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Application of an extended Kalman filter for obtaining low-strain DST shear damping ratio estimates

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ABSTRACT: The so-called FMDSMAA algorithm was developed for processing Downhole Seismic Testing (DST) datasets so that low-strain shear damping ratio (η_s) estimates could be obtained. The low-strain DST η_s values serve as reference values for laboratory test such as the Resonant Column Test (RCT). RCT can suffer from various disadvantages such as sample disturbance and sample preparation effects which can shift the estimated η_s values. The RCT results can be adjusted so that the low strain RCT η_s estimates agree with the low strain in-situ DST η_s estimates. Low strain η_s estimates are also very important for predicting and assessing ground amplification during earthquakes. The *FMDSMAA* algorithm takes into account source wave's true raypath, geometric spreading, apparent attenuation (due to mode conversion, reflection-refraction at an interface) and material losses (intrinsic attenuation or absorption). The *FMDSMAA* algorithm also addresses limitations of the spectral ratio technique such as inaccurate raypath assumptions and significant spectral ratio estimation sensitivities. An essential part of the *FMDSMAA* implementation is to identify seismic traces with either poor trace metrics or nonsensical Peak Particle Accelerations (PPAs) indicative of a nonconstant source energy output (e.g., plate slippage, poor trace quality and\or poor or variable hammer impacts). The *FMDSMAA* algorithm was initially implemented where traces were iteratively dropped due to nonsensical PPAs values (e.g., not decreasing with depth) and large *FMDSMAA* residually errors. To address this requirement a new algorithm was developed which incorporates an Extended Kalman Filter (EKF) into the *FMDSMAA* algorithm. The EKF applies a multicomponent exponential best fit to all the measured and normalized PPAs of a DST profile (so-called *EKFAE* algorithm). A multicomponent exponential best fit is utilized to ensure that the PPAs decrease with depth in case of significant measurement errors. An EKF is required because the measurement equation is nonlinear. This paper outlines the associated mathematical governing equations of the *EKFAA* algorithm. In addition, a real data example is provided which demonstrates the effectiveness of the *EKFAE* and *FMDMSAA* algorithms when processing DST datasets.

Keywords: downhole seismic testing; absorption analysis; optimal estimation; extended Kalman filter; test bed simulation.

1 INTRODUCTION

Downhole Seismic Testing (DST) techniques such as the Seismic Cone Penetration Test (SCPT) are utilized to determine the low strain viscous shear damping ratio (η_s) which is used as a reference for values derived with the Resonant Column Test (RCT) and for predicting and assessing ground amplification during earthquakes (Stewart and Campanella, 1993). The shear damping ratio is directly proportional to the absorption of a seismic wave as it travels through a soil profile. The soil acts as both an attenuator and low pass filter as a seismic wave travels through it. Attenuation of a seismic wave propagating in soils is the decay of the wave amplitude in space (Aki and Richards, 2002). Total attenuation arises from apparent attenuation (due to mode conversion, reflection-refraction at an interface), geometric spreading (due to the change in wave front), and material losses (absorption) where the source wave motion is gradually absorbed by the medium .

Baziw and Verbeek (2019) developed the Forward Modelling Downhill Simplex Method Absorption Analysis (*FMDSMAA)* algorithm for estimating low strain η_s values from DST data sets. The *FMDSMAA* is part of proprietary software and has numerous advantages over the commonly applied spectral ratio technique (Rebollar, 1984). The *FMDSMAA* algorithm utilizes several estimated in-situ parameters (source wave travel paths, density, interval velocities, and source wave amplitudes) when estimating absorption values. The algorithm (1) raypaths, which adhere to Fermat's principle and Snell's law, aree taken into account, (2) takes the soil structure into account as up to eight absorption values (eight layers) are estimated simultaneously, (3) the algorithm is applied in the time domain, which makes it less susceptible to measurement noise or source wave interference compared to frequency domain techniques such as the spectral ratio technique, (4) has the ability to estimate the geometric spreading exponent and (5) provides an error estimate.

The performance of the *FMDSMAA* algorithm was initially demonstrated (Baziw and Verbeek, 2019) by processing challenging test bed simulations. Subsequently, the algorithm was implemented on real DST data sets (Baziw and Verbeek, 2021). The *FMDSMAA* algorithm was initially implemented where traces were iteratively dropped due to nonsensical Peak Particle Accelerations (PPAs) values (e.g., not decreasing with depth (Baziw, 2024)) and large *FMDSMAA* residually errors.

