# Influence of particle crushing on the critical state line of rockfill materials

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## ABSTRACT

The mechanical behaviour of rockfill materials is strongly affected by particle breakage, which causes a continuous variation of the grain size distribution (GSD) until a stationary condition. Although the evolution of grain crushing caused among others by the initial grading, the relative density and stress level has been extensively studied, the relationship describing the influence of the current GSD on other soil properties such as the critical state line is not uniquely defined. In this study, a series of triaxial compression tests with various stress paths under monotonic loading were performed on a rockfill to examine the effect of particle breakage on the position of the critical state lines (CSLs) in the compression plane. Specimens were prepared with the same initial grading and relative density to investigate the evolution of the GSD as a result of grain breakage determined by different stress paths applied in a large triaxial apparatus. In addition, a recently proposed simplified procedure is employed to capture the evolution of the CSLs of the tested crushable rockfill as a function of a breakage parameter related to the breakage index $B_g$. Experimental results obtained for the tested rockfill demonstrate that the abovementioned procedure is capable to predict the position of the CSL linked to the GSD reached at the end of the triaxial test, also when the critical state condition, defined as the ultimate condition in which shearing could continue indefinitely without changes in volume or effective stresses, is not reached.

**Keywords:** Rockfill; grain breakage; critical state line; triaxial tests.

## 1. Introduction

Grain breakage is one of the major mechanisms that rules the stress-strain behaviour of rockfill, modifying its structure, affecting its friction angle, strength, and permeability (Marsal 1967, Marachi *et al.* 1972, McDowell and Bolton 1998, among others), as well as wetting deformation and compressibility (Oldecop and Alonso 2001, Ventini *et al.* 2019a, 2019b, 2020). It has been widely recognized in literature that the general factors influencing the grain breakage of coarse granular materials are particle strength, particle size, angularity, grain size distribution, and initial porosity (Marsal 1967, Hardin 1985, Lade *et al.* 1996, McDowell and Bolton 1998; Lirer *et al.* 2011). The current understanding is not enough to quantitatively evaluate the impact of grain breakage on some aspects of the stress-strain behaviour of rockfill such as critical state condition, *i.e.* the state of constant volume and constant shear stress at a constant mean effective stress (Roscoe *et al.* 1958).

Clearly the influence of the grading evolution on the mechanical response of granular soils is more evident in rockfill prone to crushing. Some authors (Luzzani and Coop 2002, Coop *et al.* 2004) have experimentally verified that grain breakage stops at very high strain levels, which are difficult to reach with conventional triaxial equipment. At high stress levels particles undergo breakage that results in a continuous change of soil gradation. The breakage causes additional compressibility and volume change, resulting in uncertainty in defining the critical state condition. Hence, it is expected that for the soil to reach the critical state, a stable gradation should be reached for a specific stress level (Luzzani and Coop 2002). This implies that such gradation would be more stable than the original one and could sustain a higher level of stresses without further grain breakage.

In order to highlight the effect of particle breakage on the critical state, Wood and Maeda (2008) suggested to determine a gradation state index that incorporates the evolution of the particle size distribution in the constitutive model and reproduces the parallel downward shift of the CSL in the compression plane $e$-log($p$') due to grain crushing. This hypothesis was used by Ventini *et al.* (2021) to propose a simplified conceptual model capable to predict the evolution of particle breakage along a generic stress path and the corresponding shift in the location of the initial CSL in the compression plane as a function of a breakage parameter (Fig. 1). In addition, Tong *et al.* (2022) presented a new critical state model using the particle size distribution as a variable and assuming that the CSL shifts downwards in the $v$-ln($p$) plane with increasing particle breakage under relatively low stresses. However, as confirmed by many studies (Been *et al.* 1991, Russel and Khalili 2004, Chang and Deng 2020) at high stresses particle breakage stops and all the CSLs for different grain size distributions converge to a steady state condition (Fig. 1).

