Proceedings of ISFOG 2025
5TH INTERNATIONAL SYMPOSIUM ON
FRONTIERS IN OFFSHORE GEOTECHNICS
Nantes, France | June 9-13 2025

ISBN 978-2-85782-758-0

Optimising Laboratory Test Quantities using Bayesian Statistics

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ABSTRACT: The use of Bayesian statistics is making its way into routine geotechnics because of increasing availability of regional databases of geotechnical parameter values and increasingly efficient computing. Bayesian statistics can support development of reliable probability density functions (PDF) of geotechnical parameters. These PDFs allow estimation of statistical uncertainties of parameter values, including the mean (BE), median, confidence interval (BE_L, BE_H) and the prediction interval (LE, HE) which are key inputs for design of offshore wind foundations. This paper describes the use of Bayesian statistics for optimising laboratory test quantities by leveraging existing data. The presented optimisation approach also covers dynamic updating of the (posterior) probability density function for key parameters (in this case undrained shear strength in triaxial testing for clays) and monitoring of laboratory test quantities as site-specific data become available. Achieved optimisation is compared with the conventional approach of Frequentist statistics.

Keywords: Bayesian statistics; Optimisation; Laboratory testing

1 INTRODUCTION

Sustainable development and climate resilience are currently key factors propelling countries to set targets toward carbon neutrality. As an example, the Dutch government has set out a roadmap with the required installed capacity from offshore wind farms to reach their energy supply targets. Figure 1 shows the number of turbines needed and the offshore area to be characterised to meet required installed capacity.

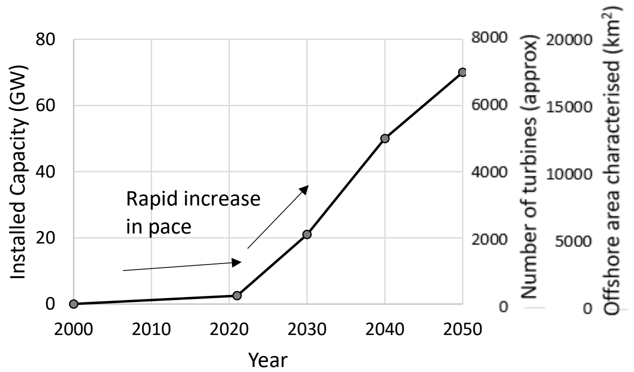

Figure 1. Sustainable development goals, Netherlands (source: RVO, 2021)

The geotechnical industry is striving to keep up with the current schedule demands. The scale of activities and the rapid change of pace, as illustrated in Figure 1, are evident. Note that the geotechnical industry typically operates some years ahead of installed capacity.

Innovation opportunities for schedule shortening are available for site characterization programmes for design of offshore structures, particularly on the topic of a good understanding of the probability density functions (PDF) of key geotechnical parameters. Figure 2 illustrates how these PDFs allow estimation of statistical uncertainties of parameter values, including the mean (BE), median, confidence interval (BE_L, BE_H, lower curve) and the prediction interval (LE, HE, upper curve).

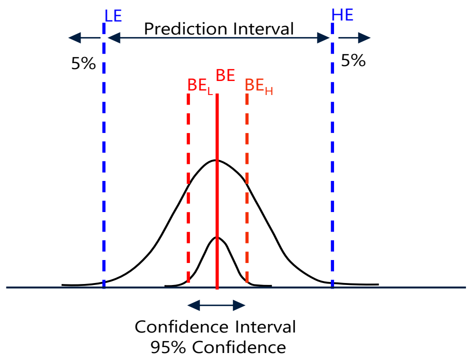

Figure 2. PDF with statistical profiles (normal distribution)

Defining these PDFs requires data, to answer the question of 'how much data is enough'. The ideal solution is to sample the population, however given the constraints of cost and schedule (and the added requirement of maintaining the same quality), Bayesian data analysis can support development of reliable PDFs.

Bayesian statistics is an accepted method as per ISO (2015). It is also certifiable, see for example DNV (2021). Bayesian statistics holds potential to leverage existing data. While the use of Bayesian statistics is not novel, it is making its way into routine geotechnics, e.g. Baecher (2017), Baecher and Christian (2005), Bozorgzadeh (2019), Bozorgzadeh (2020), Bozorgzadeh (2019). Important drivers are increasing availability of regional databases of geotechnical parameter values and increasingly efficient computing.

