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Natural state parameter for sand

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ABSTRACT

Based on the frictional state concept, the natural state parameter for sands is defined as an extension of the state parameter defined by Been and Jefferies. The proposed definition of the natural state parameter is the sum of the distance between the normalised difference of the current stress ratio and the critical frictional stress ratio and the difference between the current and critical frictional state void ratio for the same mean principal stress. Therefore, it combines the difference between the current and critical frictional state in the $q – p'$ and $e – p'$ planes and can be treated as an extension of Been and Jefferies' definition of the state parameter in the $e – p'$ plane. The results of drained triaxial compression tests for Toyoura sand and Dog's Bay sand, presented in the literature, are analysed. The values of the natural state parameter at failure for these sands are equal to zero. Therefore, the critical state in the $q – p'$ and $e – p'$ planes can be determined by analysing the conditions at failure. At failure, the deformations of the samples are almost homogeneous, and the stresses and deformations (void ratios) can be correctly determined. Additionally, the critical frictional state and critical state are very similar for these sands. The relationship between the dilatancy and the state parameter at failure, similar to that given by Been and Jefferies, was obtained directly by using the frictional state concept and the proposed definition of the natural state parameter. The natural state parameter, like the state parameter, can be used for modelling of sands in the future.

Keywords: state parameter, natural state parameter, sands.

1. Introduction

Been and Jefferies (1985) introduced the state parameter for sands as the difference between the current and the corresponding critical void ratios for the same mean normal stress ($\psi = e - e_c$). The state parameter combines the effect of the density and stress level on the sand's behaviour. Contractive behaviour for positive values ($\psi > 0$) and dilative behaviour for negative values ($\psi < 0$) are observed during shear. The dilatancy at failure and the shear strength of the sands can be simply expressed with the state parameter (Jefferies and Been 2019, Jefferies 2021). Experimental data have demonstrated that strong relationships also exist between the state parameter and the potential instability (Liu and Huang 2015) and shear wave velocity in sands (Guo and Yang 2017). The state parameter is a more meaningful parameter to represent the in-situ state of sandy soil than the relative density (Robertson 2010). Many soil models have been developed using the state parameter (e.g. Jefferies 1993, Taiebat and Dafalias 2008). Some modifications of the state parameter have been proposed to better describe the relationship between the modified state parameter and the dilatancy and shear strength of sands (Gutierrez 2007, Porcino et al. 2020).

The state parameter proposed by Been and Jefferies (1985) represents the "distance" from the current and critical state in the $e - p'$ plane without taking into account the "distance" in the $q - p'$ plane. The Frictional State Concept (FSC) proposed by Szypcio (2016, 2023) makes it possible to define a state parameter called the *natural state parameter* (ψ^o), which combines the distances from the current and critical frictional state in the $q - p'$ and $e - p'$ planes. It is assumed that for dilatant failure states $\psi^o = 0$. This condition defines the critical frictional state in the $e - p'$ plane. For the analysed drained triaxial tests of Toyoura and Dog's Bay sands the critical frictional states and critical states are very close to each other.

2. Natural state parameter

The proposed natural state parameter is the sum of two components:

$$\psi^o = \psi^o_\eta + \psi^o_e \tag{1}$$

where

$$\psi^o_\eta = (\eta - M^o)/M^o \tag{2}$$

$$\psi^o_e = e - e^o_c \tag{3}$$

The ψ^o_η and ψ^o_e represent the "distance" from the current and the corresponding critical frictional states (CFS) for the same Lode angles $\theta = \theta^o$ in the $q - p'$ and $e - p'$ planes, respectively (Fig. 1).

Points P represent the current state, and points C represent the corresponding CFS. The corresponding mean normal stress of the CFS is a function of the current stress state and the stress path (Fig. 1).

The ψ^o_η and ψ^o_e are named the stress ratio and void ratio parts of the natural state parameter, respectively.