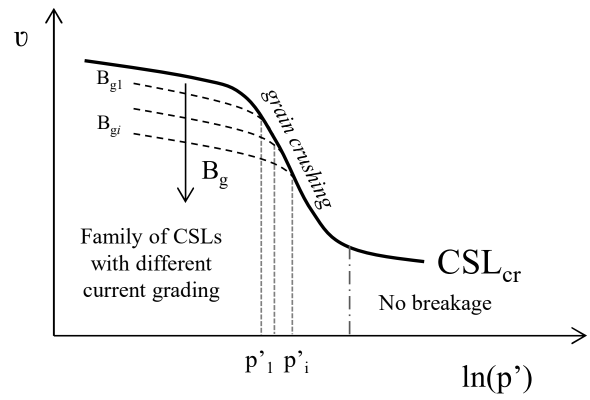


**Figure 1.** Family of CSLs changing with the current grading ($B_g$) in the compression plane (After Ventini *et al.* 2021).

Therefore, many studies reported in the literature (Russel and Khalili 2004, Bandini and Coop 2011, Kan and Taiebat 2014), showed the limited influence of the particle breakage on the critical state friction angle of granular soils. Thus, at low stresses, the CSL in the *q-p'* space can be represented by a straight line and can be assumed to be independent of particle breakage as reported in Xiao *et al.* (2016). Based on large-scale triaxial compression tests on Tacheng rockfill material, Xiao *et al.* (2016) verified that the CSL gradient in the *q-p'* space can be assumed as a constant and can be considered independent of density, pressure, gradation, and particle breakage.

In this context, the aim of this paper is to examine the effect of grain crushing on the location of the CSLs in the compression plane by means of large triaxial tests on rockfill. First, some basic observations on the stress-strain behaviour of saturated dense specimens subjected to different stress levels and stress paths are reported. Then, a model proposed in the literature is validated through some experimental results of triaxial tests.

## 2. Experimental program and procedure

### 2.1. Tested soil

The tested soil is a rockfill taken from the Marsico Nuovo dam (Italy), mainly used for irrigation of the area near the Agri river. It is a homogeneous dam with an impermeable sealing layer and a height of 68.2 m. The rockfill tested consists of grains with sharp edges and specific gravity $G_s$ equal to 2.71. The prototype grain size distribution reported in Fig. 2 was changed due to the limitation imposed by the maximum diameter $D$ of the triaxial apparatus available. It is widely known from the literature (Marachi *et al.* 1972, Indraratna *et al.* 1993, Skoglund 2002) that the maximum rate $D/d_{max}$ with $d_{max}$ the maximum grain size can vary in the range 5-7. Some modeling techniques are usually used to reduce the grain size so that the specimen prepared with smaller size particles can be representative of the prototype. Among all, the parallel gradation technique (Lowe 1964) is most used and the same has been adopted for modeling the rockfill materials from Marsico Nuovo (MN) dam. The technique consists of determining a grain size distribution parallel to the original one by fixing the maximum particle size as the one that can be accommodated in the testing equipment. The grain size distribution adopted for the tested specimens (Fig. 2) has a $d_{max}$ equal to 40 mm, a minimum diameter $d_{min}$ of 0.5 mm and a uniformity coefficient $U_c$ of 4.53. Laboratory tests to determine $e_{min}$ and $e_{max}$ of the grain size distribution tested were carried out under the standard ASTM D4253 and ASTM D4254. The values determined are 0.377 and 0.772 respectively.

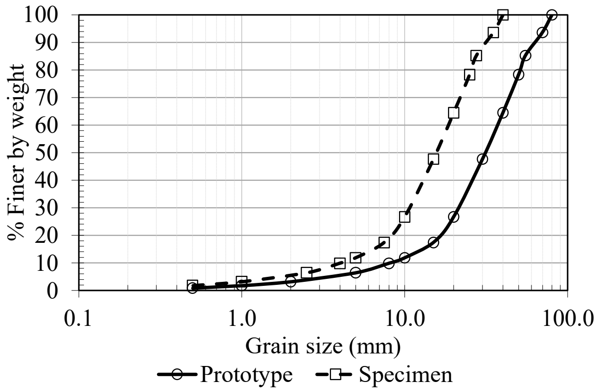


**Figure 2.** Grain size distributions of the tested rockfill at prototype and specimen scale.

### 2.2. Triaxial tests

A large stress path triaxial apparatus, which can test specimens up to 200 mm in diameter and 410 mm in height available at the University of Naples Federico II, was used to analyse the key features of the rockfill behaviour. The apparatus, able to perform tests with strain and stress control, is equipped with an internal load cell located on the top cap to avoid friction effects between the loading piston and the piston casing.

