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# Using PCE metamodels to analyse the uplift capacity of circular plate anchors in sand

A. Mentani*
*University of Bologna, Bologna, Italy*

A. Roy, S. Chow, R. Kurniadi
*The University of Melbourne, Melbourne, Australia*

**alessio.mentani2@unibo.it*

**ABSTRACT:** This paper aims to analyse the uplift capacity of circular plate anchors in sand through the use of Polynomial Chaos Expansion (PCE) metamodels. In general, a metamodel is a numerical method that surrogates the behavior of an original model to predict some selected output of interest. The PCE technique builds an approximation function to emulate the original model response, and different numerical features have been employed in this study. The metamodels for circular plate anchor capacity are developed by integrating databases from multiple sources, including centrifuge experiments and finite element analyses. The data covers anchor capacity in both loose to dense sand at embedments ranging from 1 to 12 times the anchor width, thus encompassing shallow to deep failure mechanisms of the plate. These datasets are then used to train the PCEs to capture the effect of three influencing parameters on anchor capacity. Overall, the developed PCE metamodels is able to predict the anchor capacity with good accuracies. The study also focuses on how the PCE performance can be continuously enhanced by integrating new, strategically selected finite element simulations into the metamodel. The proposed approach aims to demonstrate how the results of existing experimental and/or numerical studies can be used to develop reliable estimator tools, which can in turn be employed for a first order estimates of circular plate anchor capacities.

**Keywords:** Plate anchors, uplift capacity, sand, metamodel, Polynomial Chaos Expansion

## 1 INTRODUCTION

There has been a recent global emphasis on harnessing energy from offshore sources such as wind and wave. Such offshore energy platforms need to be moored securely to the seabed using ‘anchors’. Plate anchors are considered a promising solution due to their high capacity to weight ratio as they rely on the bearing resistance of the soil. Therefore, there is considerable interest in accurate prediction of their capacity in nearshore sandy deposits, where offshore renewable energy platforms are likely to be founded.

Plate anchor capacity can be predicted using Limit Equilibrium Methods (LEM) (Giampa et al. 2017), limit analysis theorems (upper bound and lower bound limit analyses) (Merifield et al. 2006) and through finite element (FE) analysis (Roy et al. 2021b). The performance of these analytical and numerical methods is typically validated through physical experiments (e.g. Rasulo et al. 2017, Hao et al. 2019, Roy et al. 2021a). FE models could provide accurate predictions accounting for different soil conditions, but complexities arise if sophisticated soil models need to be accurately calibrated and implemented. For this reason, analytical solutions are preferred by end-user for a preliminary design of the foundation. However, due to their approximations, such analytical solutions do not well capture the transition in anchor response from a ‘shallow’ to a ‘localised’ failure mechanism as corroborated from experimental findings (Hao et al. 2019).

In light of these uncertainties, this study employs the Polynomial Chaos Expansion (PCE) metamodelling technique (Sudret, 2008) to predict plate anchor capacity in sand. A metamodel is a model of a model; it is a mathematical model developed to emulate the response of a (physical or numerical) model. PCE is a regression-based metamodel that builds an approximation function to reproduce selected outcomes by randomly varying the input variables of the problem. The metamodel is calibrated using a database that contains pairs of input and output values from the original model. The PCE has been successfully applied to investigate the behavior of piles driven in sand (Mentani et al. 2023) and plate anchors in heterogeneous clay (Mentani et al. 2025).

In this study, the PCE is used to analyse the uplift capacity of circular plate anchors in sand, with the model trained using integrated databases from multiple sources. The aim of the study is to demonstrate how the PCE can preserve the essential information of the original models used in the calibration process, while also illustrating how incorporating new data can effectively enhance its performance.

## 2 THE APPROACH

### 2.1 The database

The present study investigates the ultimate capacity for a circular plate anchor of diameter $D$, installed with an embedment ratio $H/D$, in homogeneous sand having uniform relative density ($R_D$). Plate anchor capacity in sand is normally reported as anchor factor $N_\gamma$ ($= q_u/\sigma'_{vo}$, where $q_u$ is peak anchor capacity and $\sigma'_{vo}$ is the effective overburden stress). The three selected input (i.e. $D$, $H/D$, and $R_D$) have strong influence on $N_\gamma$, therefore any dataset chosen for developing a PCE metamodel of the plate anchor capacity should have a wide parametric variation of these variables.

