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Calibration of Undrained Shear Strength Probability Density Function of an Offshore Clay Profile in the Gulf of Mexico Using CPT Data and Slope Stability Analyses

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ABSTRACT: This paper describes the implementation of a Bayesian calibration method in slope stability analysis to develop more accurate values of undrained shear strength. Typically, one-dimensional and two-dimensional limit equilibrium analysis methods are used to calculate the factor of safety of clay slopes located upslope of planned deepwater seafloor infrastructure. The two most relevant input parameters for these analyses are the unit weight and the undrained shear strength. The unit weight and undrained shear strength can be obtained from direct measurements in the laboratory. These parameters can be characterized using probability density functions that incorporate uncertainty. When these probability density functions are used to calculate factors of safety, the results can sometimes be illogical, particularly for values of shear strength lower than the mean, indicating values lower than unity for slopes that are evidently stable. This suggests that the probability density function may include unreasonable parameter values. The Bayesian method is therefore used to adjust the probability density function to yield more accurate values of the undrained shear strength. Herein, stratigraphic information from the subbottom profiler was used to identify slopes without prior indications of slides (i.e., stable slopes) and implement that information to modify the prior distribution of the undrained shear strength that was estimated from cone penetration test data. The posterior probability distribution function was then calculated using likely factors of safety for stable slopes.

Keywords: Gulf of Mexico, Bayesian, Undrained Shear Strength, Subbottom Profiler

1 INTRODUCTION

An exploration and production (E&P) company has established development plans for an oil field in the Gulf of Mexico that include installing a seafloor architecture of wells, flowlines, manifolds, umbilicals, and risers to a floating host facility unit. Interested in developing an understanding of the risk that slope instability might pose to their seafloor-founded facilities, the E&P company commissioned an analysis that required the calculation of the factor of safety of slopes using one- and two-dimensional limit equilibrium analyses.

The inputs required for slope stability analyses include a ground model, which should include a bathymetric model, a stratigraphic model that defines the soil units and their distribution below the seafloor, and geotechnical parameters for each of the soil units identified in the stratigraphic model. Given the fine-grained nature of the soil units in this deepwater depositional environment, the relevant geotechnical parameters for limit equilibrium slope stability analyses include the unit weight and undrained shear strength of the soil.

This paper describes how a Bayesian parameter calibration method is implemented to update the prior distribution function of the undrained shear strength of a sloping deepwater offshore development area using stratigraphic information obtained from a subbottom profiler (SBP) and the absence of morphological evidence of past slides. The posterior probability distribution function is calculated using likely factors of safety of the stable slopes, which are also assessed as random variables. The results yield an updated and more accurate set of soil parameters and an improved understanding of the uncertainties associated with the parameter estimation process.

2 STRATIGRAPHIC MODEL

The water depth in the development area varies from 1,470 meters (m) to 2,580 m. The maximum depths occur in the central portion of the development area, where there is a relatively flat basin, and the shallowest areas correspond to bathymetric highs surrounding the basin. Surrounding the basin are slopes as high as 800 m and locally steeper than 30 degrees in some locations. Many of the slopes, particularly the steep areas, exhibit evidence of past rotational and translational slide activity with ensuing mass transport deposits (MTDs) that in some instances traveled significant distances. However, many long and gently sloping areas do not exhibit past evidence of slides.

The SBP data acoustically define the stratigraphy in the development area to a maximum depth of approximately 60 m below mudline (e.g., Figure 1). The data show that, as a whole, the undisturbed soils within the development area are intermittent mixtures of hemipelagic depositions and fine-grained stacked turbidites. Three prominent sequences that describe the near-surface stratigraphy were defined as Units A, B, and C.