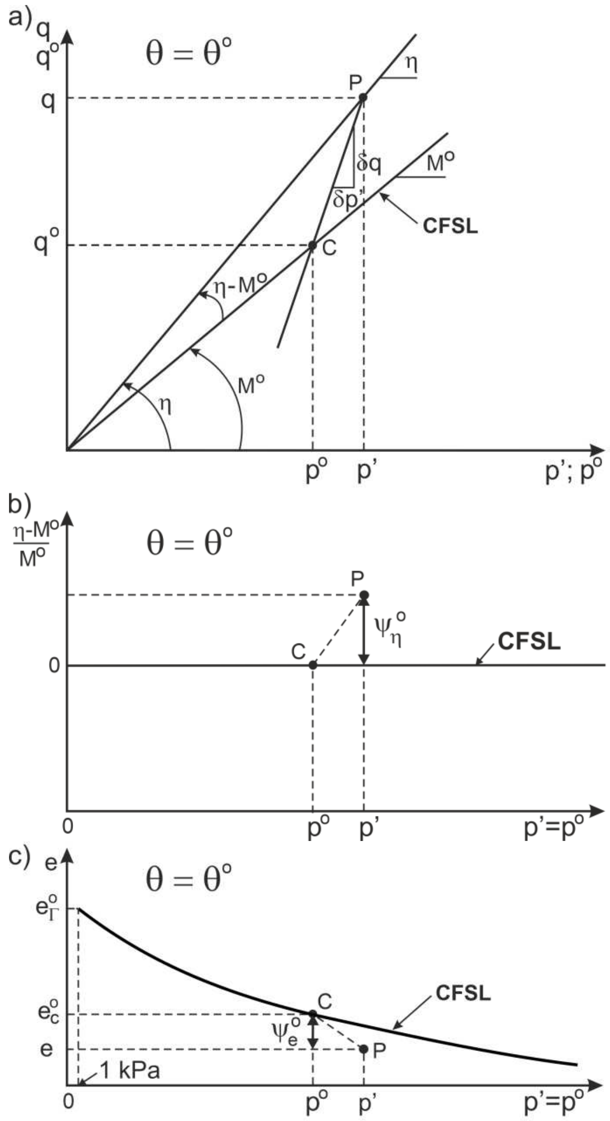

Figure 1. The natural state parameter illustration in plane: (a) $q-p'$; (b) $(\eta-M^o)/M^o-p'$; (c) $e-p'$.

$$p^o=\frac{\eta-\delta q/\delta p'}{M^o-\delta q/\delta p'}p' \tag{4}$$

where $\eta=q/p'$ is the current stress ratio, and $M^o=q^o/p^o$ is the slope of the critical frictional state line (CFSL) in the $q-p'$ plane (Szypcio 2023).

In geotechnical laboratories, conventional drained triaxial compression tests under constant confining pressure ($\sigma_c=\sigma_3'=const.$, $\delta q/\delta p'=3$) and constant mean normal stress ($p'=const.$, $\delta q/\delta p'=\infty, p^o=p'$) are most often performed. It is assumed that for dilatant failure states (Szypcio 2023) the natural state parameter equals zero.

All values in the dilatant failure states (DFS) are marked with the subscript F. Therefore,

$$\psi_F^o=\psi_{\eta F}^o+\psi_{eF}^o=0 \tag{5}$$

where

$$\psi_{\eta F}^o=(\eta_F-M^o)/M^o \tag{6}$$

$$\psi_{eF}^o=e_F-e_c^o \tag{7}$$

$$\eta_F=Q_F-A_F\,D_F^p \tag{8}$$

and

$$Q_F=M^o-\alpha_F\,A^o \tag{9}$$

$$A_F=\beta_F\,A^o \tag{10}$$

In the FSC it is assumed that for sand $\alpha_F=0$. For drained triaxial compression,

$$M^o=M_c^o=6\sin\phi^o/(3-\sin\phi^o) \tag{11}$$

$$A^o=A_c^o=1-\tfrac{1}{3}M_c^o \tag{12}$$

and

$$\phi^o=sin^{-1}(3M_c^o/(6+M_c^o)) \tag{13}$$

The M^o (ϕ^o) is determined by the intersection of the dilatant failure state line (DFSL) with the vertical axis in the $\eta-D^p$ plane (Szypcio 2023). The critical frictional state void ratio is calculated by using Eq. (5):

$$e_c^o=e_F+\psi_{\eta F}^o \tag{14}$$

The corresponding mean normal stress in the CFS is calculated by Eq. (4). Therefore, the critical frictional state can be fully determined by the dilatant failure states and the natural state parameter in these states.

