPa and n = 0.5 (Hardin and Richart 1963), and subangular–grained Toyoura sand, i.e., A = 84.0 and n = 0.5 (Kokusho 1980).

The variation of e_{max} and e_{min} with S_C (or G_C = 1-S_C) of the host sand (50% NB sand + 50% DRW sand), gravel and selected mixtures is reported in Fig. 2. By using Eq. 3, the theoretical $S_{C(th)}$ of the investigated SGMs is 0.31 (or 31%). For $0 \le S_C \le S_{C(th)}$, SGMs have a gravel–dominated structure; the global void ratio (e) of the mixtures reduces with increasing S_C as more sand particles fill the voids between the larger gravel particles. Alternatively, for $S_{C(th)} < S_C \le 1$, SGMs have a sand–

dominated structure; sand particles replace gravel grains and e of the mixtures increases with increasing S_C as more gravel particles are dispersed in the sand matrix.

The $S_{C(lim)}$ calculated using Eq. (6) for the investigated SGMs is in the range of 0.75–0.80 (or 75–80%). As shown by Fig. 2, in this study, SGMs having G_C = 0 and 10% represent the case of $S_C > S_{C(lim)}$, and those with G_C = 25, 40 and 60% represent the case of $S_{C(th)} < S_C < S_{C(lim)}$.

Table 1. Index properties and G_0 characteristics of the primary materials.

Material	G_s	e_{max}	e_{min}	G_0 (MPa) – Eq. (1)
NB sand	2.67	0.99	0.61	$69.2\frac{(2.17-e)^2}{(1+e)}\left(\frac{\sigma'_m}{p_a}\right)^{0.50}$
DRW sand	2.65	1.07	0.70	$67.8\frac{(2.17-e)^2}{(1+e)}\left(\frac{\sigma'_m}{p_a}\right)^{0.58}$
Gravel	2.66	0.69	0.52	$78.4\frac{(2.17-e)^2}{(1+e)}\left(\frac{\sigma'_m}{p_a}\right)^{0.45}$

3.2. Specimen preparation and testing procedure

The wet tamping method was adopted to create reconstituted SGM specimens with 130 mm in height and 61 mm in diameter. Firstly, the required mass of mixed materials was established based on the dry density of the mix and the specimen's size/volume; following, the mass of each primary material was calculated based on the percentage of each material in the mix. The materials were further divided into 5 equal-mass portions to produce homogeneous specimens with uniform particle size distribution and amount of each material as recommended by Ishihara (1993). The pre-weighed oven-dry host sands and gravel were mixed with deaired water with a water content of 5%. Then, the moist SGMs were compacted inside a split mould by means of a small compacting rod into 5 layers. The compacting energy was adjusted as required to achieve a uniform relative density in each layer.

After confirming the target relative density, the specimen was transferred to the triaxial cell and connected to the loading system. Initially, isotropic confining pressure of 50 kPa was applied and each specimen was consolidated for at least 30 min. To obtain G_0 for SGMs, shear wave velocity (V_S) measurement was carried out using bender elements attached to the top and bottom pedestal of the triaxial testing system. Subsequently, the isotropic confining pressure was increased to 100 kPa and followed the same consolidation process and V_S measurement procedure. The steps were repeated for 150 kPa and 200 kPa isotropic confining stress conditions. For each test, various sinusoid signals at a frequency (f) ranging from 1 to 25 kHz were used as input and the corresponding received signals were examined to identify the travel time of V_S. Eventually, in this study, the V_S result obtained using the sinusoid signal at f = 10 kHz and the peak-to-peak method of signal interpretation to estimate the G_0 of SGMs throughout the analysis of the data. Firstly, V_S was estimated using the relation; $V_S = L/T$, where L is the length of the specimen in the V_S travel direction and T is the V_S travel time. Then, G_0 was calculated using Eq. (8) as follows:

$$G_0 = \rho\, V_S^2 \tag{8}$$

where, ρ is the bulk density of the specimen.