This paper presents the use of Bayesian statistics for optimising laboratory test quantities for capturing the PDFs for key geotechnical parameters. An example is given for optimising the quantity of derived values (defined as per ISO 2014) of undrained shear strength in triaxial testing for clays.

2 BACKGROUND

Bayes theorem, first published by Thomas Bayes (Bayes, 1763), is the foundation of Bayesian statistics. Bayesian statistics is enabled by using Markov chain Monte Carlo (MCMC), which is a class of algorithms that is used to draw samples from a probability distribution through multiple iterations. The following references provide detailed reading on Bayesian data analysis (Kruschke, 2015; Gelman et al., 2013).

The approach of Bayesian data analysis to determine a geotechnical parameter value distribution, includes the following broad steps:

a. Engineering judgement and assessment by the geotechnical engineer to estimate or identify a prior understanding of the geotechnical parameter value;
b. Performing laboratory tests to obtain a site-specific derived value dataset;
c. Bayesian data analysis to obtain the posterior distribution and statistical profiles (Figure 2).

Successful implementation and accuracy of results from Bayesian statistics requires relevant prior data (Step a). Broadly encapsulated by 'engineering judgment and assessment', this step should include several important checks and procedures: assessment of ground unit similarity or a correlation analysis between prior information and site-specific information (e.g. geological setting, similarity of ground units with respect to mechanical and engineering properties), explicit definition of the parameter (e.g. reference method chosen for obtaining derived values, soil sampling type, transformations applied on the parameter values, etc) and assessments on the quality of the data.

It is also critical for any new (site-specific) data that are included in Bayesian data analysis to be of high quality and obtained consistently (e.g. s_u obtained from same reference method, sampling method etc). It is noted here that uncertainty in source data is (currently) not explicitly considered in Bayesian statistics. This can become challenging for interpretation of outcome, particularly if only a few new site-specific data points are available, as Bayesian statistics imposes a significant reliance on new data.

Statistical robustness depends on having sufficient sample size. The minimum number of samples (i.e. test count) needed can vary depending on the site, the specific statistical approach and the desired level of confidence. A low test count would probably imply reverting to an approximate verification approach of a global/ regional correlation, with no optimisation by statistical analysis.

Steps b and c from the Bayesian workflow can be performed with dynamic Bayesian updating, i.e. updating the PDF curves (Step c) as and when site-specific data become available (Step b). This would help identifying or optimising the number of laboratory tests required to capture the PDF, as further discussed in Section 3.

The approach to statistics typically used by geotechnical engineers (e.g. estimating PDFs using derived values) is termed Frequentist statistics. The fundamental difference between the two approaches of Bayesian statistics and Frequentist statistics is in how they interpret and handle probability and uncertainty, particularly:

- Use of prior information: Bayesian methods explicitly incorporate prior information into any analysis, while Frequentist methods do not;
- Probability interpretation: Frequentist statistics interprets probability as the long-term frequency of events, e.g. using a dataset of laboratory test results to estimate the average undrained shear strength. Bayesian statistics incorporates a degree of confidence which can change with new evidence or data;
- Approach to statistical parameters: Frequentist methods consider statistical parameters, such as the mean or standard deviation of a dataset to be fixed and unknown quantities which are estimated using data. Bayesian statistics considers statistical

parameters to be random variables with their own probability distributions.

Both Bayesian and Frequentist statistical approaches have their strengths and limitations and should be used in different contexts depending on the design/ assessment situation(s), parameter value and the available prior information. With a uniform prior (or no informed prior), the Bayesian posterior distribution can resemble the Frequentist estimation.

3 EXAMPLE CASE STUDY

3.1 Source Data and Data Quality

The example case study considers optimisation of laboratory test quantities for undrained shear strength, s_u, particularly within a context of characterising clay soils similar to using an N_{kt}-type approach, where $N_{kt} = q_n/s_u$ (e.g. Mayne and Peuchen 2022). A regression analysis is carried out between normalised undrained shear strength, s_u/σ'_v, and normalised net cone resistance, q_n/σ'_v, both normalised by insitu vertical effective stress, σ'_v, with the aim of assessing if the laboratory test results used to obtain the s_u-q_n relationship can be optimised. Normalised values of s_u and q_n were used to account for depth dependency which was observed to provide better results (i.e lower uncertainties) in place of the typical N_{kt} approach.