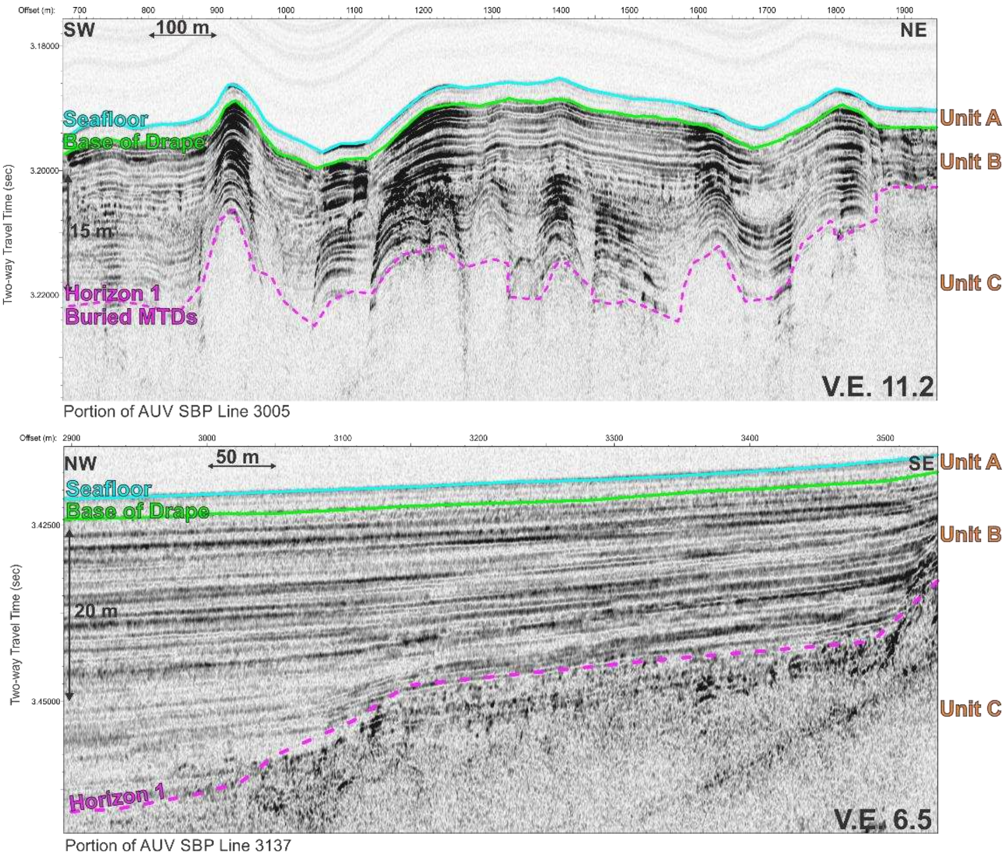

Figure 1. Portions of SBP lines displaying the shallow stratigraphy units of the project development area

Unit A is a weakly stratified sequence on the seabed. This seismically amorphous hemipelagic deposit is referred to as the hemipelagic drape deposits. Within this sequence, occasional parallel, low-impedance reflectors were noted, indicative of thin turbidites. However, evidence of past slides was not noticeable in this unit. Unit B consists of parallel to subparallel, moderate- to high-impedance reflectors and is likely composed of Late Pleistocene stacked turbidites interbedded with hemipelagic silt-clay sediment. Unit C consists of mass transport deposits (MTDs) that are interpreted from low- to high-impedance, chaotic reflectors. This unit includes variable soils consisting of large-stacked silty clay MTDs and overconsolidated soils that have been thrust upward into the shallow subsurface. For slope stability analysis and foundation analysis purposes, the soils from Unit A and Unit B were conflated into a single geotechnical unit described as Unit 1. Soils beneath Unit 1 were simply described as Unit 2.

3 GEOTECHNICAL PARAMETERS FOR SLOPE STABILITY ANALYSIS

Slope stability analysis requires an understanding of the representative ground model for the slopes of interest. Components of that ground model include seabed stratigraphy and seafloor bathymetry from the integrated geological model for the project development area, as well as geotechnical parameters and the characterization of the in situ stress regime. In marine sediments, submerged unit weight (SUW) and undrained shear strength are of principal importance for slope stability analyses. The following sections of this paper describe the geotechnical parameters that were developed for slope stability analysis.