This paper presents a new algorithm was developed which incorporates an Extended Kalman Filter (EKF (Gelb, 1978; Arulampalam et al., 2002) into the *FMDSMAA* algorithm. The EKF mitigates the requirement of iteratively dropping seismic traces. The EKF incorporated into the *FMDSMAA* algorithm is referred to as the so-called *EKFAE* (EKF Amplitude Estimation) algorithm.

2 MATHEMATICAL BACKGROUND AND *EKFAE* ALGORITHM FORMULATION

2.1 *FMDSMAA* Algorithm

The *FMDSMAA* algorithm implements an Iterative Forward Modelling (IFM) technique where synthesized source amplitudes and their associated ratios are calculated based on assumed absorption coefficients and taking in account source wave travel paths, soil density, and interval velocities. The Root Mean Square (RMS) difference between the synthesized and the measured amplitude ratios are minimized utilizing an IFM technique (iteratively adjusting absorption values). The estimated absorption values are defined as the values where a user specified IFM RMS minimum error residual threshold has been met.

2.2 Extended Kalman Filter

The Kalman Filter (KF) is a Bayesian recursive estimation filtering technique based on state-space, time-domain formulations of physical problems (Gelb, 1974; Arulampalam et al., 2002; Baziw, 2007). The KF requires that the dynamics of the system and measurement model be describable in a linear mathematical representation and probabilistic form that uniquely define the system behaviour. The KF is commonly referred to as a minimum variance estimator. The Extended KF (EKF) is implemented for the case when either the system and/or measurement model is nonlinear. In the EFK it is required to take the partial derivatives of the nonlinear model with respect to the states being estimated (Taylor series approximation (Gelb, 1974)).

The EKF equations for the case of nonlinear measurement equations are outlined in Table 1.In Table 1 $\boldsymbol{x_k}$ denotes the state to be estimated, $\boldsymbol{F_{k-1}}$ denotes the state transition matrix which describes the system dynamics, $\boldsymbol{u_{k-1}}$ the process or system noise (model uncertainty), $\boldsymbol{G_{k-1}}$ describes the relationship between $\boldsymbol{x_k}$ and $\boldsymbol{u_{k-1}}$, and $\boldsymbol{h_k}$ is a non-linear function describing the relationship between the state and the available measurement. $\boldsymbol{H_k}$ is the linearized measurement matrix.

2.3 *EKFAE* Algorithm Formulation

The *EKFAE* algorithm applies a multicomponent exponential best fit to all the measured and normalized PPAs of a DST profile. This is similar to the *DSTPolyKF* algorithm (Baziw and Verbeek, 2022) for best fitting arrival time data sets with high order polynomials. In the *EKFAE* algorithm case the best fit function is required to be a decreasing function (i.e., amplitude decreases with depth (Baziw, 2024)). The *EKFAE* algorithm best fit function and corresponding EKF measurement equation are outlined below:

$$h(d) = \sum_{i=1}^{N} e^{-d|\alpha_i|} \Big/ N \qquad (12)$$

$$z_k = (e^{-d|\alpha_1|} + \cdots + e^{-d|\alpha_N|})/N + v_k \qquad (13)$$

In Equation (12), d is the receiver depth (m) and parameters $\alpha_{i\ (i\ =\ 1\ to\ N)}$ (1/m) are the states to be estimated within the *EKFAE* algorithm. Equation 13 is the nonlinear EFK measurement equation defined by Equation 2 in Table 1 and $d = k\Delta$ where Δ is the sampling rate (m).
Equation 14 show the states that need to be defined for estimating the nth degree exponential coefficients outlined in Equations (12) and (13) within the *EKFAE* algorithm.

