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Recognising Geotechnical Uncertainty as an Asset to Site Investigation and Characterization

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ABSTRACT: Geotechnical field and laboratory data are typically presented in a factual report, as nominal values, without systematic documentation of uncertainty. These data are used in calculations, correlations and design, where error in the input data and subsequent correlations combine to a compound error that may be significant and increase risk. Uncertainty is ever present in geotechnical data, from inherent soil uncertainties (aleatory) to systematic uncertainties in methods and analysis (epistemic). We describe sources of geotechnical uncertainty in offshore geotechnical data collection, laboratory analysis and interpretation, and evaluate those uncertainties within two commonly used geotechnical parameter calculations and correlations. Through worked examples using representative tolerances from relevant standards, guidance and conventional working practice, the propagation of uncertainty is illustrated for the calculation of Undrained Shear Strength and Relative Density. The presentation of error propagation and uncertainties to end-users of data are demonstrated and discussed, highlighting the importance of recognising uncertainties.

Keywords: Uncertainty, Error Propagation, Characteristic Values.

1 INTRODUCTION

Uncertainty is presented by DNV in recommended practice C207 (DNV 2021) as either epistemic or aleatory. Epistemic uncertainty is defined as the uncertainty associated with imperfect knowledge, such as under sampling, measurement error, statistical error and more. Aleatory uncertainty is the inherent, natural random uncertainty that is associated with natural materials, and in the case of geotechnical engineering, the natural variability of soils and rock. Aleatory uncertainty is irreducible, whereas epistemic uncertainties can be reduced through action and information collection. Aleatory uncertainties are, by definition, irreducible and ever-present, and therefore the uncertainties that can typically be quantified, recognised and reduced or mitigated for, are epistemic uncertainties.

1.1 Epistemic Uncertainty in Geomaterials

In the context of offshore engineering and foundation design, epistemic uncertainties can be recognised, reduced, and mitigated through site investigations, engineering works, and design. Within epistemic uncertainty further sub-classes of uncertainty can be loosely defined. The classes used here to sub-divide epistemic uncertainty are primarily cited from NGI (Bozorgzadeh, 2024), DNV-RP-C207 (DNV 2021a), and influenced by the work of Oakley and O'Hagan (2004), and consist of:

- Measurement error
- Statistical (parameter) uncertainty
- Model error
- Parametric uncertainty
- Calculation uncertainty
- Residual error

Whilst these subdivisions are not discussed in detail here, they referred to when describing components of uncertainty quantified in the examples presented. It should be noted here that the term 'error' does not imply incorrect data collection or engineering practice; rather, it is a standard term used to describe the inherent uncertainty in measurements.

1.2 Uncertainty in Calculations

The combination of simple, linear, and statistically independent uncertainties of a variable, X, can be represented by the addition of variances (DNV 2021a):

$$s_{obs}^2 = s_X^2 + s_{meas}^2 + s_{model}^2 + s_{stat}^2 \qquad (1)$$

Where s_X^2 is the variance associated with natural variability, s_{meas}^2 is the measured variance, s_{model}^2 is the variance of any applied model, and s_{stat}^2 is the variable of any statistical calculation(s). For variables and summative soil properties that are non-linear, the sum of the partial derivatives of each property and associated uncertainties can be used (Ayyub and McCuen, 2011, Ku, 1966):

$$s_f^2 = \left(\frac{\partial_f}{\partial_X}\right)^2 s_X^2 + \left(\frac{\partial_f}{\partial_Y}\right)^2 s_Y^2 \ldots \quad (2)$$

Where s_f represents the standard deviation of the function, f, used to combine variables X, Y, and so on.