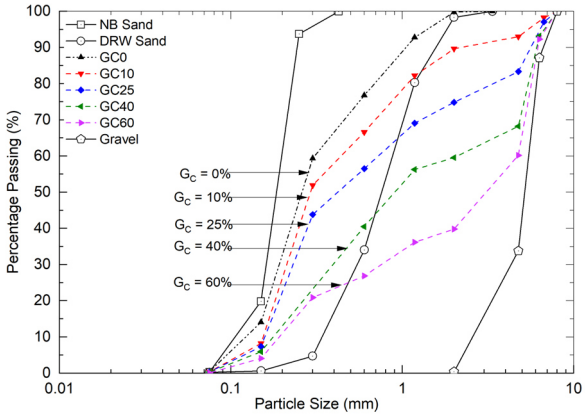

Figure 1. Particle size distribution curves for NB sand, DRW sand, gravel and SGMs.

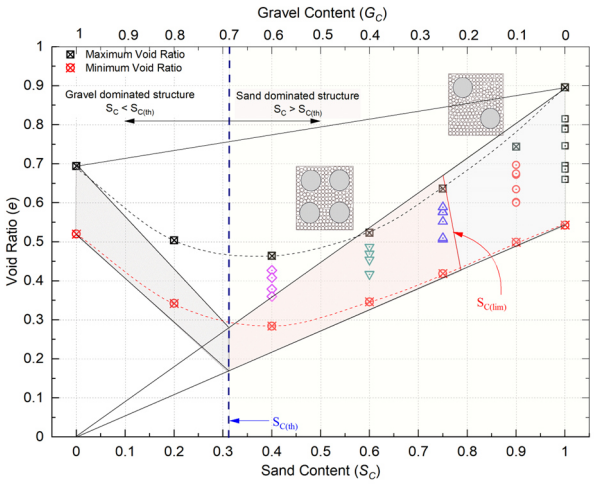

Figure 2. Variation of maximum and minimum void ratios of SGMs.

4. Test results

4.1. Variation of G_0 with G_C

The variation of G_0 with G_c at 100 kPa confining stress is illustrated in Fig. 3 for four different density states (i.e., D_r = 20, 30, 45, and 60%) of SGMs. Irrespective of the D_r value, G_0 progressively increases with increasing G_C up to 40%; yet, at G_C = 60%, G_0 is found to be almost similar or slightly lower (i.e., up to 5%) than that at G_C = 40%. In addition, the G_0 of each mixture increases with increasing D_r. From Fig. 3, it is therefore evident that G_0 of SGMs is significantly and simultaneously influenced by both G_C and D_r.

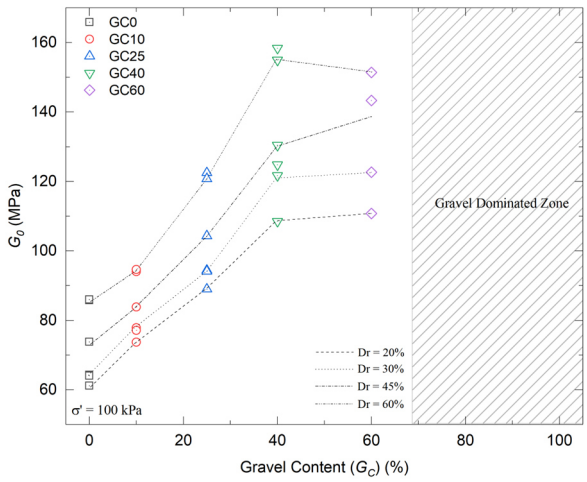

Figure 3. Variation of G_0 with G_C for SGMs at different relative density states.

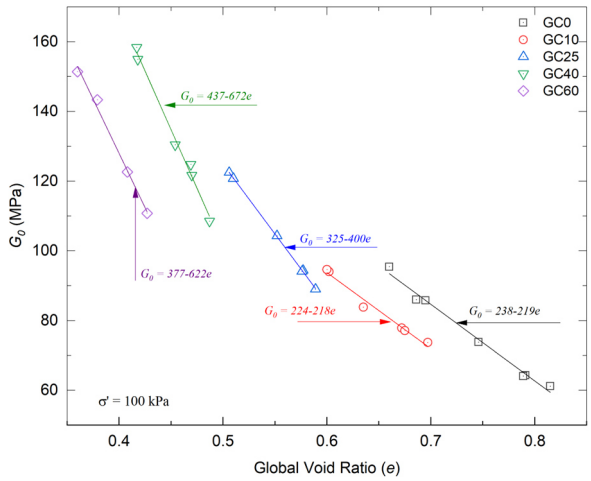

Figure 4. Variation of G_0 with global void ratio.