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} \equiv \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{n-1} \\ \alpha_N \end{bmatrix} \qquad (14)$$

The discrete system equation (Equation1) is given as

Table 1. EKF Governing Equations

DESCRIPTION	Mathematical Representation	Eq.
System equation	$x_k = F_{k-1}x_{k-1} + G_{k-1}u_{k-1}$	*(1)*
Measurement equation	$z_k = h_k(x_k) + v_k$	*(2)*
State estimate extrapolation	$\hat{x}_{k\|k-1} = F_{k-1}\hat{x}_{k-1\|k-1}$	*(3)*
Error covariance extrapolation	$P_{k\|k-1} = F_{k-1}P_{k-1\|k-1}F_{k-1}^{T} + G_{k-1}Q_{k-1\|k-1}G_{k-1}^{T}$	*(4)*
Measurement extrapolation	$\hat{z}_k = h_k(\hat{x}_{k\|k-1})$	*(5)*
Innovation	$\Delta_k = z_k - \hat{z}_k$	*(6)*
Variance of innovation	$S_k = H_k P_{k\|k-1} H_k^T + R_k$	*(7)*
Kalman gain matrix	$K_k = P_{k\|k-1} H_k (S_k)^{-1}$	*(8)*
State estimate update	$\hat{x}_{k\|k} = \hat{x}_{k\|k-1} + K_k \Delta_k$	*(9)*
Error covariance update	$P_{k\|k} = [I - K_k H_k] P_{k\|k-1}$	*(10)*
Linearized measurement matrix	$H_k = \left. \frac{\partial h(x(t),t)}{\partial x(t)} \right\|_{x(t)=\hat{x}_{k-1\|k-1}}$	*(11)*

In (1) and (2*)* $\boldsymbol{v}_k$ and $\boldsymbol{u}_k$ are *i.i.d* Gaussian zero mean white noise processes with variances of $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$, respectively (i.e., $v_k \sim N(0, R_k)$ and $u_k \sim N(0, Q_k)$).

$$\begin{bmatrix} x_{1k+1} \\ \vdots \\ x_{(n)k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1k} \\ \vdots \\ x_{(n)k} \end{bmatrix} \qquad (15)$$

The linearized measurement matrix, H_k, is given by the following equation

$$H_k = \left[-d\frac{\alpha_1}{N|\alpha_1|}e^{-d|\alpha_1|} \quad \cdots \quad -d\frac{\alpha_N}{N|\alpha_N|}e^{-d|\alpha_N|} \right] \qquad (16)$$

3 *EKFAE* ALGORITHM PERFORMANCE ASSESSMENT

The *EKFAE* algorithm was implemented on the real onshore SCPT data set processed by Baziw and Verbeek (2021). That data set outlined the estimated absorption values where seismic traces with nonsensical PPAs and relatively high residual errors were iteratively dropped. This process was somewhat laborious and resulted in several traces being dropped.

Figure 1 illustrates filtered (200Hz low pass) absolute value full waveforms and the associated PPAs for SH source wave (horizontally polarized shear wave) data acquired on the Right Side (RS) of the seismic sensor. The RS refers to applying a SH hammer impact on the RS of the sensor as opposed to the left side of the sensor. Source feature isolation was applied to the trace recorded at 5m so that it aligned with the trending peak. Table 2 outlines the estimated arrival times (reference time corrected to 55ms) and normalized PPAs for the traces illustrated in Fig 1.

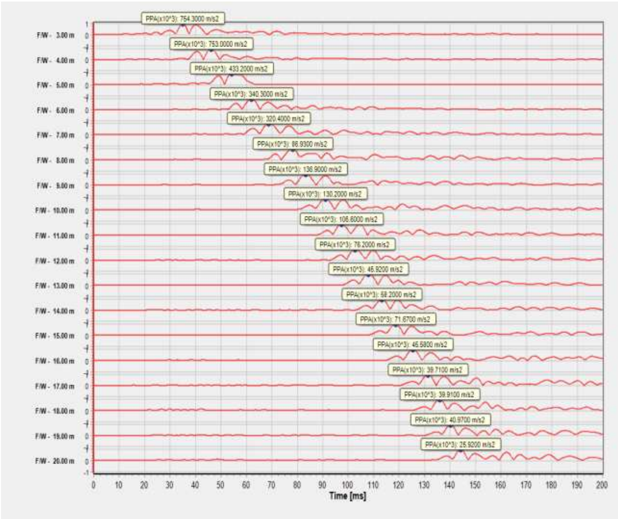

Figure 1 - Full waveforms with maximum PPAs illustrated (Baziw and Verbeek, 2021).

Table 2. Arrival times and normalized PPAs for traces illustrated in Fig 1.

