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A constitutive model accounting for grain crushing in silica sands

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ABSTRACT: Grain crushing around the pile occurs during the installation of driven piles in granular materials and soft rocks. Grain size distribution changes modify the strength and compressibility of soils, to an extent that may be significant for offshore foundation responses. To investigate those effects a new constitutive model was formulated to incorporate the effect of grain crushing on the mechanical response of granular soils. The model employs two yield surfaces: a volumetric one linking the evolution of crushing with stress history; and a shear surface, where the mobilized strength and dilatancy are function of the critical state ratio and state parameters. The mechanical effect of crushing is represented in the model through a Critical State Plane - CSP in the e-p'-B_r space (void ratio – mean effective stress – breakage index). The CSP reflects the shift downward of the Critical State Line - CSL on the compression plane as grain crushing develops. The capabilities of the proposed two-surface formulation to capture the changes in strength and dilatancy that accompany crushing is illustrated for a silica sand (Fontainebleau), but the model could be used for other crushable materials like carbonate sand.

Keywords: Grain crushing; critical state plane; breakage index; Fontainebleau sand.

1 INTRODUCTION

Particle breakage or crushing plays an important role in the soil behaviour, as it affects soil strength and compressibility (Hardin, 1985). This phenomenon is relevant for offshore foundations, as particle crushing is present in the installation of driven piles. The effect of grain crushing in offshore foundations has been related, for instance, to the low resistance mobilized during installation of monopiles in carbonate sands (OuYang et al., 2024). However, the opposite effect is also possible, with reports of underestimated soil static resistance to driving in monopiles installed in glauconitic sands (Perikleous et al., 2023).

Yang et al. (2015) studied the process of grain crushing in displacement piles using physical modelling. They found that considerable crushing takes place beneath the pile tip during the installation, and that crushed material moves radially as the pile tip advances, creating zones around the pile shaft of crushed material. The process of grain crushing in displacement piles has also been studied by some authors using numerical modelling. Although discrete element-based approaches are also possible (Lei et al. 2025) continuum-based models (e.g. Jin et al., 2018) are generally more efficient to analyse structure-scale responses.

A key ingredient of continuum-based models is a constitutive model that considers the effect of grain crushing. Broadly these models could be divided in two main groups: those in which crushing evolves by plastic strain or plastic work (Jin et al., 2018), and those where particle crushing is linked to stress history based on a crushing stress limit (Kikumoto et al., 2010). The former approach has the inconvenient of predicting continuous crushing at low mean effective stress (p') and high stress ratios (η). For quartz sands at least, this does not seem in agreement with results of experimental work (McDowell, 2002) or DEM simulation (Bono and McDowell 2018, Ciantia et al., 2019) showing that grain crushing is mostly relevant at high mean effective stresses.

In this paper a new constitutive model is formulated to incorporate the effect of grain crushing on the mechanical response of granular soils. The model is based on the work of Kikumoto et al. (2010). As in that work the model proposed links crushing to stress history and has crushing affecting the Critical State Line CSL on the compression plane. This is formulated using the concept of the Critical State Plane

CSP in the e-p'-$\mathrm{B_r}$ (void ratio – mean effective stress – breakage index) space.

The Kikumoto et al (2010) model is here modified by a) incorporating a different measure of crushing evolution b) modifying the shape of the crushing -yield surface c) changing the formulation of the shear-hardening surface, so that strength and dilatancy are function of the critical state slope and state parameter. This communication briefly illustrates how that, when compared with the original Kikumoto et al. (2010) model, the resulting new formulation is both simpler and more accurate in predicting triaxial behaviour of quartz sands undergoing crushing.

2 CONSTITUTIVE MODEL FORMULATION

2.1 Breakage Index Br

The primary effect of grain crushing is the modification of the particle size distribution - PSD. Thus, it is convenient to define a simple index that reflects grading evolution. The breakage index Br (Einav, 2007) was selected in this work, instead of the grading state index I_G (Muir Wood, 2007) used by Kikumoto et al. (2010). Br considers changes on the overall PSD from the initial grading ($B_r = 0$) to a fractal or ultimate particle size distribution ($B_r = 1$) and it allows a simpler modulation of the effect of crushing on the CSL than was not possible using I_G. The definition of these two indexes is depicted in Figure 1.