The considered database was integrated from multiple sources, including data from centrifuge experiments (Giampa et al. 2017, Rasulo et al. 2017, Hao et al. 2019, Roy et al. 2021a) and from FE simulations (Roy et al. 2021a, Kurniadi at al. 2025). It includes 128 data, which encompasses plate diameters between 0.4m and 5m, relative density ranging from 30% to 90%, and embedment ratio between 1.05 and 15. The database contains slightly more data points for shallow embedment ratios, where anchors are most likely to be located. Figure 1 presents the probability density function (PDF) of the problem input ($D$, $H/D$, and $R_D$) and output ($N_\gamma$) for the considered database. The distribution of both experimental and FE data are represented by the blue and red bars, respectively.

The centrifuge testing data on circular plates were compiled from 40 reported experiments conducted in UWA silica sand by Hao et al. (2019) and Roy et al. (2021a) involving either steel or aluminium plates. For the 21 tests reported by Hao et al. (2019), the $H/D$ ranged between 2 and 12 at acceleration levels of 20$g$ in dense sand ($R_D$ ~ 90%), whereas for the 19 tests in Roy et al. (2021a), the $H/D$ varied between 1.8 and 6 at acceleration levels between 20-100$g$ in loose ($R_D$ = 41% to 51%) and dense sand ($R_D$ = 69% to 76%). The elevated stress levels in the centrifuge allowed investigating anchor capacity for diameters ranging from 0.4m to 2m. The testing data from Rasulo et al. (2017) and Giampa et al. (2017) involved 21 medium-scale tests having embedment ratios varying from 1.8 to 4, $D$ between 0.15m and 0.40 m in sand having $R_D$ ranging from 32 to 59%.

The FE database consists of 67 validated $N_\gamma$ obtained from FE simulations in Roy et al. (2021a) and Kurniadi et al. (2025). These simulations were conducted using FE software package Abaqus using axisymmetric analyses. A bounding surface plasticity model named modified SANISAND (Roy et al. 2021b), capable of handling density and stress dependent behaviour in sand, was employed in these analyses. The analyses were conducted in sand considering $D$ = 0.4 to 5m, $H/D$ = 1.05 to 15, and $R_D$ = 30% to 85%.

### 2.2 Polynomial Chaos Expansion metamodelling

A metamodel is a mathematical function that approximates the correlation between the input and output variables of a given model. Metamodelling then refers to the methodology implemented for building the approximation function.

In this study, the Polynomial Chaos Expansion (PCE) metamodelling technique has been used. It approximates the original model function, $\boldsymbol{y} = f(\boldsymbol{x})$, through a spectral series representation, where the orthogonal polynomials, $\Psi_k(\boldsymbol{x})$, which are expressed into the $n$ input variables of the problem ($\boldsymbol{x} = \{x_1, ...,$

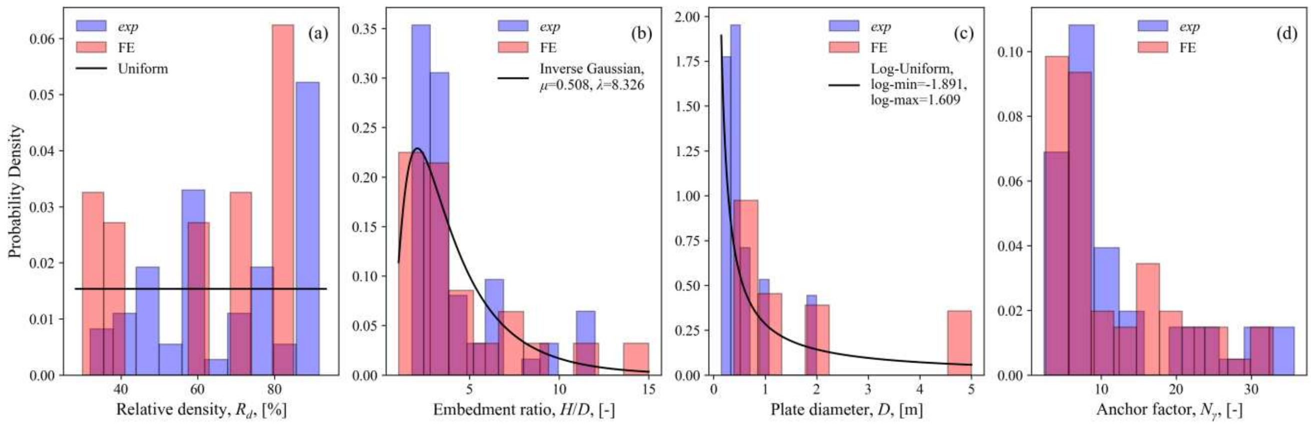


*Figure 1 – Distributions and best-fit PDF of the problem input and output: (a) sand relative density; (b) embedment ratio; (c) plate diameter; and (d) anchor factor.*