Homogeneous deformations of sheared samples are usually observed in dilative failure states, so the stresses and strains in these states can be determined more accurately than in the ultimate (critical) states. This is the main advantage of introducing the FSC and natural state parameter.

3. Critical frictional state of Toyoura and Dog's Bay sands

In this paper, the drained triaxial compression test of Toyoura sand conducted by Sun et al. (2007) and Miura and Yamanouchi (1975), and Dog's Bay sand conducted by Coop (1990) were analysed. The experimental stress–axial strain and volumetric strain–axial strain relationships presented by Sun et al. (2007) for Toyoura sand and Coop (1990) for Dog's Bay sand were segmentally approximated with high-degree polynomials. The dilatant failure states, marked with points F in Figs. 2 and 3, were identified using the definition given by Szypcio (2023). Dilatant failure states are equivalent to failure states for Toyoura sand (Fig. 2). For Dog's Bay sand, the dilatant failure states and failure states are different (Fig. 3).

For the drained triaxial compression tests analysed above, $M_c^o=1.331$ ($\phi^o=33.0^o$), $A_c^o=0.556$, $\beta_F=1.0$ and $M_c^o=1.748$ ($\phi^o=42.6^o$), $A_c^o=0.417$, $\beta_F=2.36$ respectively for Toyoura and Dog's Bay sands (Figs. 2 and 3). The initial states void ratios (e_i), void ratios after consolidation (e_0), void ratios at DFS (e_F), the parameters of stress–dilatancy relationships for DFS (p_F, η_F, β_F, $\psi_{\eta F}^o$), and the parameters of the CFS (M_c^o, ϕ^o, e_c^o, v_c^o, p^o) are presented in Table 1.

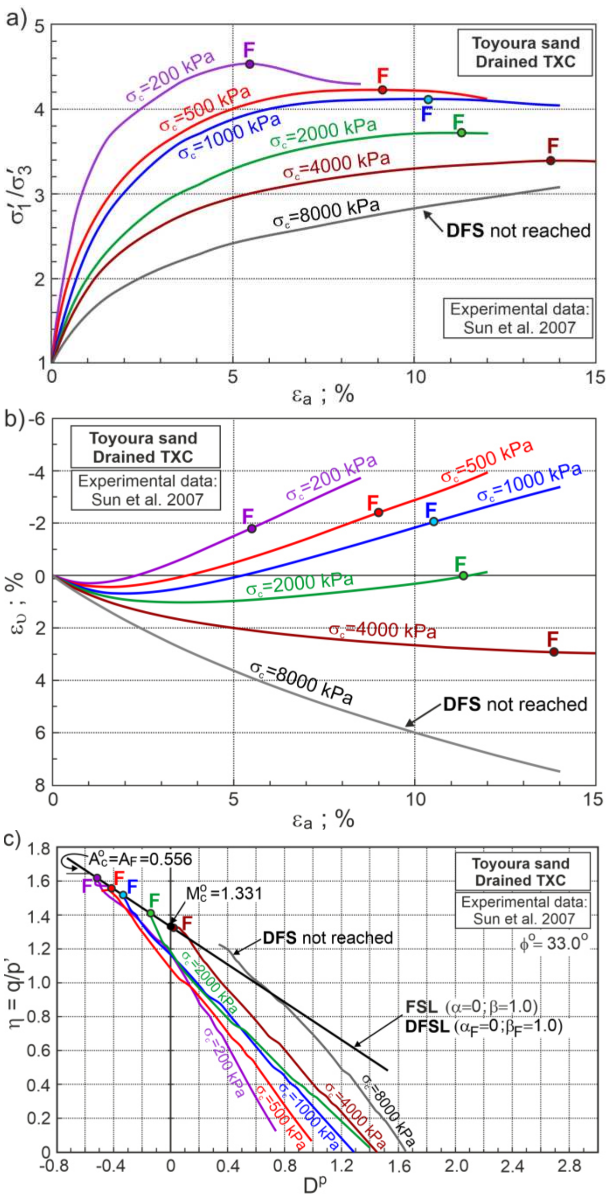

Figure 2. The relations for drained triaxial compression of Toyoura sand: (a) $(\sigma_1'/\sigma_3') - \alpha_a$; (b) $\varepsilon_v - \varepsilon_a$; (c) $\eta - D^p$ (experimental data from Sun et al. 2007).