**Table 1.** Details of consolidated drained triaxial tests

| Test ID | Isotropic compression phase | Deviatoric phase | |
|---|---|---|---|
| | **σ'c(MPa)** | **p' (MPa)** | **σ'c (MPa)** |
| **IC-MN** | Up to 0.80 | - | |
| **MN0.5** | 0.05 | variable | constant |
| **MN1P** | 0.10 | constant | variable |
| **MN2.5** | 0.25 | variable | constant |
| **MN5P** | 0.50 | constant | variable |

The experimental program, detailed in Table 1, includes 5 consolidated drained triaxial tests, of which one isotropic compression (IC) test up to the vertical effective stress of 0.80 MPa and four monotonic tests. These tests were performed at either constant effective confining stress $\sigma'_c$ or constant mean effective stress $p'$. In the latter case, the letter P was added to the test ID reported in Table 1. The confining stress applied in the isotropic compression phase of the tests varies in the range 0.05-0.80 MPa and is reported in 10·MPa in the Test ID (Table 1).

All specimens were directly compacted to a relative density $D_r$ = 75% in the triaxial cell base, with a 3.5 mm thick membrane of latex placed into a split mould. A tamping method was applied to 5 layers with the help of a Marshall hammer. Particle breakage due to compaction was proven to be not relevant by means of a grain size analysis of the specimen after compaction and the comparison with the initial grain size distribution. The prepared specimens have been saturated by flushing and the application of a backpressure until Skempton's B-value exceeded 0.98. After achieving full saturation, the specimens were isotropically consolidated to the predefined confining stresses and then the axial load was applied up to an axial strain of about 30%. The imposed axial displacement rate was 16.3 mm per hour. During the tests, axial stresses, axial strains, and volumetric strains were recorded. The stress and volumetric strain measurements were corrected for the membrane penetration, as described by Fukushima e Tatsuoka (1984) and Baldi e Nova (1984) respectively. At the end of the test the specimen was dried and sieved, and the particle breakage was measured.

## 3. Stress-strain behavior

The results of the triaxial tests performed following different stress paths are shown in Figures 3 and 4. The deviatoric stress path at a constant mean effective stress seems to determine a more pronounced dilatancy in the volumetric plane, accentuated at small effective confining stress. So dilatancy is gradually inhibited by elevated confining stress in both constant $p'$ or constant $\sigma'_c$ tests. This highlights that the loading path has a significant effect on the dilatancy. Obviously, the peak deviator stress increases with increasing applied confining pressure for both constant $p'$ and $\sigma'_c$ paths and the highest values are recorded in the constant $\sigma'_c$ tests. The peak and the critical state envelopes are uniquely determined in the stress plane (Fig. 3a). This results in a peak friction angle of 45.5° and a critical state angle of 41°.

The isotropic compression curve obtained from the IC-MN test is reported in the $\upsilon$ - $p'$ plane (Fig. 3b). As expected, it constitutes the envelope of points representing the stress state at the beginning of the deviatoric phase.

Larger strain levels were reached in all tests, but critical state conditions were identified in the $q$-$\varepsilon_a$ and $\varepsilon_v$-$\varepsilon_a$ planes only for two out of four tests (*i.e.*, MN0.5, MN1P; Fig. 4). This result confirms the well-known difficulty of determining the critical state line (CSL) in the laboratory, which is the reason why the application of the general concept of critical state to rockfill has been less successful and is still debated.