Depth [m]	Arrival time [ms]	Normalized PPAs
3	33.2672	1
4	39.2034	0.99838
5	47.0917	0.57438
6	55	0.45121
7	61.972	0.4248
8	70.9361	0.11526
9	76.3444	0.18155
10	84.153	0.17259
11	90.4378	0.14131
12	95.7365	0.10102
13	101.0353	0.06088
14	106.2743	0.07715
15	111.6527	0.09503
16	118.4554	0.06046
17	124.3716	0.05265
18	128.8935	0.05292
19	133.2162	0.05432
20	137.2301	0.03437

Figure 2 illustrates the *EKFAE* (thick blue line) best fit (0.5m resolution) for the normalized amplitudes (green dots) outlined in Table 2. Parameter *N* in Equations 12, 13 and 16 was set to 8. The EKF measurement error variance (R_k in Equation 7) was set to 100 times higher (based on minimizing residual errors) for the data acquired at 4m, 7m and 8m compared to other depths. Figure 2 also illustates the eigth associated exponential functions.

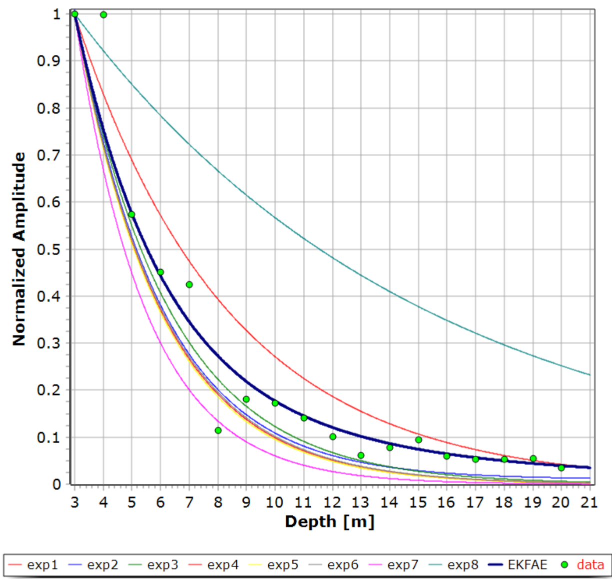

Figure 2 – PPAs (green circles), first run EKFAE eight estimated exponential and corresponding EKFAE optimal estimate (dark blue line).

The *FMDSMAA* algorithm was initially implemented on the *EKFAE* best fit estimates illustrated in Fig. 2 and the corresponding *FMDSM* interval velocities and densities outlined in Table 3. Simlar to the FMDSMAA algorithm, the *FMDSM* utilized iterative forward modeling to estimate DST interval velocities. Both algorithms implement iterative forward modelling. Column 2 of Table 3 outlines the estimated *EKFAE* normalized PPAs illustrated in Fig. 2. Column 4 of Table 3 (Baziw and Verbeek, 2019 and 2021) provides the corresponding estimated *FDMSDM* interval velocities derived from the arrival time estimates given in Table 2. Column 3 of Table 3 contains the estimated soil densities derived from CPT cone resistance, sleeve friction and pore pressure measurements and data interpolation.

The *FMDSMAA* minimum and maximum limits placed on the Q values were set to 4 $1/N_p$ and 33 $1/N_p$, respectively (N_p denotes Nepers). These minimum and maximum values were based upon typical field measurements of soil damping (Stewart and Campanella, 1993) The first run *FMDMSAA* output is illustrated in columns 5 to 8. The first run error residual at 9m was significantly large. This was attributed to a very poor measured PPA value at 8m and relatively low estimated interval velocity for the interval between 8m and 9m; therefore, the trace at 8m was dropped for the second run of the *FMDSMAA* algorithm where the output is illustrated in Table 4. The error residuals for the second run were significantly low down to a depth of 10m as outlined in Table 4 column 8. Depths 10m to 20m had relatively higher error residuals; therefore, it was decided to carry out a 3rd run of the *FMDMSAA* algorithm utilizing an *EKFAA* best fit for depths 10m to 20m. Figure 3 illustrates the second run *EKFAE* (thick blue line) best fit for the normalized amplitudes (green dots) outlined for depths 10m to 20m. Parameter N in Equations 12, 13 and 16 was again set to 8.