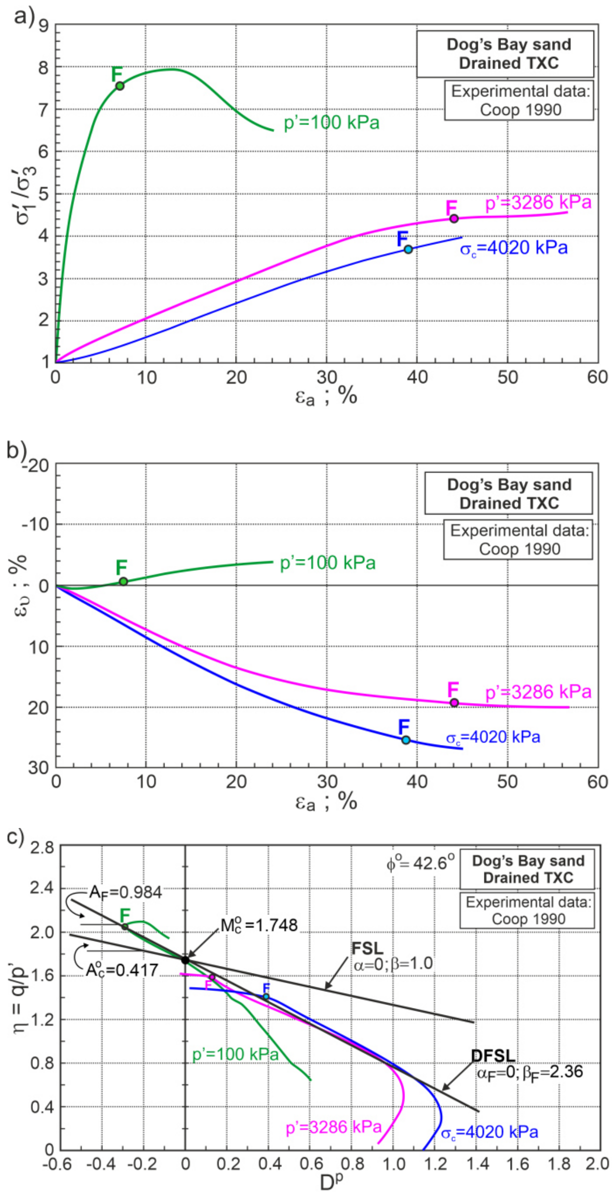

Figure 3. The relations for drained triaxial compression of Dog's Bay sand: (a) $(\sigma_1'/\sigma_3') - \alpha_a$; (b) $\varepsilon_v - \varepsilon_a$; (c) $\eta - D^p$ (experimental data from Coop 1990).

Table 1. Characteristic values at initial states, DFS and CFS

Sand	σ_c; p′	δq/δp′	e_i	e_0	e_F	η_F	D_F	β_F	M_c^o	ϕ^o	$\psi_{\eta F}^o$	e_c^o	p_F	p^o	Ref.
	kPa	–	–	–	–	–	–	–	–	deg.	–	–	kPa	kPa	
Toyoura	200			0.679	0.703	1.622	-0.520				0.220	0.920	435	351	Sun et al. (2007)
	500			0.668	0.698	1.554	-0.428				0.165	0.863	1036	898	
	1000	3	0.680	0.661	0.690	1.528	-0.357	1.0	1.331	33.0	0.146	0.836	2038	1797	
	2000			0.655	0.648	1.406	-0.147				0.071	0.719	3764	3595	
	4000			0.651	0.601	1.325	0.053				-0.002	0.598	7164	7190	
	2500			0.581	0.583	1.502	-0.187				0.110	0.693	5007	4494	Miura & Yamanouchi (1975)
	5000			0.573	0.559	1.357	0.028				0.003	0.562	9130	8988	
	7500			0.564	0.479	1.283	0.130				-0.052	0.427	13104	13482	
	10000			0.558	0.448	1.287	0.130				-0.049	0.399	17513	17981	
	15000	3	0.610	0.544	0.377	1.276	0.118	1.0	1.353	33.5	-0.057	0.320	26102	26970	
	20000			0.530	0.321	1.310	0.095				-0.043	0.287	35503	35940	
	30000			0.484	0.255	1.332	0.050				-0.021	0.234	53957	53972	
	50000			0.422	0.218	1.330	0.000				-0.017	0.201	89820	89800	
Dog's Bay	100	∞	–	1.584	1.712	2.052	-0.287				0.174	1.886	100	100	Coop (1990)
	3286		–	1.075	0.714	1.586	0.125	2.36	1.748	42.6	-0.091	0.653	3286	3286	
	4020	3	–	0.991	0.552	1.407	0.381				-0.195	0.357	7571	9633	