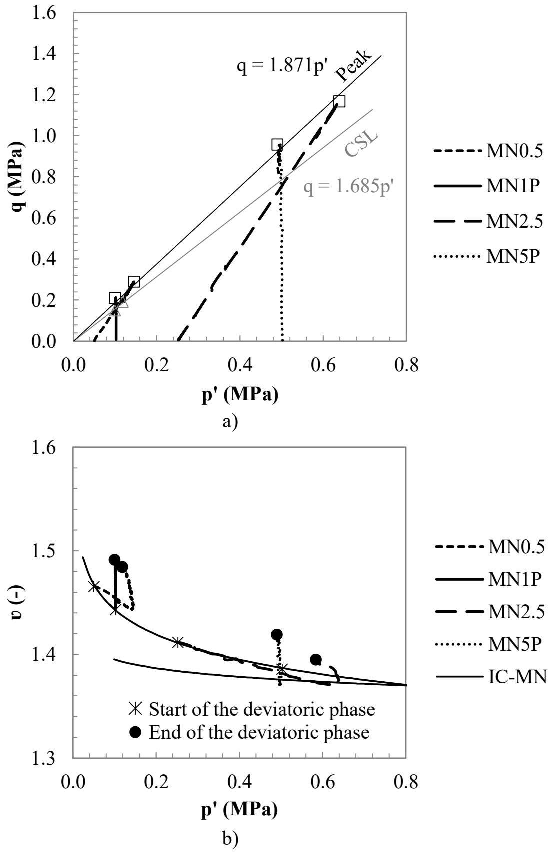


**Figure 3.** Stress-strain-volume change relationship in the a) q - p' plane and b) υ - p' plane.

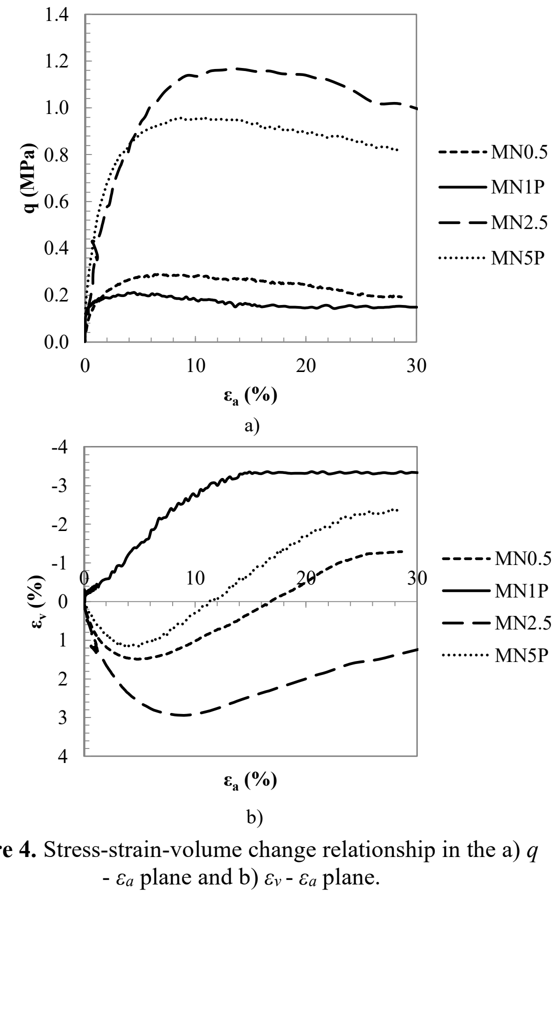


**Figure 4.** Stress-strain-volume change relationship in the a) $q$ - $\varepsilon_a$ plane and b) $\varepsilon_v$ - $\varepsilon_a$ plane.

## 4. Changing CSL due to particle breakage

### 4.1. Literature model validation

Several studies have been carried out to quantify the amount of particle breakage that takes place during a laboratory test (Marsal 1967, Lee and Farhoomand 1967, Hardin 1985, Einav 2007 and many others). In particular, Marsal (1967) suggested the use of a breakage index based on the changes in individual particle sizes between the initial and final grain-size distributions. Marsal's breakage factor, $B_g$, is the sum of the differences in the percentage retained computed for each sieve size having the same sign. This index has been recently adopted in Ventini *et al.* (2021) to propose the dimensionless breakage parameter $B_{gn}$ as a function of $B_g$, as follows:

$$B_{gn} = B_g/(1 + e_{cons}) \quad (1)$$

where $e_{cons}$ is the void ratio at the end of the isotropic compression phase of the triaxial tests.