Specific guidance on the use of Equation 1 is provided in DNV-RP-C207 (DNV 2021a) and DNV-RP-C212 (DNV 2021b) and is used in this paper for the calculation of single variable uncertainties, such as soil properties. Equation 1 is used for the calculation of undrained shear strength, S_u, and relative density, D_r. It is possible to rearrange Equation 1 and 2 to estimate s_X^2, however caution must be used when interpreting the estimated values. The term s_X^2 is a complex parameter that is interpreted here as the combination of presently unknown epistemic uncertainties that are yet to be quantified, and irreducible aleatory uncertainties. Estimation of s_X^2 by rearrangement can become a circular argument, as is cautioned in DNV-RP-C207. For this reason, we only consider epistemic uncertainties in calculations and is a representation of minimum uncertainty.

Table 1. Uncertainty values associated with data types used in this work with citation sources.

Variable	Uncertainty	Citation
Sample Depth	0.5 m	ISO 22476-2:2022[1]
CPTu: qc	0.5 - 5 % or 15 - 200 kPa	ISO 22476-1:2022[2]
CPTu: fs	1 - 10 % or 5 - 25 kPa	ISO 22476-1:2022[2]
CPTu: u2	0.5 - 5 % or 3 - 50 kPa	ISO 22476-1:2022[2]
CPTu: Depth	0.1 - 0.2 m, or 1 - 2 %	ISO 22476-1:2022, ISO 22476-2:2022
Bulk Density	25 kg/m^3	ASTM D7263, ISO 11508, Rocchi and Coop (2014)
UU Triaxial	5% of Su	ASTM D2850, BS1377-7
Min-Max Unit Weight	15 - 40 kg/m3	ASTM D4254, Lunne et al., (2019)

1.3 Epistemic Uncertainty Quantification in Geotechnical Data

All geotechnical data contains uncertainty as a product of aleatory uncertainty and the previously defined epistemic uncertainty sources. In the examples presented here, the uncertainties of the parameters used in calculations to determine undrained shear strength and relative density were considered by referring to relevant standards and published work. These are presented in Table 1.

2 ERROR PROPAGATION IN GEOTECHNICAL CALCULATIONS

2.1 Correlations used to determine soil properties

The CPTu zero-reference total stress, σ_{V0} (MPa), can be estimated by:

$$\sigma_{V0} = \int_{z_i}^{z} \rho_B(z_i) g \Delta z \qquad (3)$$

Where z is depth (m), ρ_B is the bulk in-situ density (kg/m^3), and g (m/s^2) is gravity, taken as 9.81 m/s^2. The water column is not considered as CPTu measurements are zeroed at the mudline. Bulk density is represented by an average profile for a soil unit (Section 3.2).

In-situ undrained shear strength, S_u (kPa), is estimated from CPTu data using:

$$S_u = q_{net}/N_{kt} \qquad (4)$$

Where q_{net} (kPa) is net cone resistance, and N_{kt} (-) is a cone-factor relating q_{net} to S_u. Net cone resistance is calculated from corrected total cone resistance, q_t (kPa), by subtracting the CPTu zero-reference total stress, σ_{V0} (kPa). Total stress is calculated using an average density profile for a soil unit.

In-situ relative density, D_r (%), is estimated using one of two relationships proposed by Jamiolkowski et al (2003), the first of which is:

$$D_r = \frac{1}{C_2} \ln\left[\frac{q_c/p_a}{C_0(\sigma_m{}'/p_a)^{C_1}}\right] \qquad (5)$$

Where q_c (kPa) is measured cone resistance, $\sigma_m{}'$, (kPa) is mean effective stress calculated as $\sigma_m' = \frac{1+2k_0}{3}$, p_a (kPa) is a reference atmospheric pressure (kPa), and C_0, C_1, C_2 are average fitting parameters for unaged, uncemented normally consolidated Ticino, Toyoura and Hokksund sands. When the stress is defined as total effective stress, these parameters are: C_0 = 17.68, C_1 = 0.5, C_2 = 3.1. When the stress is defined as mean effective stress, they are: C_0 = 24.94, C_1 = 0.46, C_2 = 2.96. The second commonly used relationship, originally proposed by Lancellotta (1983), but modified by Jamiolkowski (2003) is:

$$D_r = -A_0 + B_0 \ln[(q_c)(\sigma_{v0}')^{-0.5}] \qquad (6)$$

Where Jamiolkowski suggested values of coefficients A_0 and B_0 of 1.292 and 0.268. Site specific correlations to obtain relative density can be estimated from min-max tests and CPTu data by regression analysis to estimate coefficients A_0 and B_0 (Trandafir et al., 2023).