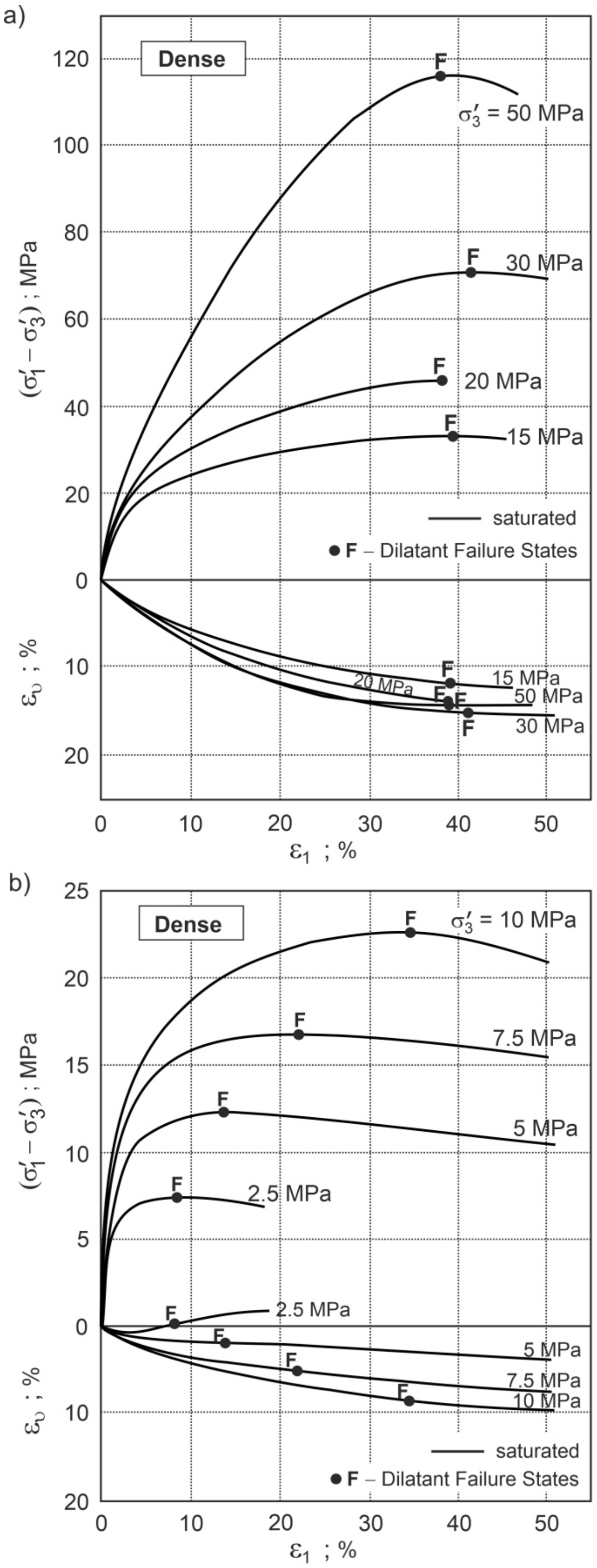

Figure 4. Experimental curves $(\sigma_1' - \sigma_3') - \varepsilon_1$ and $\varepsilon_v - \varepsilon_a$ for drained triaxial compression of Toyoura sand under high pressure: (a) $\sigma_3' = 15{\sim}50\ MPa$; (b) $\sigma_3' = 2.5{\sim}10\ MPa$ (adapted from Miura and Yamanouchi 1975).