$x_n\}^T$), are weighted by coefficients, $\alpha_k$, as:

$$\boldsymbol{y}^* = g(\boldsymbol{x}) = \sum_{k=1}^{K} \alpha_k \psi_k(\boldsymbol{x}) \approx f(\boldsymbol{x}) \quad (1)$$

where $g(\boldsymbol{x})$ is the PCE function that generates the approximated model response vector, $\boldsymbol{y}^*$; and $K$ represents the number of terms in the equation.

The generation of the PCE requires the identification of the type of polynomials; the definition of a truncation rule to set the size of the equation; and the adoption of a regression method for computing the coefficients.

### 2.2.1 PCE mathematical features

The selection of the family of polynomials is linked to the distribution type of the sampled input (Figure 2). The PCE works in the Hilbert space for what the type of polynomial basis associated to standard distribution is known, as detailed by Sudret (2008).

Two different methods were considered to set the size of the equation: the standard and the hyperbolic truncation scheme. The first considers all the multivariate polynomials which total degree is smaller or equal to $p$, therefore the total number of terms is givens by:

$$K = \frac{(n+p)!}{n!\,p!} \quad (2)$$

This value is limited by the size, $N$, of the available training database for allowing the computation of the coefficients. A rule of thumb of $N = 2 \div 3K$ could be considered for the problem to be well-posed.

The multivariate polynomials have generally smaller coefficients compared to the univariate basis and the hyperbolic scheme could be used with the aim of removing some terms in the expansion. Specifically, it uses a hyperbolic function to gradually remove the coefficients of polynomials with higher-order interactions. A power function governed by the parameter $h$ varied within the range [0, 1] is used to truncate the terms, as detailed by Blatman and Sudret (2010). This approach ensures the expansion remains computationally feasible and it was demonstrated that sufficient accuracy can be reached for $h \geq 0.4$, while for $h = 1$ the equation reduces to the standard truncation scheme.

The computation of the unknown coefficients was carried out following two methods. First, a least square minimisation (LSM) was implemented as:

$$\boldsymbol{\alpha} = \arg\min \left\| \boldsymbol{y}^{(i)} - \sum_{k=1}^{K} \alpha_k \psi_k\left(\boldsymbol{x}^{(i)}\right) \right\|_N^2 \quad (3)$$

where the coefficients minimise the quadratic error between the original model evaluations, $\boldsymbol{y}^{(i)}$, and the PCE functional approximation.

As alternative method, the least angle regression (LAR) approach was also implemented. The method aims at considering only the polynomials that have the largest influence on the PCE performance, by discarding the non-significant terms. The method is based on an iterative procedure. A PCE of zero-order is first generated and the accuracy score $Q^2$ is computed as:

$$Q^2 = 1 - \frac{\frac{1}{N}\sum_{i=1}^{N}\left[\boldsymbol{y}^{(i)} - g\left(\boldsymbol{x}^{(i)}\right)\right]^2}{Var(\boldsymbol{Y})} \quad (4)$$

The PCE equation is then enriched by increasing the order of the polynomials, eventually retaining those terms which significantly increase $Q^2$, based on a cut-off value, $\varepsilon_{cut}$. The operation continues until $c = \min(K, N\text{-}1)$ coefficients have been entered in the equation. This procedure allows to exploit larger degrees of multivariate polynomials, if required, without recurring to a priori removal of the terms with only the hyperbolic scheme implemented. A detailed description of the procedure can be found in Blatman and Sudret (2010).

### 2.2.2 PCE accuracy measure

The metamodel performance was assessed through the accuracy score $Q^2$ using the Leave-One-Out (LOO) cross validation method. In this method, the PCE is firstly generated with a training sample of size $N$-1. The $i^{th}$ discarded input combination, $x^{(i)}$, is then used to generate the PCE prediction which is compared to the original model evaluation, $y^{(i)}$. The procedure is iterated over the sample of size $N$ for computing $Q^2$.