In the following sections worked calculations are presented for S_u and D_r. The first calculation does not consider uncertainty and treats all calculated values as nominal without uncertainty. The second calculation propagates all uncertainties through calculations and presents uncertainty at each calculation step. Where appropriate, all calculations are presented using characteristic values advised by DNV-RP-C207 and DNV-RP-C212 and referred to as the A-E estimates. These values are calculated using 5^{th} and 95^{th} percentiles at 95% confidence (A, E), and the mean (C) with a 95% confidence interval (B, D).

These are then compared and discussed. In the uncertainty taxonomy outlined in Section 1.1 both geotechnical-property correlations are classified as models that can be used to derive engineering parameters from field data. Both models contain input data with measurement error, statistical uncertainty associated with the determination of bulk unit properties, computational, and a residual error as it is impossible to fit these models perfectly to each sample point (nor should they be). The relative density correlations contain an inherent parametric uncertainty associated with the fitting parameters, and derivation of the relationships from specific experimental data.

3 SITE DATASET

3.1 Data and Outliers

The data presented in this paper is from an anonymised site that is currently in the detailed design phase of development and represents a relatively complete and standard site investigation dataset for Front End Engineering Design (FEED) purposes. The two shallowest soil units were selected for use in calculations of S_u and D_r with uncertainty, and are anonymised as Soil Unit A, and Soil Unit B. Soil Unit A occurs close to the seabed and comprises of normally consolidated marine clays with some silty partings. Soil Unit B comprises of marine deposited, clean, normally consolidated quartz sands, and occurs typically at the same stratigraphic depth range, or above Soil Unit A at the seabed. The available data for the soil units are presented in Table 2.

Sample data outliers were identified and removed from the data using a two-standard deviation threshold as recommended by DNV-RP-C207. The guidance in DNV-RP-C207 presents two approaches to estimating characteristic values of soil properties: population-based with no-depth dependency, and regression-based for parameters that are depth-dependent (DNV 2021a). The inputs in Table 2 for each soil unit were tested for normality using the Shapiro-Wilk Test (Shapiro and Wilk, 1965), and depth dependency by slope testing. None of the data types in Table 2 demonstrated any noticeable depth-dependency and were interpreted as weakly normally distributed. For this reason, population-based A-E estimates have been used throughout, unless otherwise stated.

Table 2. Summary of data used, with Soil Unit denoted in brackets.

Test Type	No. of tests	Cumulative Depth of CPTu Data (m)	Outliers
Seabed CPT (A)	129	670.80	
Downhole CPT (A)	23	15.76	
Seabed CPT (B)	129	497.30	
Downhole CPT (B)	4	22.38	
Bulk density (A)	53		12
Bulk density (B)	56		10
UU Triaxial (A)	29		4
Min-Max Density Lab Tests(B)	17		

3.2 CPTu Data Sampling

The CPTu data available for this site is sampled at an interval of 0.02 m, and recovered samples vary in size and recovery from 0.15 to 0.5 m, with an average recovery of 0.23 m. An averaging approach was taken to compare sample data and CPTu measurements rather than single depth points. The recovered samples were depth-matched to the CPTu data and averaged over a 0.2 m window. The number of samples, standard deviation and median were also recorded and used as part of the uncertainties. This was then combined with the uncertainties presented in Table 1.

3.3 Bulk Density and Total Stress Determination

An average bulk density profile was estimated for Soil Unit A and Soil Unit B by considering the uncertainties outlined in Table 1. The average density profile, uncertainties and A-E estimates are presented for Soil unit A in Figure 1. The calculated values for Soil Unit A and B are presented in Table 3.