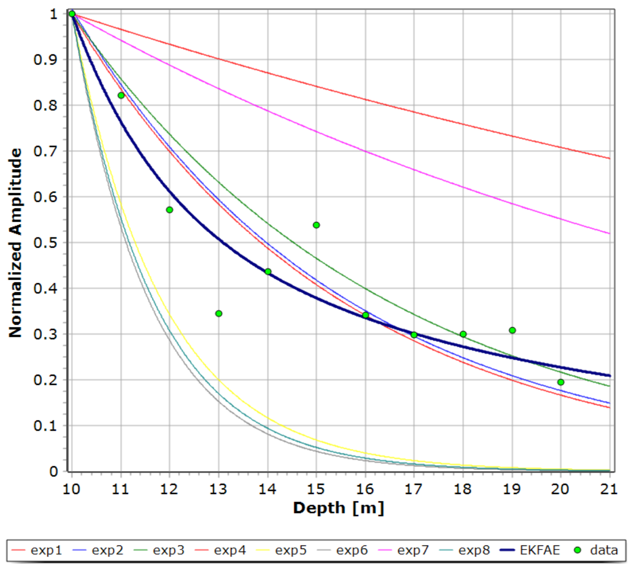

Figure 3 – PPAs (green circles), second run EKFAE eight estimated exponential and corresponding EKFAE optimal estimate (dark blue line).

Table 5 outlines the corresponding *FMDSMAA* estimates of absorption (α), *Q*, damping ratio (η_s = *1/2Q*), and amplitude ratio residual for the *FMDSMAA* second run (results in black) and third run (results highlighted in red). In the *FMDSMAA* second run the depth interval 3m to 10m was treated as a half space. As is evident from Table 5, the combined estimated values have very low associated error residuals for depths 3m to 16m. The larger *Q* values for depths 17m to 20m are consistent for increasing interval velocities and deeper soil layers.

4 CONCLUSIONS

The *FMDSMAA* algorithm was developed for estimating low strain η_s values from DST data sets.

The *FMDSMAA* algorithm was initially implemented where traces were iteratively dropped due to nonsensical Peak Particle Accelerations (PPAs) values. This proved to be a time consuming and somewhat arbitrary process. To overcome this process a new technique was the developed, the so-called *EKFAE* algorithm, which utilizes an extended Kalman filter. The *EKFAE* algorithm applies a multicomponent exponential best fit to all the measured and normalized PPAs of a DST profile. This assured that the PPAs values inputted into the *FMDSMAA* algorithm decrease with depth and facilitates the ability to utilize all PPAs in obtaining optimal estimates of low strain DST η_s values. This paper outlined the mathematical details of the *EKFAE* algorithm and provided the results from processing a real data DST data set. The performance of the *FMDSMAA* algorithm with incorporation of the *EKFAE* algorithm proved promising. It is the intention of the authors to carry out further test of the *EKFAE* algorithm and develop enhancements such as constraining the filter for known and/or highly accurate PPA values.

AUTHOR CONTRIBUTION STATEMENT

Erick Baziw: Conceptualization, software, formal analysis, writing-original draft preparation. **Gerald Verbeek**: writing - review and editing.

Table 3. FMDSMAA first run output after inputting the estimates from the estimated arrival times, FMDSM and EKFAE algorithms..

Depth [m]	***EKFAE* Normalized PPAs**	**Density (ρ) [kg/m^3]**	**Interval Velocity [m/s]**	**Absorption (α) [1/m]**	**Q [1/ Np]**	**Damping (η_s) [% Np]**	**Amplitude Ratio Residual**
3	1	1661	108.4	0.5772	5	12.5	N/A
4	0.75235	1726	135	0.11398	16.31	3.07	7.34E-07
5	0.57346	1637	115.5	0.21147	10.32	4.84	1.92E-06
6	0.44197	1757	118.4	0.08722	24.34	2.05	4.34E-06
7	0.34471	1764	135.1	0.06981	26.63	1.88	1.99E-06
8	0.27225	1734	108.2	0.22715	10.24	4.88	1.23E-07
9	0.21782	1841	175.5	0.04348	32.99	1.52	0.149
10	0.17658	1763	124.9	0.27654	7.28	6.87	4.83E-06
11	0.14502	1812	155.2	0.04913	32.96	1.52	0.0518
12	0.12062	1836	183.4	0.04158	32.99	1.52	0.023
13	0.10154	1819	184.2	0.06959	19.63	2.55	0.0243
14	0.08644	1850	187.1	0.06908	19.43	2.57	0.00405
15	0.07434	1833	183	0.06981	19.65	2.54	0.0256
16	0.06454	1744	145.4	0.20779	8.31	6.02	1.61E-05
17	0.05648	1725	167.6	0.04556	32.99	1.52	0.0332
18	0.0498	1525	218.7	0.03488	32.99	1.52	0.0308
19	0.04418	1541	228.4	0.03965	27.7	1.81	2.96E-06
20	0.0394	1601	246.1	0.03091	33	1.52	0.021