4.2. Variation of G_0 with global void ratio (e)

The data points shown in Fig. 3, are replotted in Fig. 4 in terms of G_0 vs. global void ratio (e) relationships. The results indicated that the G_0 of each mixture decreases linearly with increasing e. A similar finding was reported by Kokusho and Yoshida (1997) for V_S of gravelly soils. Furthermore, G_0 significantly increases with increasing G_C as the SGMs become coarser and more well-graded.

The independent G_0–e relationships obtaind of each SGM configuration is clear evidence that G_0 of these mixed soils cannot be described by a unique function of D_r or global void ratio (e). Thus, other correlations between G_0 with state parameters are analyzed hereafter.

4.3. Variation of G_0 with effective mean stress (σ_m')

The values of G_0 normalized by the Hardin's void ratio function (Eqn. 2) are plotted in Fig. 5 as a function of normalised confining stress. The G_0 is found to increase with increasing confining stress for all the SGM configurations, evidently reflecting the stress dependence of G_0. The material constant A and exponent n (Eqn. 1) are obtained by regression analysis for each G_C and their values are summarised in Table 2. The exponent n is found to be affected by the G_C as it increases with increasing G_C. Alternatively, there is not a clear trend for the material constant A, although Figs. 3 and 4 would suggest a dependence of A from G_C to reflect the increasing value of G_0 with increasing G_C. Yet, the empirical Eqn. (1) was found to be appropriate for all the tested SGMs as it yields an R^2 value equal to or higher than 0.90 for all G_C conditions.

Table 2. Constant parameter for normalized G_{max}

Test No.	G_C (%)	A	n	R^2
1	0	64.1	0.506	0.90
2	10	59.1	0.509	0.98
3	25	61.6	0.52	0.95
4	40	65.3	0.53	0.90
5	60	58.2	0.58	0.93

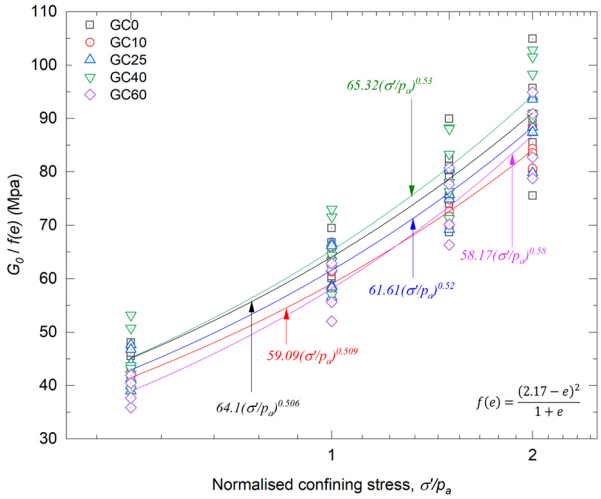

Figure 5. Void ratio normalized G_0 as a function of normalised confining stress.

4.4. Relationship between G_0 and equivalent void ration ($e_{f(eq)}$)

The variation of stress-normalized G_0 with the equivalent void ratio ($e_{f(eq)}$) is illustrated in Fig. 6. Stress- normalized G_0 was calculated using the exponent n obtained from the regression analysis of the G_0 results of each SGMs at different confining stress levels. The $e_{f(eq)}$ for G_C = 0 and 10% was calculated using Eqn. (7), while that for G_C = 25, 40 and 60% was determined using Eqn. (6). The contact index value "m" indicated in Eqn. (6) was back-calculated based on the experimental results. By regression analysis of all the data, an arbitrary value of 0.3 provided the highest fitting R-squared (R^2) value of 0.97. Fig. 6 shows that, for all the SGMs, the G_0 values fit into a narrow band with a relative error of ±10%, producing a unique $G_0 - e_{f(eq)}$ correlation as indicated by the black solid line. The G_0 is, therefore, found to decrease non-linearly with increasing $e_{f(eq)}$. The decreasing correlation obtained for all the SGMs can be expressed by Eqn. (9):