Table 1 presents information on the data sources used for this case study. The site name abbreviations are Hollandse Kust Noord (HKN), Hollandse Kust West (HKW) and IJmuiden Ver (IJV) Sites Alpha and Beta wind farm zones. The test count refers to paired values of s_u and q_n associated with test specimen depth (or corresponding value of σ'_v) from multiple investigation locations and various depths below seafloor.

The data sources listed in Table 1 include consistently obtained derived values. The reference method for s_u is a laboratory single-stage undrained triaxial compression test on an undisturbed clay specimen obtained by means of thin-walled push sampling, anisotropically consolidated to estimated in situ vertical and radial effective stress conditions, monotonically sheared, and s_u calculated by maximum value of principal stress ratio within an axial strain range of 0 to 15 %. The reference method for q_n is according to ISO (2014).

The reference reports of Table 1 include detailed procedures implemented for data quality review, pairing (between CPT and laboratory data) and other relevant information required for Step a from the workflow presented in Section 2. These detailed procedures are key to validating the suitability of source data.

Table 1. Source data for example case study.

Site name	Report reference	Plate reference	Test count
HKN	Fugro (2019)	Plate B.6-1	7
HKW	Fugro (2020)	Plate E7-4	33
IJV	Fugro (2023)	Plate C.2-66	38

The three sites show predominantly sandy soils with some clay layers. A correlation assessment (Step a from the Bayesian workflow) was carried out on the datasets presented in Table 1 on establishing similarity of ground units particularly the clay layers. Based on the results of the assessment, geological setting and understanding of the site, the datasets presented in Table 1 can be considered to have similar engineering and geotechnical properties for a significant range of (over)consolidation settings indicated by q_n/σ'_v ranging from about 5 to 45.

3.2 Optimisation Approach

The example case study (or regression analysis) focusses on the IJV site (with 38 datapoints). This regression analysis was performed in multiple ways and a comparison of the statistical profiles (and the PDF curve) was carried out.

Table 2 presents the approaches used for the regression analysis.

Table 2. Overview of data analyses.

Approach	Description	Test count
FDA-complete	Frequentist data analysis	38
BDA-partial	Bayesian data analysis with no informed prior and partial IJV dataset	15 (/38)
BDA-complete	Bayesian data analysis with no informed prior and complete IJV dataset	38
BDA-optimised	Bayesian data analysis with informed prior (HKW + HKN) and partial IJV dataset	15 (/38)

Approaches *BDA-partial* and *BDA-optimised* make use of a partial dataset, i.e. by using 15 IJV test results out of the existing 38 test results available. The 15 test results were randomly selected. The choice in the number (or count) of tests results used, i.e. 15 test results (as opposed to a higher or lower number) was also random and intended for illustration of the optimisation approach. The optimization approach presented in this paper does not include dynamic Bayesian updating as described in Section 2.

The regression models were analysed by visual inspection and by use of root mean square error (RMSE) values. RMSE is typically used in statistics to assess how well a model's predictions match the actual data. A low RMSE value indicates a better fit.

3.3 Frequentist Data Analysis (FDA-complete)

Figure 3 presents results from Frequentist analysis, *FDA-complete*. It shows a best-fit linear regression on 38 test results from IJV, shown by black circles. For reference, the figure also includes the HKN and HKW data points shown by grey circles. The blue dashed lines are LE and HE profiles. The red solid line is the BE and the red dashed lines represent BE_L and BE_H.

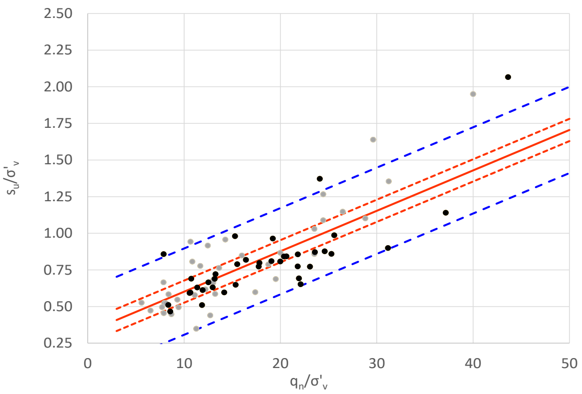

Figure 3. Results of FDA-complete analysis

Equation 3 presents the selected linear regression relationship, where m and c are statistical parameters refering to the slope and intercept respectively.

$$\frac{s_u}{\sigma'_v} = m\frac{q_n}{\sigma'_v} + c \quad (3)$$

A linear representation (versus non-linear) was observed to be the best-fit regression relationship, with RMSE = 0.165.