3.1 Unit Weight

SUW profiles were developed by interpreting available geotechnical laboratory testing results. The data used were from jumbo piston cores (JPCs) that were selected based on their proximity to the areas selected for slope stability analysis. The SUW within the upper 16 m (i.e., the maximum penetration depth of the JPCs) of the soil profile was estimated using direct measurements from laboratory test data and indirect estimates from water content (Figure 2). In lieu of direct measurements below a depth of about 16 m, the SUW up to a depth of 45 m was derived by extrapolation, with adjustments made to consider the void ratio and effective stress relationship that can be derived from consolidation test data.

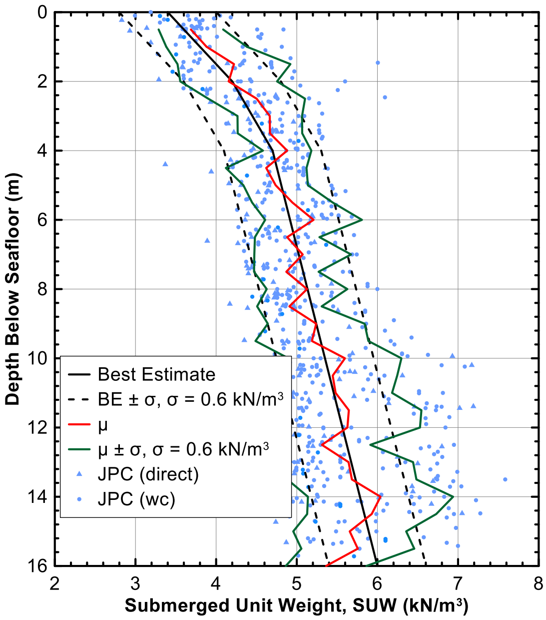

Figure 2. Relationship between SUW and depth below seafloor

A statistical evaluation was used to transform the scattered discrete SUW interpretations into a continuous profile. The mean (μ) and the standard deviation (σ) of the SUW were calculated within each 0.5 m depth interval. Figure 2 shows the mean and the mean plus and minus one standard deviation (μ ± σ) lines relative to the discrete SUW data. The figure also includes a smoothed-line best estimate (BE) at about the mean value and the lower estimate (LE) and upper estimate (UE) calculated from BE plus or minus the average standard deviation of discrete values, respectively. This average standard deviation was set as 0.6 kilonewtons per cubic meter (kN/m^3).

3.2 Undrained Shear Strength

Direct measurements of undrained shear strength were obtained by using Torvane and laboratory miniature vane shear tests on JPC subsamples from depths of 0 to 16 m and by conducting direct simple shear (DSS) laboratory tests. Indirect measurement of undrained shear strength was obtained in situ from CPT measurements.

3.2.1 CPT Total Tip Resistance

Data from CPTs were selected based on their proximity to areas critical for slope stability analyses. The arithmetic mean (μ) of the total tip resistance was calculated by subdividing the profile into 0.5 m intervals to develop a central tendency of the data set. To express the variability and dispersion in the data set and to measure confidence in statistical estimates, the standard deviation of the mean was calculated for those same 0.5 m intervals as the unit weight.

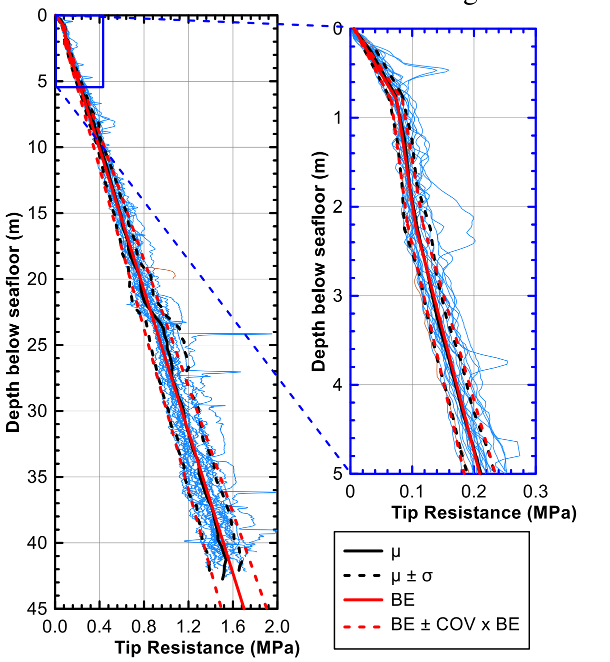

Figure 3. Statistical estimate of CPT total tip resistance (q_t) in Unit 1 layer

Figure 3 shows the mean (μ) and the mean plus and minus one standard deviation (μ ± σ) of the total tip resistance data. Noticeably higher values of cone tip resistance on Figure 3 are due to localized differences in material strength.