Table 3. Estimated Bulk Density Characteristic Values with uncertainty, and stress gradient calculated using the unbiased mean with uncertainty (C value).

Characteristic Value	Bulk Density (kg/m^3)	
	Soil Unit A	Soil Unit B
A	1685.04 ± 97.43	1768.19 ± 88.26

B	1880.09 ± 96.68	1945.85 ± 87.66
C	1891.58 ± 96.47	1955.44 ± 87.47
D	1902.28 ± 96.68	1965.03 ± 87.66
E	2098.13 ± 97.44	2142.69 ± 88.29
Stress Gradient (kPa/m)	18.56 ± 0.95	19.18 ± 0.86

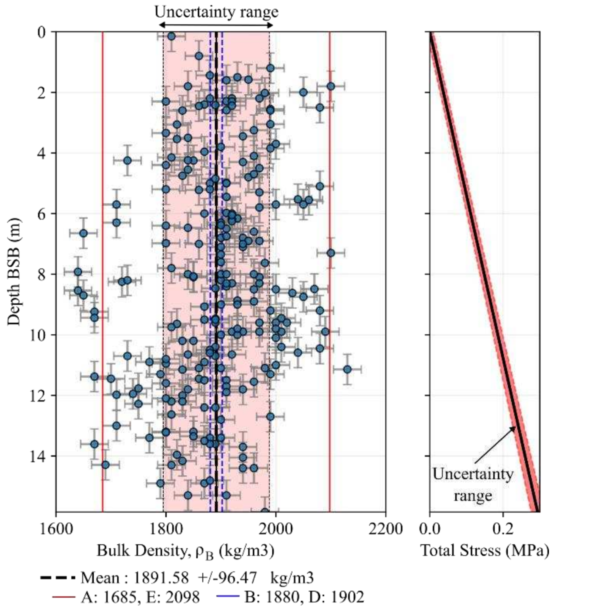

Figure 1. Soil Unit A bulk density with mean bulk density and associated uncertainty, A-E estimates, and calculated CPTu-relative total stress with uncertainty in red. Uncertainty range is only shown for the mean.

Whilst Figure 1 illustrates that the uncertainty range is greater than the B-D estimates, it would be erroneous to interpret this as an inadequacy of either approach. The characteristic values, as defined by DNV (2021a), are estimates at a defined confidence interval and therefore represent a type of uncertainty. This uncertainty is relative to the precision of the estimate from the sample population used to estimate the characteristic value. Each A-E estimate presented in Table 3 has a total uncertainty and should any characteristic value of a property be required for the next calculation; it can be carried forward. In our examples we have carried forward the unbiased mean with uncertainty, characteristic value C, into subsequent calculations requiring bulk density and stress, and used the uncertainty associated with this estimate. It is at this point at where common working practice can begin to diverge, with some practitioners opting to use each characteristic value estimate in subsequent calculations, and others using the central estimate, then calculating A-E estimates on the subsequent property distribution. Unless clearly defined by a relevant standard or code, the choice of which approach to use requires careful consideration of the final requirements, use and available data.

4 ESTIMATION OF SOIL PARAMETERS

4.1 Undrained Shear Strength

The semi-empirical cone factor, N_{kt}, was calculated by fitting a zero-intercept slope model to a scatter plot of S_u versus q_{net} (Figure 2). In the first case, no uncertainty was considered in calculation of the N_{kt} slope. In the second, the uncertainties in both S_u and q_{net} were considered as weights in the fit of N_{kt}.

Characteristic values, A-E, were estimated using the fit N_{kt}, the 95 % confidence interval around the line (B and D), and the 5th and 95th percentiles at 95 % confidence of the predicted X and Y values (A and E).