3.4 Bayesian Data Analysis

3.4.1 Regression models

The selected models for Bayesian regression are according to Kahlsrom and Borgorgzadeh (2022). Details are as follows.

Equation 4 shows a generic empirical correlation, considering a dataset with N observations. Equation 5 presents the same empirical correlation on log-scale for each observation i, where $ln(c)$ is the intercept, m is the slope, μ_i is the mean of $ln(s_u/\sigma'_v)$ and ε_i is the error normally distributed with zero mean and standard deviation SD.

$$\frac{s_u}{\sigma'_v} = c\left(\frac{q_n}{\sigma'_v}\right)^m \quad (4)$$

$$ln\left(\frac{s_u}{\sigma'_{vi}}\right) = ln(c) + m.ln\left(\frac{q_n}{\sigma'_{vi}}\right) + \varepsilon_i = \mu_i + \varepsilon_i$$
$$i = 1, 2, 3, .., N \quad (5)$$

Equation 5 translates that $ln(s_u/\sigma'_v) \sim Normal(\mu_i, SD)$ for which the statistical parameters in Bayesian data analysis are $ln(c)$, m and SD.

In the case of no informed prior (*BDA-partial* and *BDA-complete* approaches), the statistical parameters do not have any prior information and are inferred from sufficiently dispersed distributions:

- $ln(c) \sim Normal(0, 10)$
- $m \sim Normal(0, 10)$
- $SD \sim Cauchy(0, 5)$

In the case of an informed prior (*BDA-optimised* approach), the statistical parameters still follow a multi variate normal distribution (MVN) but also consider the statistical parameters from the prior dataset (HKN and HKW data). This is written mathematically as below:

$$\begin{pmatrix}\ln(c)\\ m\end{pmatrix} \sim MVN\left(\begin{pmatrix}\mu_{\ln(c)}\\ \mu_m\end{pmatrix}, \begin{pmatrix}Var(\ln c) & Cov(\ln c, m)\\ Cov(m, \ln c) & Var(m)\end{pmatrix}\right)$$
$$SD \sim Lognormal(\mu_{SD}, SD_{SD})$$

The parameters Var and Cov stand for variance and covariance of statistical parameters, respectively, inferred from the prior dataset.

Computation for posterior distributions is according to the MCMC method for all regression models.

Table 3. Posterior statistical parameters and root mean square errors of predictions.

Approach	$\ln(c)$		m		SD		RMSE	
	Mean $\mu_{\ln(c)}$	Variance $Var(\ln c)$	Mean μ_m	Variance $Var(m)$	Mean $\mu_{\ln SD}$	Variance $Var(\ln SD)$	Complete IJV dataset	Blind test dataset
FDA-complete	-	-	-	-	-	-	0.165	-
BDA-partial	-1.25	0.25	0.36	0.03	-1.41	0.05	0.040	0.045
BDA-complete	-1.72	0.04	0.51	0.005	-1.69	0.01	0.033	-
BDA-optimised	-1.96	0.023	0.63	0.003	-1.49	0.01	0.036	0.035

3.4.2 Results

Figures 4 to 6 and Table 3 present results. For comparison, Table 3 includes RMSE values from *FDA-complete* analysis. Note that the slope and intercept from FDA-complete are not directly comparable with those from BDA analyses, because of linear versus logarithmic regression.