A smoothed representation of the mean value of tip resistance is represented by the solid red BE line. The data set dispersion and the estimated standard deviation at 0.5 m intervals exhibited an approximately linearly increasing variability. The coefficient of variation (COV; i.e., the ratio of the standard deviation and the mean) of 12% was estimated to encompass the overall statistical variability of tip resistance. The red dashed lines in

Figure 3 present the LE and UE lines, which were calculated respectively as the BE minus one standard deviation and the mean plus one standard deviation (which is the same as the BE times the COV).

3.2.2 Undrained Shear Strength Interpreted from CPT

Undrained shear strength was estimated from CPT net tip resistance (q_{net}). At shallow depths near the seafloor, where samples are too soft to be tested in the laboratory, N_{kt} was calibrated by comparing the CPT data to the miniature vane shear and Torvane undrained shear strength measurements. At greater depths, N_{kt} = 17.5 was considered based on DSS tests and consistency with typical foundation design values used for the Gulf of Mexico (e.g., Cheon et al. 2015). Based on these parameters, a linear N_{kt} model was developed where $N_{kt} = 22.5 - 0.2 \times z$ from 0 to 25 m below mudline and N_{kt} = 17.5 from 25 to 45 m.

3.2.3 Development of Undrained Shear Strength Profile for Slope Stability Analysis

The statistical variability in both tip resistance and vertical stress was considered to develop undrained shear strength profiles for slope stability analysis. This was done by first calculating the UE and LE of tip resistance and vertical stress to represent the statistical variability in the undrained shear strength profile. The UE of undrained shear strength corresponds to the mean plus one standard deviation, and LE corresponds to the mean minus one standard deviation. The BE of the undrained shear strength was taken as the average of the UEs and LEs.

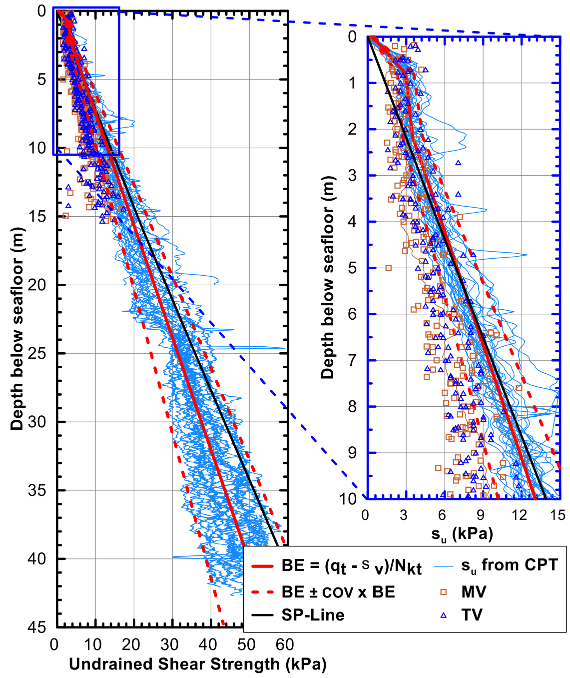

Figure 4. Statistical estimate of undrained shear strength

Figure 4 shows the direct measurements of undrained shear strength by miniature vane shear and Torvane, the undrained shear strength relationship as a function of vertical effective stress (SP-line) of $S = 0.455 \cdot \sigma'_v{}^{-0.113}$ (per test data not presented herein), the undrained shear strength of individual CPTs interpreted using the approach described above, and the BE, LE, and UE developed from the combined data.

As may be noted on the plot, the selected mean value that is primarily based on the SP-line typically plots higher than the miniature vane and Torvane measurements.