As shown above, the DFS and failure states are equivalent for Toyoura sand. Therefore, for this sand, a simpler procedure for determining the CFS parameters can be used. This procedure was used to analyse the results of the drained triaxial compression of saturated Toyoura sand presented by Miura and Yamanouchi (1975). The initial void ratio of all tested samples was $e_i = 0.61$. Post-consolidation void ratios (e_0) were obtained using isotropic compression curves for Toyoura sand presented by Pestana et al. (2002). The DFS (failure state) values q_F, volumetric strains ε_{vF}, and ratios of the increments of volumetric and axial strain $\delta\varepsilon_v/\delta\varepsilon_a$ were read directly from the presented $q - \varepsilon_a$ and $\varepsilon_v - \varepsilon_a$ curves (Fig. 4). The other values for DFS are:

$$e_F = e_0 - \varepsilon_{vF}(1 + e_0) \quad (15)$$

$$p_F = \sigma_c + q_F/3 \quad (16)$$

$$\eta_F = 3q_F/(q_F + 3\sigma_c) \quad (17)$$

$$D_F = 3(\delta\varepsilon_v/\delta\varepsilon_a)_F/(3 - (\delta\varepsilon_v/\delta\varepsilon_a)_F) \quad (18)$$

where σ_c is a constant confining pressure. The values of $M_c^o = 1.353$ $(\phi^o = 33.5^o)$, $A_c^o = 0.549$ and $\beta_F = 1.0$ were obtained from Fig. 5.

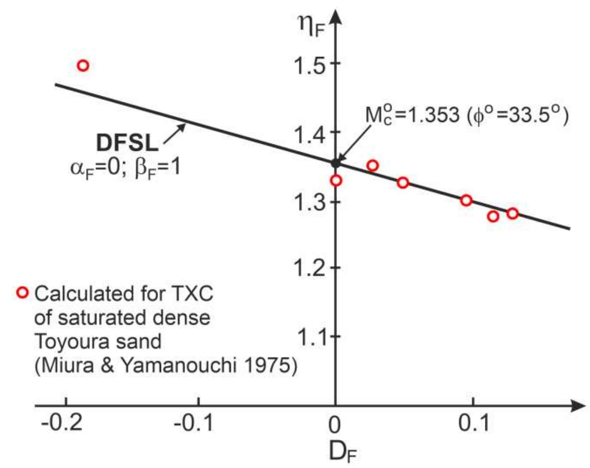

Figure 5. The stress ratio–dilatancy relationships at failure for drained triaxial compression of Toyoura sand under high pressure.

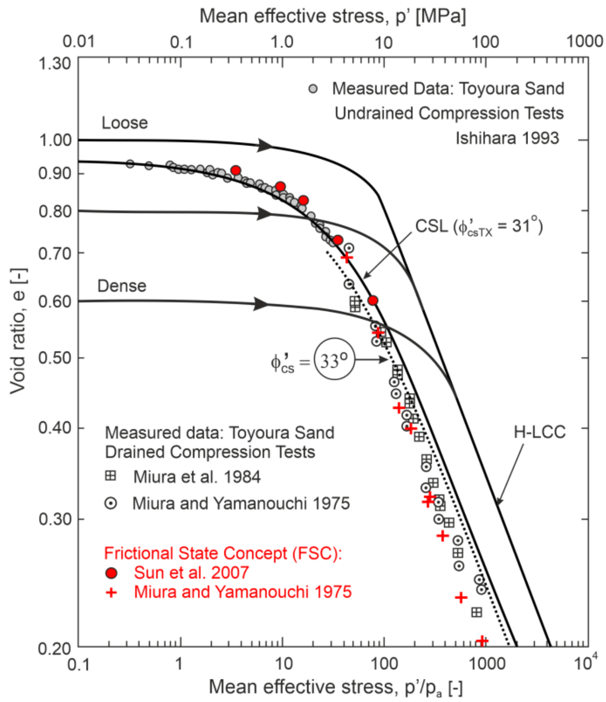

Figure 6. Critical state and critical frictional state for Toyoura sand (adapted from Pestana et al. 2002).