# 3 TRAINING METAMODELS

## 3.1 PCE types

The case study of the plate anchor described in Section 2 consists of $n = 3$ input variables and $N_\gamma$ as problem output. The training database for the PCE development consisted of $N = 128$ input/output pairs.

The statistics of the three input variables were analysed to determine the PDF that best-fits each distribution, as shown in Figure 1 with black curves. This allowed the association of each input variable with the relevant polynomial family.

The relative density was rather homogeneous over its range and a uniform distribution was adopted to fit the data (Figure 1a), with Legendre polynomial basis used for this variable. The same choice was made for the polynomials associated with the plate diameter $D$, which was described by a log-uniform PDF (Figure 1c). An inverse Gaussian distribution was used to fit the statistics of the embedment ratio (Figure 1b), with parameters $\mu$ and $\lambda$ representing the mean and shape parameter of the PDF, whose general form is:

$$p(x) = \sqrt{\frac{\lambda}{2\pi x^3}} \exp\left[-\frac{\lambda(x-\mu)}{2\mu^2 x}\right] \quad (5)$$

As detailed in Figure 1b, values of $\mu = 0.508$ and $\lambda = 8.326$ were calibrated to fit the distribution, and the polynomials of the Hermite family were associated to this variable.

The other mathematical features of the PCE have been opportunely implemented and combined for considering different metamodelling strategies. Four PCE types were employed in the study, as reported in Table 1, combining the two truncation schemes and regression methods. When the hyperbolic scheme was implemented, the parameter $h$ was iteratively adjusted from 0.2 to 0.9 in steps of 0.1, while the cut-off for the LAR was set to $\varepsilon_{cut} = 1*10^{-4}$.

*Table 1. PCE types.*

| PCE name | Truncation scheme | Regression method |
|---|---|---|
| $PCE_1$ | standard | LSM |
| $PCE_2$ | hyperbolic | LSM |
| $PCE_3$ | standard | LAR |
| $PCE_4$ | hyperbolic | LAR |

## 3.2 Results

Figure 2 shows the accuracy scores of the four developed PCEs, calculated using the LOO cross-validation method, and plotted as function of the total degree, $p$, of the polynomials implemented in Eq. 1.

The performance of $PCE_1$ rapidly increases up to $p = 3$, with a reached peak of $Q^2 = 0.961$, before sharply declining for higher degrees. This was expected as the LSM regression method with the standard truncation and limited training size did not support reliable solutions for larger $p$.

$PCE_3$ also using LSM, but with the hyperbolic truncation scheme, showed a similar trend. The latter allowed for the exploitation of high-order polùynomials, however an accuracy drop was observed when $p > 9$, with a best performance computed as $Q^2 = 0.962$, at $p = 5$ and $h = 0.7$.

$PCE_3$ and $PCE_4$ featured the LAR approach for the computation of the coefficicents which supported higher order polynomials, as highlighted in Figure 2b. The accuracy of $PCE_3$ stabilised after peaking at $Q^2 = 0.954$ for $p = 3$. Conversely, $PCE_4$ constantly improved its accuracy when high-order polynomials were implemented, reaching the highest accuracy $Q^2 = 0.961$ with $p = 21$ and $h = 0.4$.

The $N_\gamma$ predicted by the best-performing metamodels were compared to the database observations (Figure 3a, 3b, 3c, and 3d). The four PCE showed similar and good performances as evidenced by the high and rather unvaried $Q^2$. On a side, the measures reflect the statistics of the database used to calibrate the metamodels, with relatively high coefficient of variation and variance of the output (i.e. $COV(N_\gamma) = 0.749$, and $Var(N_\gamma) = 73.05$) if compared to the reduced number of data, noting the second term enters in the accuracy measure of Eq. 4.

The dashed lines in Figure 3 represents bounds of 20% of the prediction errors, while the red dots highlight the estimations with errors larger than 50%. The highest residuals were observed at low anchor capacity (i.e. $N_\gamma < 10$), and mainly for anchors with small embedment ratio ($H/D < 2$). $PCE_3$ –which was the worst-performing metamodel – experienced the larger number of bad predictions, with nine cases computing errors > 50%, but this reduced to two cases for $PCE_4$ (six for $PCE_1$, and seven for $PCE_2$). However, the average of all the prediction errors was consistently about 1% for the four PCEs, with a maximum deviation of 23.3% calculated for $PCE_3$ reducing to 17.6% for $PCE_4$.