As shown in Table 2, specimens tested at constant mean effective stress break less or equal than specimens tested at constant confining stress in the deviatoric phase, even when higher confining stresses are applied. This confirms that the deviatoric component of the loading path is the major factor responsible for particle breakage, as assumed by Ventini *et al.* (2021).

As previously mentioned, grading evolution has also a remarkable effect on the location of the critical state line (CSL) in the compression plane. Many authors have rearranged the well know analytical expression for crushable soils as follows:

$$\upsilon = \Gamma(B_g) - \lambda \cdot \ln(p') \quad (2)$$

where the parameter $\Gamma$ is a function of the level of grain crushing.

Ventini *et al.* (2021) used the breakage parameter $B_{gn}$ for the definition of a degradation law of the CSL in the compression plane:

$$\Gamma(B_{gn}) = \Gamma_{min} + (\Gamma_{max} - \Gamma_{min}) \cdot \left(1 + B_{gn}^{\alpha}\right)^{-1} \quad (3)$$

where $\alpha$ is a soil parameter that can be determined by means of a best-fitting procedure.

Equation (2) assumes that the current critical state line (CSL) falls within two possible extreme CSLs expressed by the following equations:

$$\upsilon = \Gamma_{min} - \lambda \cdot \ln(p') \quad (4)$$

and

$$\upsilon = \Gamma_{max} - \lambda \cdot \ln(p') \quad (5)$$

where the intercepts $\Gamma_{min}$ and $\Gamma_{max}$ may be linked to the minimum and maximum specific volumes of the initial grading. As stated by Wood and Maeda (2008) and Tong *et al.* (2022), it was assumed in Ventini *et al.* (2021) that particle breakage only causes a downward shift of the critical state line [$\lambda$ = constant; Eq. (4) and (5) and Fig. 5b], so the gradient $\lambda$ is regarded as an intrinsic material parameter. Such an assumption is reasonable due to the low stress levels applied to the specimens in Ventini *et al.* (2021) and adopted in the large triaxial tests performed on Marsico Nuovo rockfill (Tab. 1).

The simplified model to identify the CSL for crushable soils (Eq. 3) proposed in Ventini *et al.* (2021) undertakes that since grain breakage is affected by both deviatoric and isotropic stress components, it is reasonable to assume the existence of iso-$B_{gn}$ elliptical loci in the deviatoric $q - p'$ plane. In other words, the authors suggest that in the $q - p'$ plane there are elliptic curves with equal value of the particle crushing ($B_{g,i}$), centered in the origin of the axes and with a major axis $a$ larger than the minor one $b$. The greater the ratio $a/b$, the smaller the influence of $p'$ on particle breakage. Therefore, during a generic stress path (line AF, Fig. 5a) the specimen undergoes a continuous evolution of the grain size distribution but for stress paths that fail on the same iso-$B_{gn}$ elliptical loci, the amount of grain crushing is the same (*e.g.* OABC and OAF in Fig. 5). As particle breakage increases, the size of the ellipse (represented, for example, by the $a$ value) will grow being, therefore, a state parameter, while its shape (*i.e.* the $a/b$ value) can be considered constant and assumed to be an intrinsic material parameter.

The model proposed by Ventini *et al.* (2021), based on five easily determinable parameters (*i.e.* $\Gamma_{min}$, $\Gamma_{max}$, $\alpha$, $\lambda$, $a/b$) allows to determine the values of the specific volume in critical state conditions and the positions of the activated CSLs for crushable soils even for tests that did not reach this condition. In order to validate this new model, it has been applied to the triaxial test results on Marsico Nuovo rockfill.