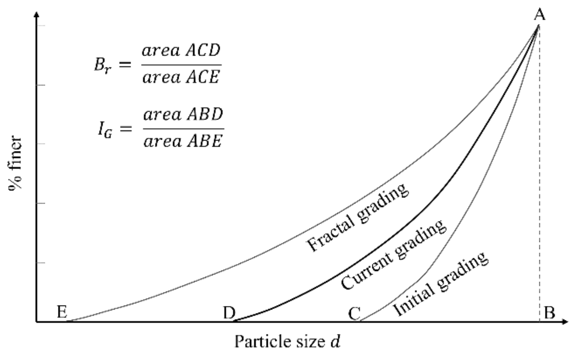

Figure 1. Definition of the Breakage Index B_r and Grading State Index I_G.

2.2 Critical State Plane

Traditionally, the CSL is defined as a semi logarithmic line in the compression plane (p'-e). However, in this work an exponential form of the CSL following Gudehus (1997) is used, because it has a greater range of applicability for sands. The exponential form is slightly modified following the work of Kikumoto et al. (2010) to maintain positive values of critical void ratio at high stresses:

$$e_c = e_{min} + (e_{max} - e_{min})\, exp\left[-\left(\frac{p'}{p_{cs}}\right)^{k_1}\right] \quad (1)$$

where, e_{min} and e_{max} are the limits void ratio of the critical state line, and p_{cs} and k_1 are material parameters. We follow Kikumoto et al. (2010) for the elasticity of the model, defined using a similar non-linear relationship of Equation (1) but with a different reference pressure p_{is} instead of p_{cs}.

It has been found experimentally (Ghafghazi et al., 2014) and by means of DEM simulations (Muir Wood and Maeda, 2008; Ciantia et al., 2019) that the CSL varies with the gradation of the material. Then, for a crushable material, the CSL is continuously changing as crushing takes place and gradation evolves. In order to consider this effect in the model, the concept of a Critical State Plane - CSP (Muir Wood and Maeda, 2008) in the *e-p'-B_r* space (void ratio-mean effective stress – breakage index), that reflects the shift downward of the critical state line - CSL on the compression plane as grain crushing develops is used. This is achieved by considering that e_{min} and e_{max} vary linearly with B_r.

$$e_{min} = e^{i}_{min} - e^{i}_{min}\, B_r \quad (2a)$$

$$e_{max} = e^{i}_{max} - e^{i}_{min}\, B_r \quad (2b)$$

where e^{i}_{min} and e^{i}_{max} are the limits of the CSL for the initial grading. Equation (2) states that, at the ultimate grading ($B_r = 1$) the minimum critical void ratio is zero, and that e_{min} and e_{max} vary by the same amount as crushing is produced, so the CSL moves downwards in the compression plane without changing its shape.

As illustrated in Figure 2, the selected shape of the CSL and the linear variation with B_r allow for the fitting of the different CSLs found by Ciantia et al. (2019) using DEM for a Fontainebleau sand. The parameters used for this fit are presented in Table 1.

2.3 Crushing surface

Some models have already used a crushing surface to consider the evolution of crushing (Kikumoto et al., 2010). Typically, those models employ the crushing surface proposed by Hardin (1985), which was defined based on experimental results from quarried granite sands. As depicted in Figure 3, the shape of this surface does not match the evolution of breakage in different stress paths found by Ciantia et al. (2019)

using a DEM model of Fontainebleau sand. To overcome this limitation, the yield surface of the Clay and Sand Model – CASM (Yu, 1998) was used in this work with the flow rule of the Modified Cam-Clay model. This yield surface is expressed as:

$$f_v = \left(\frac{q}{pM_p}\right)^n + \frac{log(p'/p_c)}{log(r)} \tag{3}$$

where, M_p is the mobilized peak stress ratio, p_c is the isotropic breakage stress, and n and r are parameters that define the shape of the surface.

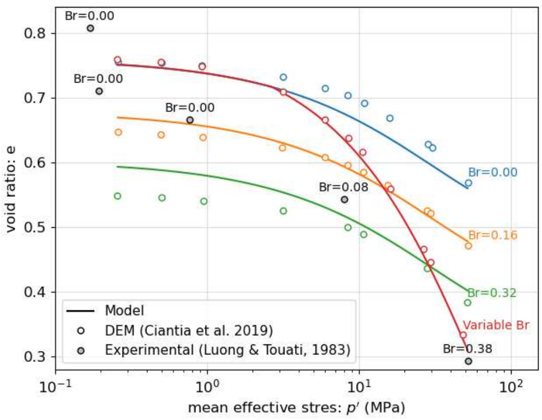

Figure 2. Critical State Lines for different Breakage Index Br for Fontainebleau Sand.