$$G_0 = 37.53\, e_{f(eq)}^{-2.38} \left(\frac{\sigma'}{p_a}\right)^n \quad (9)$$

Therefore, $e_{f(eq)}$ appears to be a promising parameter to capture the effect of G_C on the G_0 of SGMs, when S_C is higher than $S_{c(th)}$, i.e., for sand–dominated microstructures of SGMs. The proposed $\frac{G_0}{(\sigma'/p_a)^n} - e_{f(eq)}$ relation might be useful to estimate the G_0 of sand–dominated SGMs for any G_C conditions.

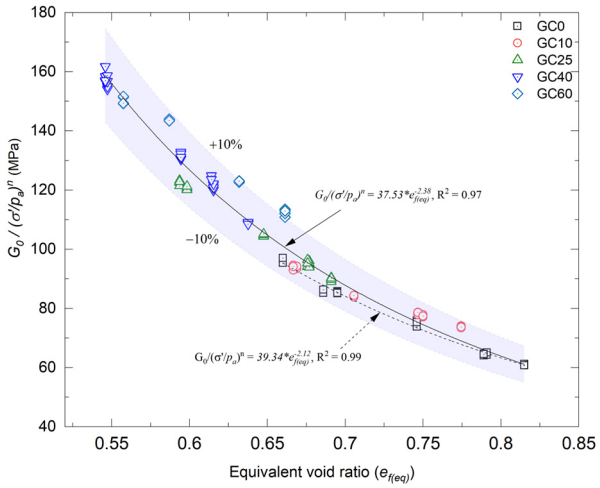

Figure 6. Relationship between stress-normalized G_0 and equiavalent void raio ($e_{f(eq)}$).

4.5. Relationship between $e_{f(eq)}$- normalised G_0 and normalised confining stress

The G_0 values were normalized by $e_{f(eq)}$ instead of the void ratio (e) in the Hardin's void ratio function (Eqn. 2). The $e_{f(eq)}$–normalized G_0 as a function of normalized confining stress is presented in the Fig. 7. The material constant A and the exponent n in Eqn. (1) obtained by regression analysis for each G_C conditions and their values are summarised in Table 3. Both the material constant A and the exponent n increase with increasing G_C. Such increasing trends clearly reflect the effect of G_C on the G_0 of SGMs.

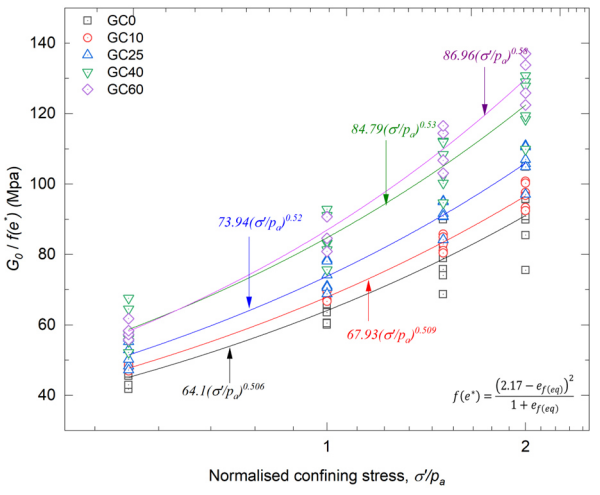

Figure 7. $e_{f(eq)}$ normalized G_0 as a function of normalised confining stress.

Table 3. Constant parameter for $e_{f(eq)}$-normalized G_0

S.No	G_C (%)	A	n	R^2
1	0	64.10	0.506	0.90
2	10	67.93	0.509	0.99
3	25	73.84	0.52	0.96
4	40	84.79	0.53	0.93
5	60	86.96	0.58	0.97

4.6. Application of equivalent void ratio

For the host sand matrix (i.e., G_C = 0%), the global void ratio (e) is equal to the equivalent void ratio ($e_{f(eq)}$). Therefore, from the regression analysis of G_0 data points for only the sand matrix, as indicated by the dotted fitting line in Fig. 6, the necessary soil parameters can be obtained. The non-linear equation for the sand matrix can be written as indicated by Eqn. (10):

$$G_0 = 39.34\, e_{f(eq)}^{-2.12} \left(\frac{\sigma'}{p_a}\right)^n \quad (10)$$