4 PRELIMINARY EVALUATION OF SLOPE STABILITY

Slope stability analyses began with the evaluation of a section through one of the slopes surrounding the basin. The section was chosen because of its location directly upslope of the planned seafloor infrastructure and because of the presence of seafloor expressions of faulting that can behave as weak planes for slides to develop. The initial analyses focused on the use of infinite slope stability, which is described this way because it considers that sliding occurs along a relatively long sliding plane that is parallel to the inclination of the seafloor. The assumptions involved in an infinite slope stability analysis are understood to apply if the length of the slide is at least 10 times the thickness of the slide. Therefore, the infinite slope stability analysis should be used in areas with uniform seafloor inclination over long and wide areas.

The use of infinite slope stability analyses to evaluate the stability of offshore slopes has been described by Nadim et al. (2003) for the Gulf of Mexico and by Dimmock et al. (2012) for the West Nile Delta.

The infinite slope stability analysis considering the effect of gravity but not the effect of other outside forces such as earthquakes uses the relatively simple equation (Equation 3)

$$FS = \frac{s_u(z)}{0.5 \cdot SUW(z) \cdot z \cdot \sin(2\beta)} \quad (3)$$

where the factor of safety (FS) at a given depth z (i.e., the depth that defines the thickness of the slide on a slope inclined at an angle β) is described as the ratio of the resisting force and the driving force. In this equation, the value of $s_u(z)$ corresponds to the shear strength at the depth of the sliding plane.

To calculate the factor of safety, the mean values of SUW and undrained shear strength profiles described above were used with slope angles of 13, 15, 17, and 20 degrees. Figure 5 presents the results, where the factor of safety decreases with increasing depth of the sliding plane and is less than 1 for sliding planes deeper than 5 m on 20-degree slopes, 10 m on 17-degree slopes, and 16 m on 15-degree slopes.

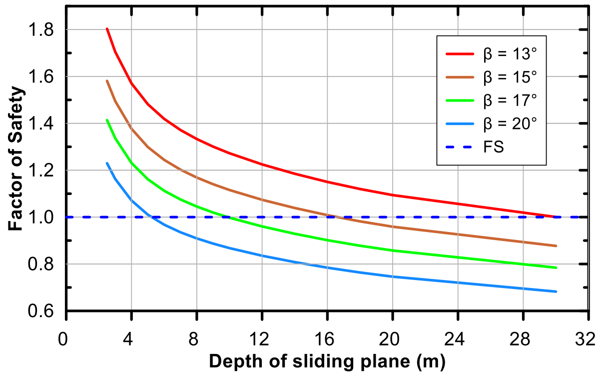

Figure 5. Infinite slope factor of safety as a function of the depth of the sliding plane for slopes at 13, 15, 17, and 20 degrees using prior distribution values

The absence of any slides in many of the planar slopes around the basin that exhibit slope angles greater than 15 degrees indicates that the results of the analyses are incongruent with the seafloor morphology. Moreover, the analyses were conducted using mean values of undrained shear strength, which indicates that 50% of the possible strength values lead to higher factor-of-safety values and 50% of the possible strength values lead to lower factor-of-safety values. As such, it was necessary to revisit the shear strength profile and the statistical distribution of the values of undrained shear strength that had been developed. A Bayesian updating approach, described in the following section, was selected for this purpose.

5 CALIBRATION OF UNDRAINED SHEAR STRENGTH USING BAYESIAN UPDATING

Considering that the undrained shear strength has been defined by a normally distributed probability density function (PDF), the total probability of sliding, P(FS < 1), can be estimated based on the theorem of total probability (Ang and Tang 1975). The theorem of total probability is described by Equation 4, where the term $\mathrm{P(FS < 1}|s_{u_\mathrm{i}})$ is the probability of a factor of safety less than 1 given a value of undrained shear strength, s_{u_i}; and $\mathrm{P}(s_{u_\mathrm{i}})$ is the probability for that value of undrained shear strength.

$$\mathrm{P(FS < 1)} = \textstyle\sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{P(FS < 1}|s_{u_\mathrm{i}}) \cdot \mathrm{P}(s_{u_\mathrm{i}}) \qquad (4)$$