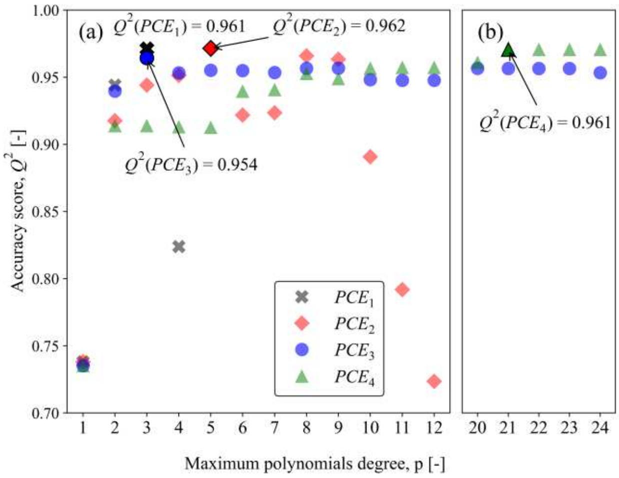


*Figure 2 – Accuracy of the PCE metamodels as function of the maximum degree of the polynomials.*

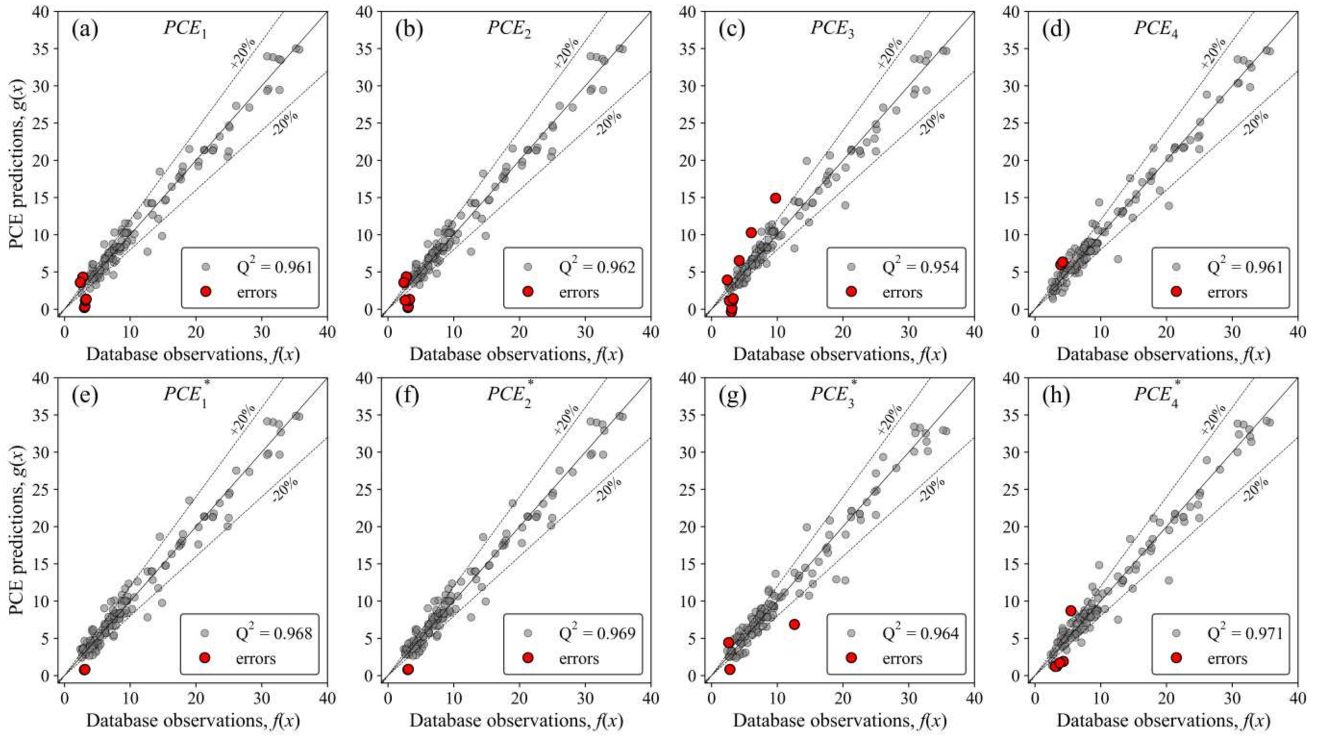

Figure 3 – PCE metamodels predictions Vs measurements of the original and enhanced databases for: (a) $PCE_1$; (b) $PCE_2$; (c) $PCE_3$; (d) $PCE_4$; (e) $PCE_1^*$;(f) $PCE_2^*$;(g) $PCE_3^*$;(h) $PCE_4^*$