The use of uncertainty weights when fitting a slope, as presented in Figure 2, results in different A-E estimates, and facilitates the calculation of an uncertainty range around each slope estimate. The uncertainty propagated A and E estimates are closer to the C estimate, and the C estimate is counter-intuitively reduced compared to a non-weighted fit that does not consider the uncertainties of the input datapoints. This reduction is likely to be a result of the greater uncertainty in the S_u measurements from UU-triaxial tests, which will have increased the weighting in the slope calculation towards this axis. Whilst this fit appears visually worse than the unweighted fit, it represents a fit that honours the uncertainty of the input datapoints.

Conversely it may indicate that the assignment of locations and soil data to Soil Unit A could be improved, however it should be noted that these soils were grouped together using similar CPTu profiles, seismic facies and geological interpretation. At this point, it is difficult to evaluate model-misfit, poor unitization, or inherent geological variability of the property of S_u within Soil Unit A.

Despite this, it is still of relevance to consider the uncertainties that have been carried through the calculations as the A-E estimates are different, and the process of considering uncertainty has highlighted potential model misfit, the requirement for reunitisation, or the requirement for more data collection.

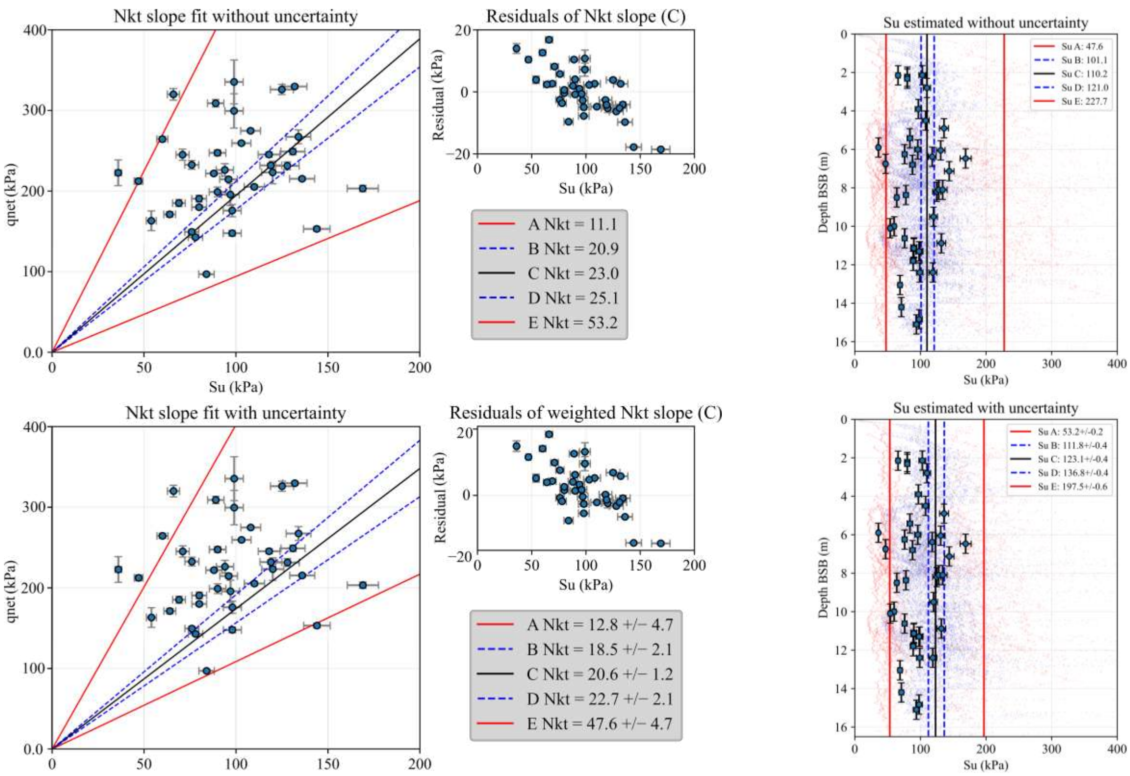

Figure 2. Semi-empirical cone factor, N_{kt}, determined by fitting a slope to S_u versus q_{net} (upper) and with a weighted slope (lower), and resulting A-E N_{kt} estimates for Soil Unit A. Profiles of Undrained Shear Strength and resulting A-E characteristic values were calculating using estimated N_{kt} values. Uncertainties are calculated by multiplying through the N_{kt} uncertainties into the calculation of S_u, and consider statistical uncertainty as well as the uncertainty of the N_{kt} slope model.