Estimation of P(FS < 1) greater than zero for gravity-induced triggers calculated with Equation 4 is inconsistent with the absence of morphological expressions of sliding on the seafloor (i.e., seafloor expressions of slides were not observed in areas used for this analysis); therefore, the undrained shear strength probability mass function (PMF) needs to be adjusted. This adjustment is best done by implementing Bayes's theorem as described by Equation 5 (Ang and Tang 1975), where the term $P(s_{u_\mathrm{i}})$ is the prior probability of undrained shear strength equal to s_{u_i}, $P\left(FS < 1\middle|s_{u_i}\right)$ is the probability of FS < 1 given s_{u_i}, and $P(FS < 1)$ is calculated as described in Equation 4.

$$P\left(s_{u_\mathrm{i}}\middle|FS \geq 1\right) = \frac{\left[1-P\left(FS < 1\middle|s_{u_i}\right)\right] \times P(s_{u_\mathrm{i}})}{1-P(FS<1)} \qquad (5)$$

Application of Bayes's theorem to problems involving slope stability analysis has been extensively used in practice to update the shear strength (e.g., Gilbert et al. 1998, Sancio et al. 2017) or to update the factor of safety (e.g., Nadim et al. 2014, Haneberg 2015).

5.1 Evaluation of Slope Inclination and Soil Thickness

The SBP data were examined to identify sloping planar areas (i.e., sloping areas where the width and length are at least 10 times the thickness of Unit 1). Pairs of average slope angle (β) and average thickness of Unit 1 (t) in the planar areas identified within the basin were compiled as they describe the sliding planes that should exhibit a factor of safety greater than one.

The β-t combinations were then used to develop three different analytical models to capture model uncertainty (Figure 6).

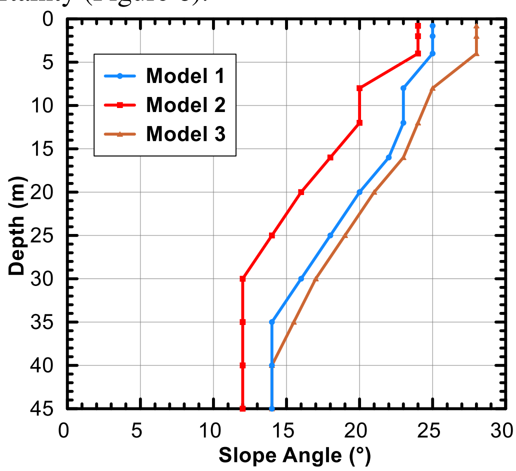

Figure 6. Slope inclination and depth of the sliding plane models derived from slope-thickness analysis

5.2 Logic Tree and Strength Updating Calculations

A logic-tree approach was used to calculate the probability of FS < 1. The logic tree is used to incorporate aleatory and epistemic uncertainty into the selection of input parameters to calculate the factor of safety and, in turn, the updated undrained shear strength that satisfies FS > 1. Two different logic trees (Logic Tree 1 and 2) were developed that incorporate uncertainty as follows:

1) The aleatory variability of the SUW was incorporated by using the mean minus one standard deviation, the mean, and the mean plus one standard deviation values. Given that the SUW is not independent of the undrained shear strength, greater weight was given to the mean value. In Logic Tree 1, the weight distribution was 0.20, 0.70, and 0.10. In Logic Tree 2, the weight distribution was 0.25, 0.50, and 0.25.

2) The overall uncertainty in the β-t model was captured by using the three models developed and plotted in Figure 6. In Logic Tree 1, all weight was assigned to the lowest model (Model 2) to avoid rendering an undrained shear strength that is too high. In Logic Tree 2, most weight was assigned to Model 2 (W = 0.70), but a weight of 0.15 was also assigned to Models 1 and 3.

3) The epistemic uncertainty in the application of the infinite slope stability analysis method was considered by using a target factor of safety of either 0.95 or 1.0. In both logic trees, a weight of 0.25 was applied for FS = 0.95, and a weight of 0.75 for FS = 1.0.