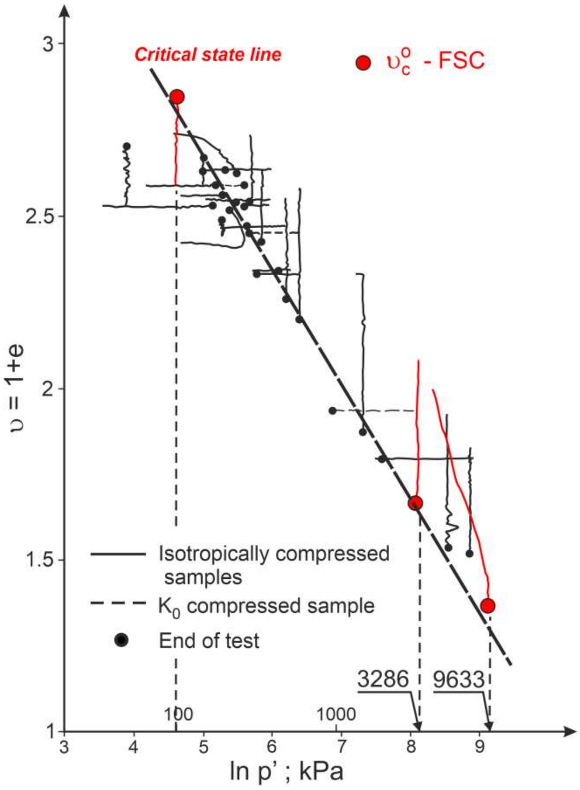

Figure 7. Critical state and critical frictional state for Dog's Bay sand (adapted from Coop 1990).

The values p_0, $\psi^o_{\eta F}$, and e^o_c of the CFS were calculated from Eq. (4), (6), and (14). All the characteristic values for Toyoura sand triaxially compressed by Miura and Yamanouchi (1975) are also presented in Table 1.

The critical frictional state and critical states in the $e - p'$ plane for Toyoura and Dog's Bay sands are shown in Figs. 6 and 7, respectively.

The e^o_c and e_c values are very close to each other. The critical state angle of Toyoura sand tested by Sun et al. (2007) $\phi'_{cs} = 33.4^o$ (Yao et al. 2008), tested by Miura and Yamanouchi (1975) $\phi'_{cs} = 33.0^o$ (Pestana et al. 2002) and Dog's Bay sand $\phi'_{cs} = 40.3^o$ (Coop 1993). Thus the CFS values of the analysed sands are very close to the critical states.

4. Dilatancy at dilatant failure state

Assuming that $D^p = D$ for the dilatant failure state, Eq. (5) after some algebra can be written as

$$D = \chi_c \psi^o_{eF} \quad (19)$$

where

$$\chi_c = M^o_c / (\beta_F A^o_c) \quad (20)$$

for drained triaxial compression. For Toyoura sand tested by Sun et al. (2007) $\chi_c = 2.394$ and tested by Miura and Yamanouchi (1975) $\chi_c = 2.464$ (Fig. 8). These values are similar to those proposed for sands by Jefferies and Been (2019). For Dog's Bay sand tested by Coop (1993) $\chi_c = 1.776$ (Fig. 8).

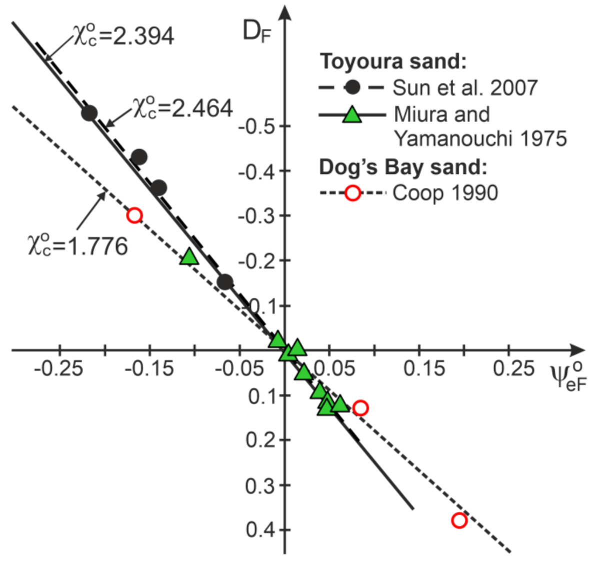

Figure 8. $D_F - \psi^o_{eF}$ relationships for Toyoura and Dog's Bay sand.

For calcareous sand tested by Giretti et al. (2018) of $M^o_c = M^o = 1.65$ $(\phi^o = \phi'_{cs} = 40.3^o)$, $A^o_c = 0.45$, $\beta_F = 2.0$ the value $\chi_c = 1.883$ was obtained from Eq. (5) and $\chi_c = 1.8$ from the experiment (Fig. 9).