Figure 3 shows the adjustment of the B_r isocountours obtained by Ciantia et al. (2019), using the CASM surface, with $n = 2.0$ and $r = 2.75$. The inclusion of these two shape parameters adds versatility to the formulation.

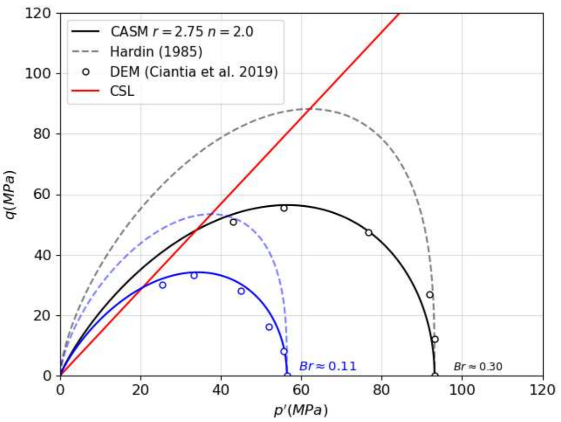

Figure 3. Adjustment of the CASM as a crushing surface for Fontainebleau sand.

Following Kikumoto et al. (2010), the breakage stress p_c in Equation (3) is linked with the breakage index B_r by an exponential equation, and it is assumed that all the volumetric plastic strains predicted with this mechanism are due to crushing.

$$B_r = 1 - exp\left[-\left(\frac{p_c - p_{c0}}{p_r}\right)^{k_2}\right] \tag{4}$$

$$\delta\varepsilon_v^p = \frac{K_c e_{min}^i}{1+e}\,\delta B_r \tag{5}$$

where, p_{c0} is the initial breakage stress, p_r and k_2 are materials constants, and K_c is an internal variable that controls the amount of plastic volumetric compression due to crushing. The evolution of the breakage index for isotropic loading and critical state using Equations (3) and (4) for the Fontainebleau sand parameters in Table 1 is presented in Figure 4.

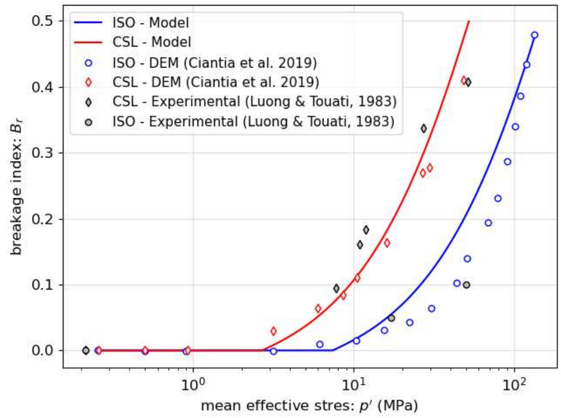

Figure 4. Evolution of the Breakage index for isotropic condition and critical state for Fontainebleau sand.

2.4 Shear surface

In order to describe the shear behaviour, the volumetric surface is used alongside the Simple Sand model developed by Jin et al. (2018). This is a critical state-based shear model with a nonlinear plastic hardening, in which the yield surface is linear in the p' - q space.

$$f_s = \frac{q}{p'} - \frac{M_p\,\varepsilon_d^p}{k_p + \varepsilon_d^p} \tag{6}$$

where, M_p is the available peak stress ratio, ε_d^p is the deviatoric plastic strain and k_p is a material parameter that controls the plastic stiffness.

The model uses a nonlinear stress-dilatancy relationship generalizing a Roscoe-type dilatancy rule.

$$\frac{\delta\varepsilon_v^p}{\delta\varepsilon_d^p} = A_d\left(M_{pt} - \frac{q}{p'}\right) \tag{7}$$

where, A_d is a parameter that controls the magnitude of stress-dilatancy and M_{pt} is the slope of the phase transformation line. M_p and M_{pt} are defined to be function of the critical state ratio M_c and the state parameter ψ of Been and Jefferies (1985) using the following exponential expressions:

$$M_p = M_c\, exp(-n_p \psi) \quad (8a)$$

$$M_{pt} = M_c\, exp(n_d \psi) \quad (8b)$$

where, n_p and n_d are materials parameters. It should be noted that as the strength and dilatancy are function of ψ, those are affected by the position of the CSL and so by the evolution of the B_r.

3 SIMULATIONS

The proposed model requires a calibration of 15 parameters: two parameters for elasticity, five parameters for the CSL, five parameters for the crushing evolution, and four parameters for the shear surface.