A 97-bin PMF equally spaced from the mean minus 4 standard deviations to mean plus 5.7 standard deviations was developed from the normally distributed PDF for the undrained shear strength at each depth of interest (2, 4, 8, 12, 16, 20, 25, 30, 35, 40, and 45 m). An example of the PMF is shown on Figure 7 (upper panel).

5.3 Results

Calculations were carried out following each of the logic trees described above. Each logic tree required 18 combinations of parameters for static analyses. Each of the combinations included analyses for 11 depths (2, 4, 8, 12, 16, 20, 25, 30, 35, 40, and 45 m). For each of those 11 depths, the PMF of the undrained shear strength included 57 bins. Figure 7 (upper panel) shows the prior and posterior PMF of the undrained shear strength for static analyses at a depth of 4 m for one of the branches of Logic Tree 2, where the mean value of SUW was used together with Model 2 and FS = 1.0. In this example, the prior value of the mean undrained shear strength was 5.7 kilopascals (kPa), but after the Bayesian updating, the mean value became 5.8 kPa and all values lower than 4.04 kPa exhibited zero probability. The Bayesian updating process increased the likelihood for all values greater than the minimum (4.04 kPa).

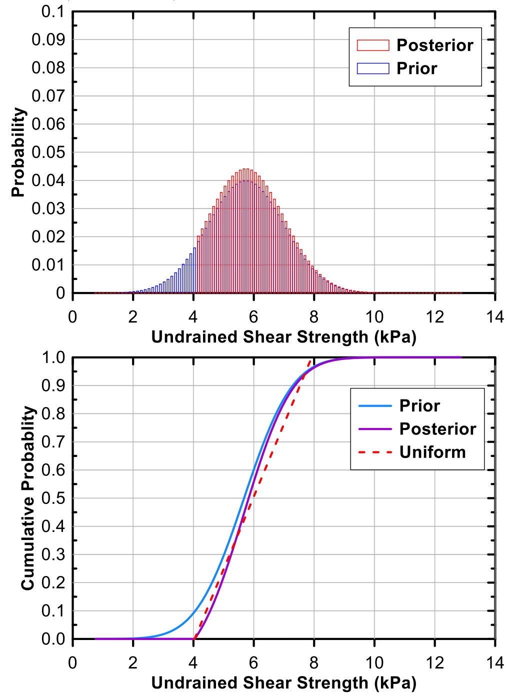

Figure 7. Example of prior and posterior distributions and implementation of uniform distribution for Unit 1 soil at a depth of 4 m

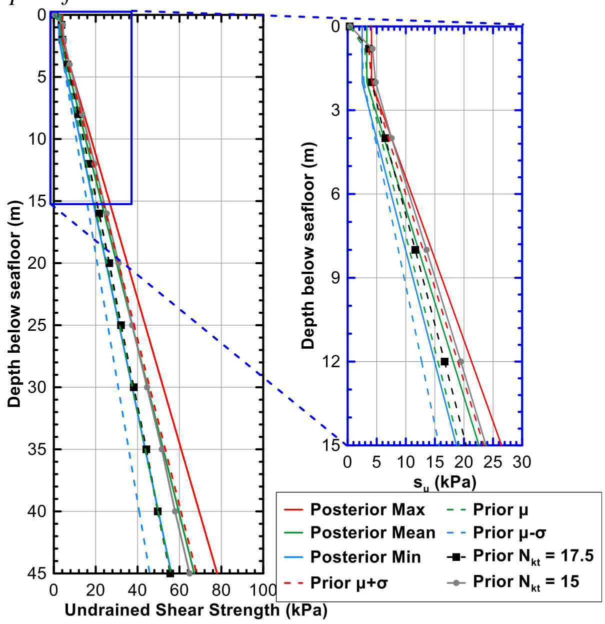

Figure 8. Comparison between updated static strength profile and prior strength profiles for Unit 1

Figure 7 (lower panel) shows the prior and posterior cumulative probability function for the PMF shown in the upper panel. To avoid using a skewed PMF for future slope stability analyses, a simple uniform PDF was assigned to the undrained shear strengths. The uniform PDF is defined by a minimum and maximum value. The minimum value was selected to be equal to the minimum from the Bayesian analysis, and the maximum value equal to the mean plus the difference between the mean and the minimum. The undrained shear strengths after Bayesian updating for static conditions are plotted on Figure 8.