Then, using Eqn. (10), the G_0 values for any other G_C conditions can be predicted. The predicted G_0 versus laboratory–measured G_0 values are plotted in Fig. 8 for comparison. To calculate the $e_{f(eq)}$, back–calculated "m" value of 0.3 was used in Eqn. (6). The predicted G_0 is in good agreement with the measured G_0. Thus, the $G_0 - e$ relation of only sand seems to be a useful proxy to estimate the G_0 of other SGMs irrespective of gravel content in the mixtures.

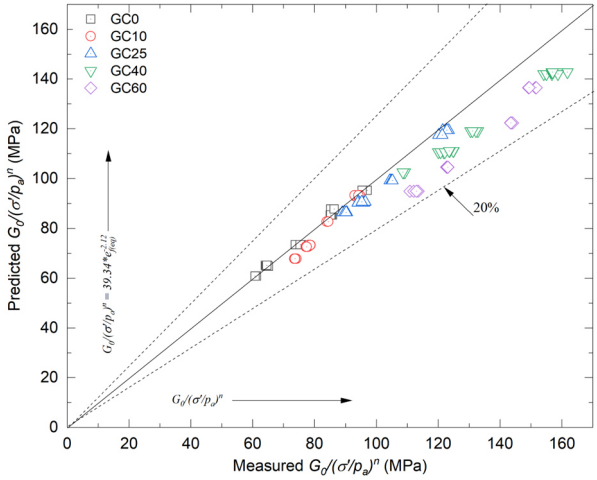

Figure 8. Comparison between measured and predicted G_0 using sand–based $G_0 - e_{f(eq)}$ relation.

5. Conclusions

To estimate the small-strain shear stiffness (G_0) of sand-gravel mixtures (SGMs), a series of S-wave tests by means of bender element was conducted on five selected SGMs having gravel content (G_C) = 0, 10, 25, 40 and 60%, prepared at a relative density (D_r) of 20, 30, 45 and 60%, and tested under σ_m' = 50, 100, 150 and 200 kPa confining pressure. The effects of σ_m', G_C and density state on the G_0 of SGMs were evaluated and the suitability of developing a framework to uniquely describe the G_0 of SGMs was explored.

The following main conclusions can be drawn from this experimental study:

- G_0 of SGMs increases with increasing $\sigma_m{}'$, G_C and D_r. Nevertheless, the effect of G_C would be marginal to significant depending on the amount of G_C on the specimen.
- The effects of G_C and D_r on the G_0 of SGMs cannot be considered independently but should be seen as a combined effect. In this regard, this study indicates that the equivalent void ratio $e_{f(eq)}$ is a promising parameter to uniquely describe the small-strain shear stiffness of SGMs, since it makes it possible to suitably combine the effects of G_C and D_r.
- The $G_0 - e$ relation obtained for only sand is useful to estimate G_0 for any gravel content of SGMs, if the microstructure is sand–dominated.

Abbreviations;

SGMs	Sand–gravel mixtures
G_C	Gravel content
S_C	Sand content
$S_{C(th)}$	Threshold sand content
$S_{C(lim)}$	Limiting sand content
D_r	Global relative density
e	Global void ratio
e_{max}	Maximum void ratio
e_{min}	Minimum void ratio
G_S	Specific gravity
G_0	Small strain shear modulus
e_f^*	Inter–fine skeleton void ratio
$e_{c(eq)}$	Inter–coarse equivalent void ratio
$e_{f(eq)}$	Inter–fine equivalent void ratio
d_{50}	Mean diameter of sand
D_{50}	Mean diameter of gravel
D_{10}	10% gravel particles finer than D_{10}
σ'	Effective confining pressure
p_a	Atmospheric pressure
n	Function of effective confining stress
WT	Wet tamping method
R_d	Particle disparity ratio (D_{50} / d_{50})
b	Sand fraction that participates in gravel structure
m	Gravel fraction that participates in sand structure

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