4.2 Relative Density

Model uncertainty was approached when estimating D_r using Equations 5 and 6, as these equations are commonly used without modification or semi-empirical scaling. Jamiolkowski (2003) reported the standard errors associated with each correlation, and therefore these models can be considered to contain a model uncertainty. The total uncertainty was calculated by comparing the observed data for Soil Unit B, calculating residuals of the observed data from the estimated D_r and combining this with all other uncertainties. This is presented for both D_r models, Equation 5 and 6, in Figure 3. The associated uncertainty calculated for Soil Unit B for Equation 5 is ± 26.8 %, and for Equation 6 is ± 23.81 %.

It is apparent from Figure 3 that neither model is a good fit to the observed data, and both models result in large uncertainty ranges, which would result in relatively uncertain estimates of D_r when used nominally at a site. Furthermore, both models result in slightly different calculated values of D_r with neither seemingly fitting the data suitably well. This suggests that neither model is appropriate when used this way, a conclusion which has been similarly proposed by Hamidi et al (2013) and others. Despite this being relatively common knowledge, these correlations are still commonly used in site characterisation. We propose that in this case, an honest treatment of these correlations as relatively uncertain models is prudent.

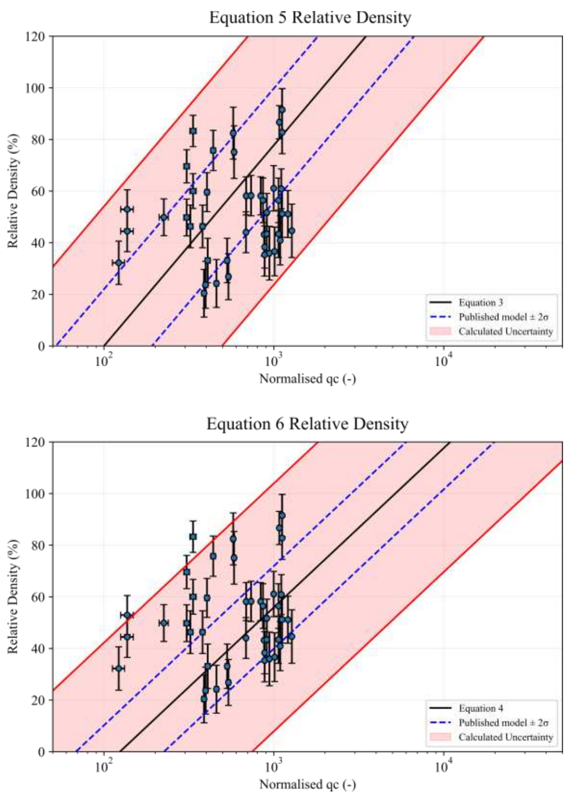

Figure 3. Uncertainties for estimated Relative Density.

5 CONCLUSIONS

We have demonstrated that by recognising and propagating uncertainty through multi-stage correlated property estimates that even when measurement uncertainties seem low, the final uncertainty of a multi-stage calculation can be greater than anticipated. Characteristic values of design-relevant properties, such as S_u, can, and in our examples, were different when uncertainties are carried forward. Calculating an 'honest' uncertainty when using a model-based correlation can demonstrate model inappropriateness.

AUTHOR CONTRIBUTION STATEMENT

Kieran Blacker: Data curation, Formal Analysis, Writing, Figures. **Neil Dyer**: Review, Supervision. **Erica Tataki & Sean Kilburn**: Review, Discussion.

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