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Table 4. FMDSMAA second run output after inputting the estimates from the estimated arrival times, FMDSM and EKFAE algorithms..

Depth [m]	***EKFAE* Normalized PPAs**	**Density (ρ) [kg/m^3]**	**Interval Velocity [m/s]**	**Absorption (α) [1/m]**	**Q [1/ Np]**	**Damping (η_s) [% Np]**	**Amplitude Ratio Residual**
3	1	1661	108.4	0.57527	5.02	12.44	N/A
4	0.75235	1726	135	0.1135	16.38	3.05	9.38E-07
5	0.57346	1637	115.5	0.21126	10.33	4.84	4.17E-09
6	0.44197	1757	118.4	0.0871	24.37	2.05	4.04E-06
7	0.34471	1764	135.1	0.06972	26.66	1.88	3.06E-06
9	0.21782	1841	133.9	0.11375	16.54	3.02	5.37E-06
10	0.17658	1763	125.1	0.16746	12.03	4.16	5.44E-06
11	0.14502	1812	155.1	0.05128	31.58	1.58	0.0507
12	0.12062	1836	183.3	0.0416	32.98	1.52	0.0222
13	0.10154	1819	184.1	0.08521	16.03	3.12	0.0112
14	0.08644	1850	187	0.06972	19.26	2.6	0.00368
15	0.07434	1833	182.9	0.0852	16.1	3.11	0.0122
16	0.06454	1744	145.4	0.21135	8.17	6.12	0.00342
17	0.05648	1725	167.6	0.08522	17.64	2.83	0.0657
18	0.0498	1525	218.6	0.03488	32.99	1.52	0.0307
19	0.04418	1541	228.4	0.03957	27.76	1.8	3.85E-06
20	0.0394	1601	246.1	0.03091	33	1.52	0.0209

Table 5. FMDSMAA third run output after inputting the estimates from the estimated arrival times, FMDSM and EKFAE algorithms..

Depth [m]	***EKFAE* Normalized PPAs**	**Density (ρ) [kg/m^3]**	**Interval Velocity [m/s]**	**Absorption (α) [1/m]**	**Q [1/ Np]**	**Damping (η_s) [% Np]**	**Amplitude Ratio Residual**
3	1	1661	108.4	0.57527	5.02	12.44	N/A
4	0.75235	1726	135	0.1135	16.38	3.05	9.38E-07
5	0.57346	1637	115.5	0.21126	10.33	4.84	4.17E-09
6	0.44197	1757	118.4	0.0871	24.37	2.05	4.04E-06
7	0.34471	1764	135.1	0.06972	26.66	1.88	3.06E-06
9	0.21782	1841	133.9	0.11375	16.54	3.02	5.37E-06
10	0.17658	1763	125.1	0.1182	16.61	3.01	3.91E-07
11	0.134577	1812	155.1	0.05964	27.15	1.84	3.09E-07
12	0.107714	1836	183.3	0.05226	26.25	1.9	3.03E-07
13	0.089487	1819	184.1	0.11077	12.33	4.06	6.50E-07
14	0.076495	1850	187	0.0697	19.26	2.6	3.97E-07
15	0.066788	1833	182.9	0.0841	16.31	3.07	3.12E-07
16	0.059222	1744	145.4	0.18572	9.29	5.38	3.02E-06
17	0.053119	1725	167.6	0.04556	32.99	1.52	0.0547
18	0.048058	1525	218.6	0.03488	32.99	1.52	0.0533
19	0.043775	1541	228.4	0.03341	32.99	1.52	0.0183
20	0.040092	1601	246.1	0.03091	33	1.52	0.0453