## 4 IMPROVING PCE PERFORMANCES

The accuracies computed for each PCE showed that the developed metamodels can be used as quick tools to estimate plate anchor capacity. However, some high residuals were observed with errors larger than 50%. A reasonable strategy for improving the PCE accuracy could be to separate the available data into different clusters (e.g. shallow and deep anchors). However, due to the limited amount of available data, this approach did not result in a clear improvement in computed accuracies.

Nevertheless, as demonstrated in the following, integrating new, strategically selected data into the training database proved beneficial in enhancing PCE accuracy. The new training data involved four additional FE simulations using the FE model validated by Roy et al. (2021b), as summarised in Table 2. The input variables were opportunely selected to both fill the gap of the database ($D$ = 1.5m, and $D$ = 4m), and to aid the PCE performance (e.g. $H/D < 2$).

*Table 2. Additional FE simulations: inputs and results.*

| $D$ | $H/D$ | $R_D$ | $N_\gamma$ |
|---|---|---|---|
| 2 | 1.4 | 77.5 | 4.28 |
| 1.5 | 1.8 | 40 | 3.74 |
| 4 | 2.5 | 80 | 7.06 |
| 4.5 | 1.8 | 35 | 4.23 |

New metamodels, named $PCE_i^*$, were developed considering the enlarged database and the results of the best-performing $PCE_i^*$ were shown in Figure 3e, 3f, 3g, and 3h. In general, their accuracies were slightly improved with computed $Q^2$ ranging from 0.964 for $PCE_3^*$, to 0.971 for $PCE_4^*$ (versus $Q^2 = 0.954$ for $PCE_3$ and $Q^2$ = 0.961 for $PCE_4$). The case of $PCE_3$ was specifically analysed in Table 3. The five worst predictions of $PCE_3$ were reported, showing errors larger than 60% and up to 111%. The same metamodel, developed with the enlarged database (i.e. $PCE_3^*$), was able to reduce the maximum error of the predictions to 39.7%, with three over five cases reduced to errors lower than 20%. It is worth noticing that the worst prediction of $PCE_3$ was reduced to an almost nill error in $PCE_3^*$, as this case was specifically addressed by the first new run reported in Table 2.

*Table 3. Five worst predictions of $PCE_3$ with original ($N$ = 128) and enlarged ($N^*$ = 132) database.*

| **Input and output** | | | | **Predictions and errors** | | | |
|---|---|---|---|---|---|---|---|
| $R_D$ [%] | $H/D$ [-] | $D$ [m] | $N\gamma$ [-] | $PCE_3$ [-] | error [%] | $PCE_3^*$ [-] | error [%] |
| 70 | 1.05 | 2 | 3.01 | -0.33 | -111 | 3.00 | -0.4 |
| 70 | 1.05 | 1 | 3.09 | 0.104 | -97 | 3.39 | 9.8 |
| 48.7 | 3.9 | 0.6 | 6.04 | 10.29 | 70.4 | 7.83 | 29.6 |
| 47.4 | 1.9 | 0.6 | 2.39 | 3.95 | 65.1 | 3.34 | 39.7 |
| 85 | 1.05 | 2 | 3.17 | 1.15 | -64 | 2.65 | -16.4 |

The improved PCE* still computed prediction errors exceeding 50%, but this occured in only two cases. This was further reduced to a single case for $PCE_1^*$ and $PCE_2^*$ (Figure 3). However, a worse trend was observed for $PCE_4^*$, where the number of poor predictions increased to four cases, with a maximum error of 60.3%. Despite this, $PCE_4^*$ still achieved the highest global accuracy score, with $Q^2 = 0.971$.

## 5 CONCLUSION

The paper investigated the use of PCE metamodelling technique for analysing uplift capacity of circular plate anchors in sand. A database from multiple sources (centrifuge and FE results) was used to train four different PCE, which were developed by combining two truncation schemes and two regression methods.