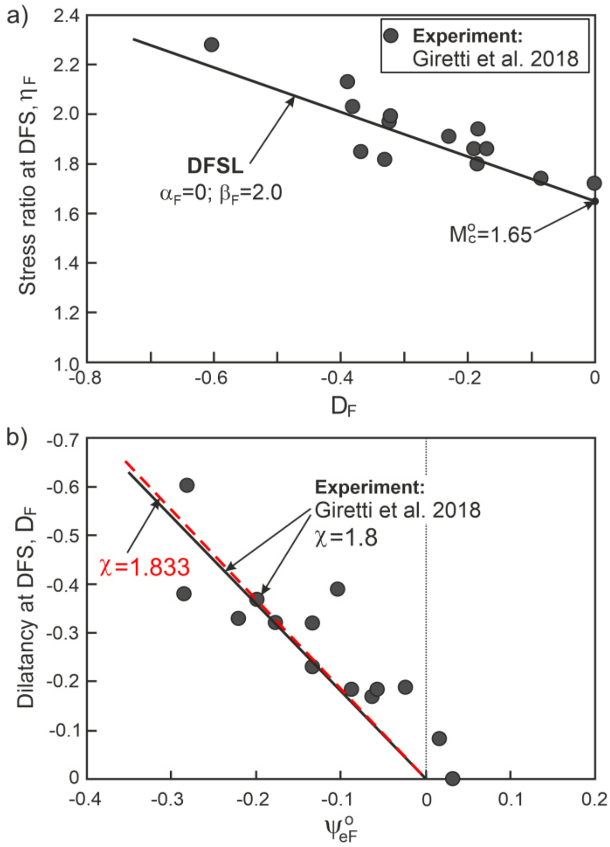

Figure 9. Characteristic relations for calcareous sand studied by Giretti et al. 2018: (a) $\eta_F - D_F$; (b) $D_F - \psi^o_{eF}$ (adapted from Giretti et al. 2018).

The ratios of the volumetric and axial strain increments at failure as a function of $\psi^o_{eF} = \psi$ calculated using the CFS concept and experimental relationships collected by Jefferies and Been (2019) are shown in Fig. 10.

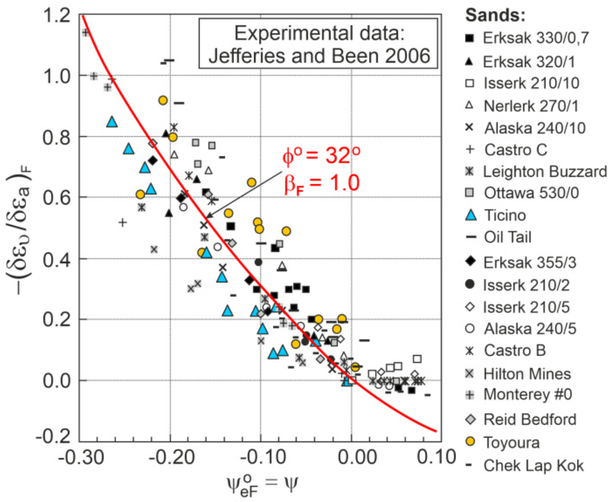

Figure 10. The relationship $(\delta\varepsilon_{\upsilon}/\delta\varepsilon_{a})_{F} - \psi^{o}_{eF}$ for sands (adapted from Prearo 2015)

Theoretical relationship (19) describes the experimental relationships for sands well.

5. Conclusions

The proposed natural state parameter can be treated as an extension of the state parameter defined by Been and Jefferies. It combines the distance from the current and critical frictional states in the $q - p'$ and $e - p'$ planes.

The dilatant failure states and assumed zero values of the natural state parameter in these states fully define the critical frictional state.

For the analysed drained triaxial compression of Toyoura and Dog's Bay sands, the critical frictional states and critical states are very close to each other.

Assuming that for sands $\psi^{o}_{eF} = \psi$, the relation between the dilatancy in dilatant failure states and the state parameter is directly determined by the natural state parameter.

The correctness of the proposed critical frictional state and the natural state parameter for other soils must be confirmed in the future by experimental and theoretical research.

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