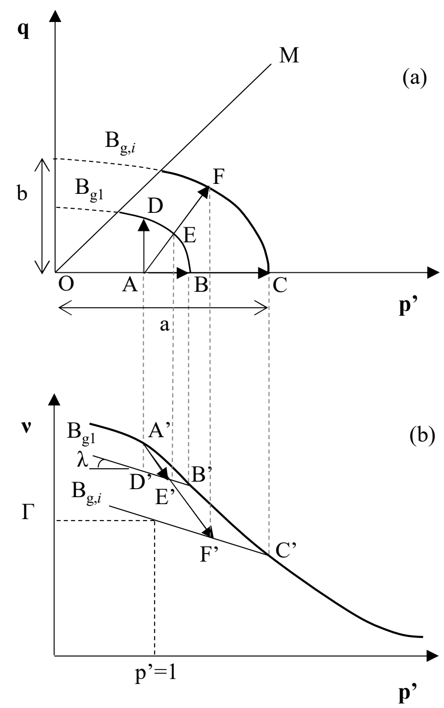


**Figure 5.** Conceptual framework: iso-$B_{gn}$ loci a) in the deviatoric plan and b) in the compression plane (after Ventini *et al.* 2021).

As stated before (eqs. 4 and 5), the $\Gamma_{min}$ and $\Gamma_{max}$ parameters, determined from the compaction tests are 1.377 and 1.772 respectively while the $a/b$ parameter, easily determined by means of the equation of the ellipse passing through data points at equal $B_{gn}$ in the $q - p'$ plane, is equal to 4.72 for the MN rockfill. Specifically, the *a/b* parameter was obtained by considering the results of tests IC-MN, MN0.5 and MN1P that returned similar $B_{gn}$ values (Tab. 2). Known the value of $a$ (*i.e.* 0.80) for the isotropic compression test IC-MN, which must be the same for the other two tests lying on the same ellipse, an objective function was used to obtain *a/b* for the two deviatoric tests. The iso-$B_{gn}$ elliptical loci obtained by considered the *a/b* parameter as a constant for the rockfill are reported in Fig. 6.

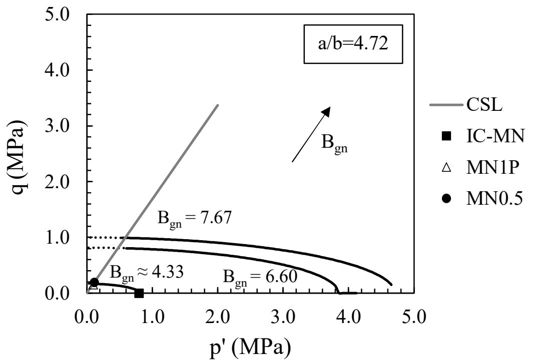


**Figure 6.** Iso-$B_{gn}$ elliptical loci for *a/b* = 4.72.

**Table 2.** Breakage factors

| Test ID | σ'c (MPa) | Bg (%) | Bgn (%) |
|---|---|---|---|
| **IC-MN** | Up to 0.80 | 6.42 | 4.35 |
| **MN0.5** | 0.05 | 6.32 | 4.31 |
| **MN1P** | 0.10 | 6.25 | 4.33 |
| **MN2.5** | 0.25 | 10.82 | 7.67 |
| **MN5P** | 0.50 | 9.14 | 6.60 |

Interesting evidence arises from Fig. 7, in which the experimental data for $B_{gn}$ pertaining to MN rockfill were plotted against the value of the major ellipse axis $a$ and compared with the same data from Redisole and Coreno rockfill materials. For the latter, results from tests on specimens with different initial grain size distributions are interpolated with a single power regression, confirming that the *a/b* value is an intrinsic material property.

In order to determine the intercept $\Gamma$ of each CSL activated during all the tests performed, Eq. (3) has been used. The downward shift of the CSL provided by the model is represented in Fig. 8 in terms of values of $\Delta\Gamma$ ($\Delta\Gamma = \Gamma_{max} - \Gamma_{model}$) versus $B_{gn}$. The comparison of the MN data with data from Ventini *et al.* (2021) confirms that the shift of the critical state line in the compression plane (represented by the $\Delta\Gamma$ parameter) is affected by the initial soil grain size distribution (represented by the $d_{max}$ and $U_c$ parameters) and the intrinsic rockfill properties (such as particle tensile strength, angularity, and shape).