The study showed that PCE can replicate the original model responses with good accuracy, providing reliable estimations of anchor capacity. Accuracy scores ranging from 0.954 to 0.962 were computed for the four developed PCE. However, nine predictions exceeded 50% errors in the worst case (i.e. $PCE_3$), and a new set of FE run was carried out to increase the training database and improve the metamodel performance. With just four newly-added data, all PCE showed consistent global accuracy improvements, with accuracy scores now ranging from 0.964 to 0.971. In particular, $PCE_3^*$ reduced its worst predictions to only two cases.

A key conclusion is that a PCE metamodel – opportunely trained with numerical and/or experimental data – can provide quick and reliable estimates of circular anchor uplift capacity in sand, without the need for costly experimental investigations or time-consuming FE run. The PCE metamodel could be particularly useful in the preliminary design stage, where limited information is available and rapid decisions are required. The number of bad predictions (i.e., errors larger than 50%) indicates that the developed metamodel is not yet suitable for improving current design practice. Additionally, an ideal size of the database required to train suitable PCE metamodels cannot be determined a priori, as it depends on the non-linearity of the predicted outcomes. However, the PCE accuracy can be further enhanced by incorporating new data made available from future studies, ensuring continuous improvement and adaptability.

## AUTHOR CONTRIBUTION STATEMENT

**Mentani A.**: Methodology, Formal Analysis, Writing- Original draft, Funding acquisition. **Roy A.**: Data curation, Writing- Original draft. **Chow S.**: Conceptualization, Writing- Reviewing and Editing. **Kurniadi R.**: Data curation.

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## REFERENCES

Blatman, G., & Sudret, B. (2010). Adaptive sparse polynomial chaos expansion based on least angle regression. *J. Comput. Phys.*, 230: 2345-2367. https://doi.org/10.1016/j.jcp.2010.12.021

Giampa, J.R., Bradshaw, A.S., & Schneider, J.A. (2017). Influence of Dilation Angle on Drained Shallow Circular Anchor Uplift Capacity. *Int. J. Geomech., 17*(2), 04016056. https://doi.org/10.1061/(ASCE)GM.1943-5622.0000725

Hao, D., Wang, D., O'Loughlin, C., Gaudin, C. (2019). Tensile monotonic capacity of helical anchors in sand: interaction between helices. *Can. Geotech. J., 56*. https://doi.org/10.1139/cgj-2018-0202

Kurniadi, R., Roy, A., Chow, S.H. & Cassidy, M.J. (2025). Drained uplift capacity of deeply embedded horizontal circular plate anchors in sand. Manuscript submitted for publication.

Mentani, A., Govoni, L., Bourrier, F., & Zabatta, R. (2023). Metamodelling of the load-displacement response of offshore piles in sand. *Comput. Geotech.* 159(6):105490. https://doi.org/10.1016/j.compgeo.2023.105490

Mentani A., Govoni L., Gaudin C., Watson P., Tian Y. (2025) Predicting the uniaxial capacity of plate anchors in spatially variable clay using metamodels. *Comput. Geotech.* 182:107157. https://doi.org/10.1016/j.compgeo.2025.107157

Merifield, R., Lyamin, A., & Sloan, S. (2006). Three dimensional lower bound solutions for the stability of plate anchors in sand. *Géotechnique*, 56(2):123-132. https://doi.org/10.1680/geot.2006.56.2.123

Rasulo, M. P., Schneider, J. A., Newgard, J. T., & Aubeny, C. P. (2017). Transition depths for deep circular anchors in saturated sand. *Offshore Site Investigation Geotechnics 8th Int. Conf. Proc.* 987-994.

Roy, A., Chow, S.H., O'Loughlin, C.D., Randolph, M.F. (2021a). Towards a simple and reliable method for calculating uplift capacity of plate anchors in sand. *Can. Geotech. J., 58*(9), 1314-1333. https://doi.org/10.1139/cgj-2020-0280

Roy, A., Chow, S.H., O'Loughlin, C.D., Randolph, M.F., Whyte, S. (2021b). Use of a bounding surface model in predicting element tests and capacity in boundary value problems. *Can. Geotech. J., 58*(6), 782-799. https://doi.org/10.1139/cgj-2019-0841

Sudret, B. (2008). Global sensitivity analysis using polynomial chaos expansions. *Reliab. Engng. Syst. Saf.* 93(7): 964–979. https://doi.org/10.1016/j.ress.2007.04.002