6 DISCUSSION

The prior values of undrained shear strength were calculated using methods typical for the analysis of foundation capacity in the Gulf of Mexico. Foundations tend to be placed on flat or gently sloping seafloors (i.e., slopes of less than 5 degrees), whereas many of the slopes surrounding the basin of the project development area are steeper. As such, the clay on the slopes has deposited with a permanent static shear stress. This initial stress leads to a higher peak strength and a lower strain at peak strength (e.g., Pestana et al. 2000).

The need to upwardly adjust the undrained shear strength profile was made evident when slope stability analyses showed that the undrained shear strengths were too low and, therefore, unreasonable factors of safety were calculated given the absence of evidence of past slides in some of the areas of the analysis. Moreover, adjusting the PDF from normally distributed to uniformly distributed is appropriate to limit the low values of undrained shear strength that are improbable (i.e., have low probability of occurrence) but nonetheless could lead to impossible values of the factor of safety.

The results of the analysis presented above show that the updated mean undrained shear strength is approximately the same as the undrained shear strength that would be obtained if N_{kt} = 15 were used, and the minimum values below a depth of approximately 25 m would be equivalent to using N_{kt} = 17.5 (Figure 8). Therefore, despite the impression that the updated strength is significantly larger than the prior strength, the N_{kt} values for the mean are still consistent with typical practice in the Gulf of Mexico. However, it may be prudent for practitioners conducting deterministic slope stability analyses in clayey soils of the Gulf of Mexico to consider N_{kt} = 15 to estimate the mean value of the undrained shear strength for slope stability analysis purposes.

AUTHOR CONTRIBUTION STATEMENT

R. Sancio: Conceptualization, Data Curation, Methodology, Formal Analysis, Writing - Original Draft. **P. Varela and L. Brant**: Methodology, Formal Analysis, Writing - Reviewing and Editing.

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REFERENCES

Ang, A.H.-S., and W.H. Tang. 1975. *Probability Concepts in Engineering Planning and Design*. New York: John Wiley & Sons.

Cheon, J.Y., D.R. Spikula, A.G. Young, R.B. Gilbert, and P. Jeanjean. 2015. “A Perspective on Selecting Design Strength: Gulf of Mexico Deepwater Clay.” In *Frontiers in Offshore Geotechnics III*, edited by Vaughan Meyer, 1349–54. London: Routledge.

Dimmock, P., B. Mackenzie, and A.J. Mills. 2012. “Probabilistic Slope Stability Analysis in the West Nile Delta, Offshore Egypt.” In *Proceedings of Offshore Site Investigation and Geotechnics Conference, London, UK, 12–14 September*, 535–542.

Gilbert, R.B., S.G. Wright, and E. Liedtke. 1998. “Uncertainty in Back Analysis of Slopes: Kettleman Hills Case History.” *Journal of Geotechnical and Geoenvironmental Engineering* 124(12):1167–76.

Haneberg, W.C. 2015. “Understanding the Element of Time in Probabilistic Submarine Slope Stability Analyses.” In *Frontiers in Offshore Geotechnics III*, edited by Vaughan Meyer, 963–68. London: Routledge.

Nadim, F., D. Krunic, and P. Jean. 2003. “Probabilistic Slope Stability Analyses of the Sigsbee Escarpment.” OTC Paper 15203. Offshore Technology Conference, Houston, Texas, USA May 5–8.

Pestana, J.M., G. Biscontin, F. Nadim., and K. Andersen. 2000. “Modeling Cyclic Behavior of Lightly Overconsolidated Clays in Simple Shear.” *Soil Dynamics and Earthquake Engineering* 19:501–519.

Sancio, R., P. Rao, C. Hunt, A. Greene, and S. Misra. 2017. “Hazard Quantification of Seismically Induced Tsunamigenic Subaerial/Submarine Mass Movements.” OTC Paper 27628-MS. Offshore Technology Conference, Houston, Texas, USA, May 1–4.