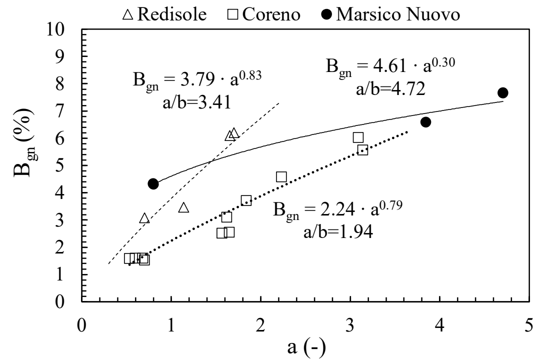


**Figure 7.** Values of $B_{gn}$ versus ellipse major axis $a$ for the tested MN rockfill compared with Coreno and Redisole ones (data from Ventini *et al.* 2021).

The $\lambda$ parameter can be identified once the critical state line in the $v$ - $p'$ plane passing through two or more experimental points that return the same $B_{gn}$ value is known (Fig. 3b). As stated before, during the MN0.5 and MN1P tests critical state conditions and the same level of grain crushing ($B_{gn}$) were reached. Consequently, it is appropriate to assume that the two relative points lie on the same CSL in the $v$ - $p'$ plane (Fig. 3b). At this point it is possible to consider several CSLs, all having the same slope $\lambda = 0.041$. This value is higher than that of Ranjit Sagar rockfill ($0.005 < \lambda < 0.026$) and lower than those of Coreno rockfill ($0.14 < \lambda < 0.19$) reported in Ventini *et al.* (2021), confirming the authors' statement that poorly graded soils (such as the Coreno rockfill) tend to have steeper CSLs than well-graded ones such as the MN rockfill, which has a greater $U_c$ than Coreno but lower than Ranjit Sagar rockfill. Then, knowing $\Gamma$ for the same two tests (*i.e.* MN0.5 and MN1P), the value of $\alpha$ in Eq. (3) was determined by means of a best-fitting procedure ($\alpha = 1.97$).

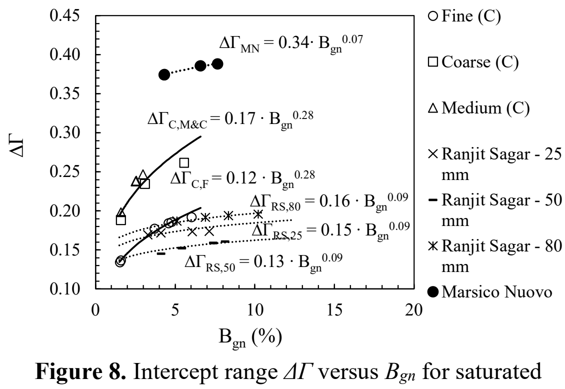


**Figure 8.** Intercept range $\Delta\Gamma$ versus $B_{gn}$ for saturated specimens of MN, Coreno and Ranjit Sagar rockfills (data from Ventini *et al.* 2021).

The data from MN rockfill are an experimental confirmation of the effectiveness of the model proposed by Ventini *et al.* (2021), which provides a degradation law for CSL due to grain breakage, and with the calibration of only five parameters, allows the position of the CSL to be determined for any stress path the specimen has been subjected to.

## 5. Conclusions

The evolution of the grain size distribution significantly affects the stress-strain behaviour of rockfill and makes it difficult to achieve critical state conditions for the strain levels usually reached with conventional equipment. A series of large triaxial compression tests were performed on saturated specimens to investigate the influence of particle breakage on the position of the critical state line (CSL) for a rockfill material. It was found that, for the low stress levels investigated:

- a constant gradient of the CSL in the $v$ - $p'$ plane can be considered even if the grain size distribution changes;
- the intercept $\Gamma$ of the CSL in the compression plane is a variable strictly related to a breakage factor that measures the amount of particles crushing.

To describe the effect of particle breakage on the location of the CSL, a simple degradation law of CSL in the $v$ - $p'$ plane proposed by Ventini *et al.* (2021) was applied. The model parameters were calibrated against experimental results and the simplified procedure was successfully validated. Comparison with literature data allowed for the determination of the $v$ - $p'$-$B_{gn}$ space for the rockfill tested, even although critical state conditions were not reached in all tests performed.

Finally, it emerged that the main advantages of the simplified model proposed by Ventini *et al.* (2021) are its small number of parameters, ease of calibration, and the effective identification of the position of the critical state line based on a well-known and widely used breakage index in the literature.

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