The parameters of this constitutive model for Fontainebleau sand are summarized in Table 1. The parameters related to the CSL, and the crushing evolution were calibrated based on the experimental tests and DEM simulations presented in Figure 2 to Figure 4. The parameters for the elasticity and the shear surface were calibrated based on the high stress oedometric tests and low stress triaxial tests presented in the following sections.

3.1 Oedometric test

A series of high-pressure oedometric compression tests were performed by Ciantia et al. (2019) at different maximum stresses and measuring the PSDs after unloading the samples. These tests were used to calibrate the parameters that control the breakage evolution and the elasticity.

The simulation with the proposed constitutive model of an oedometric test with a maximum stress level of 100 MPa is presented in Figure 5. The correspondence between the simulation and the experimental results can serve as a first validation of the model formulation. It may be observed that the loading path is better adjusted than the unloading one, pointing to some difficulties with the original elastic formulation of the model. There is ongoing work.

Table 1. Parameters for Fontainebleau sand.

	Parameter	Value
Elas	Reference stress for elasticity $\boldsymbol{p_{ic}}$ (kPa)	6.0x10^4
	Poisson ratio: $\boldsymbol{\nu}$	0.30
CSL	Reference stress for CSL $\boldsymbol{p_{cs}}$ (kPa)	3.0x10^4
	Exponent of the CSL $\boldsymbol{k_1}$	0.70
	Initial minimum void ratio $\boldsymbol{e^i_{min}}$	0.50
	Initial minimum void ratio $\boldsymbol{e^i_{max}}$	0.76
	Critical friction angle $\boldsymbol{\phi_c}$ (°)	33.0
Crushing	Initial breakage stress $\boldsymbol{p_{c0}}$ (kPa)	7.4x10^3
	Reference stress for breakage $\boldsymbol{p_r}$ (kPa)	2.0x10^5
	Exponent of the breakage evolution $\boldsymbol{k_2}$	0.95
	CASM spacing ratio: $\boldsymbol{r}$	2.75
	CASM shape parameter: $\boldsymbol{n}$	2.0
Shear	Plastic modulus $\boldsymbol{k_p}$	0.001
	Exponent of mobilized peak ratio $\boldsymbol{n_p}$	1.5
	Exponent of dilatancy ratio $\boldsymbol{n_d}$	1.4
	Stress-dilatancy parameter $\boldsymbol{A_d}$	0.65

3.2 Low-pressure triaxial test

The calibration of the shear behaviour of the model was done by the adjustment of the low-pressure drained triaxial test performed by Seif El Dine et al. (2010). Those tests achieved p' lower than 1 MPa, and no crushing was observed after the test ended.

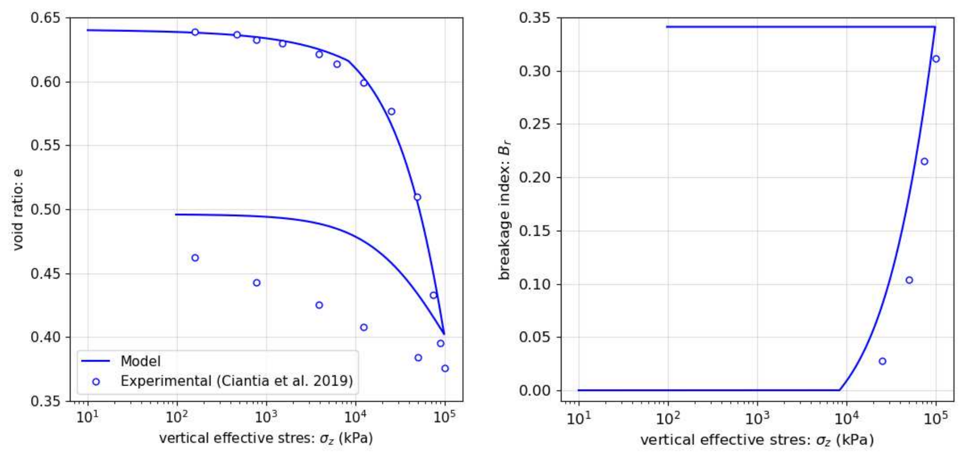

Figure 5 – Simulation of high-pressure oedometric compression test in Fontainebleau sand

Figure 6 shows the comparation between the model simulations and the experimental results. The good agreement highlights the ability of the model formulation to predict the shear behaviour of sands at different initial densities with the same set of parameters.

3.3 High-pressure triaxial test

As a validation of the model formulation and the calibration done for a Fontainebleau sand, a class A prediction of the high-pressure drained triaxial test performed by Luong and Touati (1983) is depicted in Figure 7.