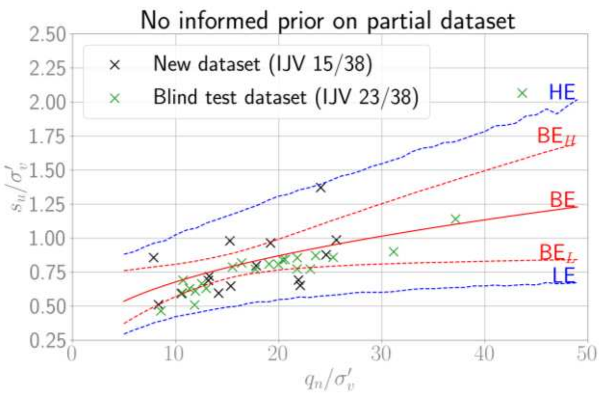

Figure 4. Results of BDA-partial analysis

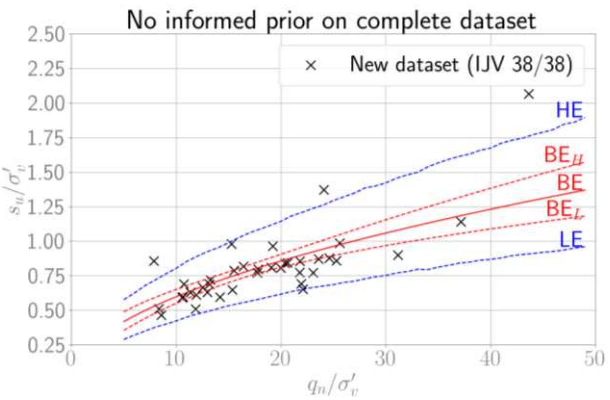

Figure 5. Results of BDA-complete analysis

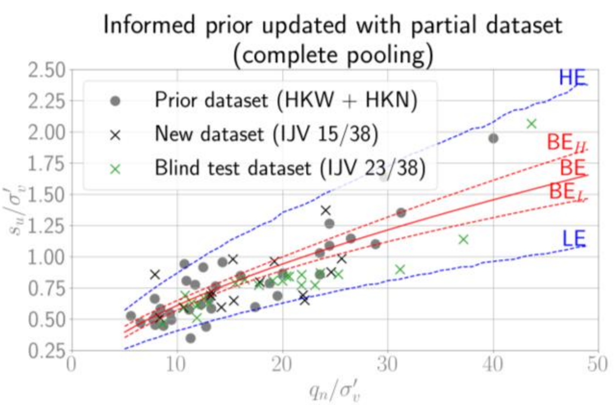

Figure 6. Results of BDA-optimised analysis

The 'Blind test dataset' RMSE value is only applicable to *BDA-partial* and *BDA-optimised* approaches as they used a partial dataset (15 out of 38 laboratory test results). The RMSE values refer to the error of the regression models (i.e. approaches *BDA-partial* and *BDA-optimised*) in predicting the remaining 23 test results.

The results indicate better predictions (lower RMSE values, lower uncertainty ranges) obtained by use of Bayesian analysis as compared to Frequentist analysis. The added value of prior data can be observed in the difference in predictions from *BDA-optimised* analysis compared to *BDA-partial* analysis. Results from *BDA-complete* analysis presented the best predictions.

4 DISCUSSION AND CONCLUSION

It can be challenging to balance the quantity of data required for statistical robustness (regardless of approach) and optimization for schedule.

The use of Bayesian statistics for optimising laboratory test quantities by leveraging existing data is promising and offers significant potential. The presented case study indicated a possibility of using less than half of the undrained shear strength test results available for analysis. This finding on optimisation is supported by results from multiple Bayesian data analyses and the conventional approach of Frequentist data analysis.

Some key concluding points on implementing Bayesian data analysis are summarised below:

- Availability and selection of high-quality prior data is key for successful implementation and accuracy of results from Bayesian statistics. Correlation analysis on establishing similarity (refer to Section 2) between prior data and site-specific data is critical to avoid misinterpretation of results and/ or overconfidence in conclusions.
- Statistical uncertainty assessment (refer Figure 2) is built-in to the Bayesian approach and is thereby better characterised. It is important to note here that a statistical approach does not compensate for data (quantity), however good prior knowledge can and Bayesian statistics offers a sound, data-driven approach to incorporate it.
- Many open-source softwares and libraries are available for Bayesian statistics (e.g. Python, R). Nevertheless, a high level of statistical and computational expertise is required to understand and implement the statistical model, which can be associated with significant algorithmic complexity. Additionally, the underlying assumptions must be carefully considered to ensure robust and credible outcomes.
- Further refinements can enhance results from the Bayesian approach. Examples are use of alterna-

tive priors, use of hierarchical models (e.g. accounting for spatial variation in datasets), accounting for source data uncertainty and use of multiple validation methods (e.g. Watanabe-Akaike Information Criterion, Leave-One-Out Cross-Validation etc.).

AUTHOR CONTRIBUTION STATEMENT

Madhuri Murali: Conceptualization, Data curation, Methodology, Formal analysis (partial), Writing - Original draft. **Joek Peuchen**: Supervision, Funding acquisition, Writing - Reviewing and Editing. **Matthieu Constant**: Formal analysis, Software (scripting), Visualization, Review. **Benoît Spinewine**: Supervision, Review. **Michel Vrolijk**: Review. **Peter-Paul Lebbink**: Review.

ACKNOWLEDGEMENTS

The authors are grateful to Nezam Bozorgzadeh (and NGI) for their support and discussions on Bayesian data analysis.

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