As shown in the Figure 7, it is necessary to include grain crushing in the model to achieve a good prediction. It is worth highlighting that the drained triaxial tests correspond to initial denser than critical materials, but those behave as a loose material because as grain crushing is produced and the CSL is moving downwards the strength and the dilatancy are reduced.

4 CONCLUSIONS

A critical state-based model has been developed that considers the mechanical effect of grain crushing in sandy soil. The model uses two yield surfaces: a crushing surface used to link the evolution of crushing with stress history and a shear surface for predicting the shear strength and dilatancy. The important aspect of the model is the existence of a CSP, defined within physical void ratio limits, that reflect the shift downwards of the CSL on the compression plane as grain crushing develops.

The model was calibrated and validated for Fontainebleau sand, showing an appropriate prediction of the material behaviour for different densities, mean effective stresses and stress paths.

The good performance of the model reported here provides confidence that it can be extended to large deformations and employed to simulate pile installation effects.

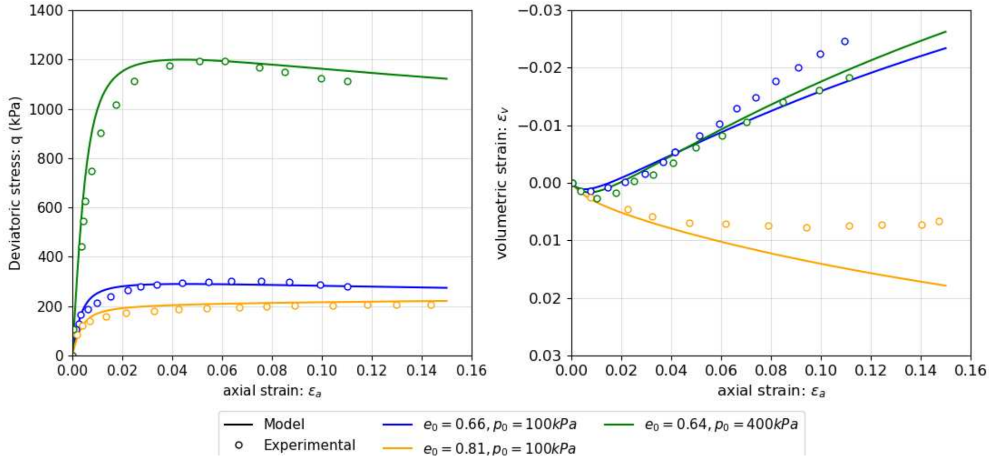

Figure 6 – Simulation of low-pressure drained triaxial test in Fontainebleau sand.

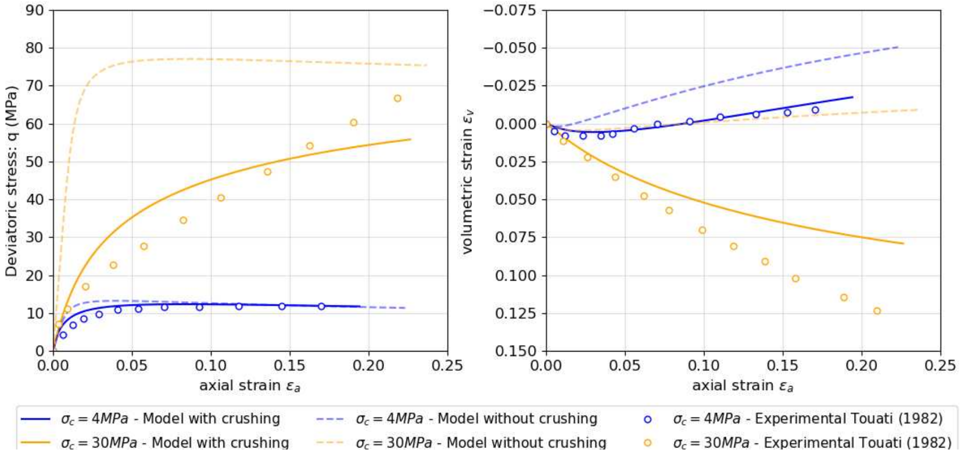

Figure 7 – Simulation of high-pressure drained triaxial test in Fontainebleau sand.

AUTHOR CONTRIBUTION STATEMENT

D. León-Vanegas: Conceptualization, Methodology, Software, Writing – original draft. **L. Monforte**.: Conceptualization, Methodology, Writing – review & editting. **M. Arroyo**: Conceptualization, Funding acquisition, Writing – review & editting. **A. Gens**: Conceptualization, Writing- reviewing and editing.

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