Limitations of Terzaghi consolidation theory in soft soil practice

W. David Carrier, III [1*]

[1] Argila Enterprises, Inc., Lakeland, Florida, USA

1 Introduction

All geotechnical engineers know that Terzaghi [1] developed the consolidation theory a century ago. Many consider this to be the birth of modern soil mechanics, now known as geotechnical engineering. Terzaghi developed his eponymous theory by making a series of assumptions about the behavior of clay until he arrived at a second order partial differential equation that was equivalent to the one-dimensional Fourier heat flow equation; and, hence, had a known solution. The Terzaghi assumptions were: (1) The soil is weightless (i.e., no self-weight consolidation); (2) The change in void ratio, e, is negligible (i.e., small strain); (3) The coefficient of compressibility, a_v, is constant (i.e., linear compressibility); and (4) The coefficient of permeability, k, is constant.

The consequences of those four assumptions are well known to geotechnical engineers: (1) The time rate of consolidation is proportional to the square of the length of the drainage path; i.e., H^2. Thus, if a layer of clay has both top and bottom drainage ("double drainage"), it will consolidate four times faster than if it only has top drainage ("single drainage"). (2) The degree of settlement is equal to the degree of excess pore pressure dissipation. And (3), If any specific time on the settlement curve is known, then the rest of the curve is known; i.e., all settlement-time curves are the same (non-dimensionally). Thus, the Casagrande method refers to t_{50} (the time to 50% consolidation) and the Taylor method uses t_{90}. Under Terzaghi's assumptions the ratio between the two is constant, with t_{90} / t_{50} = constant = 4.33.

Another consequence of his four assumptions is that Terzaghi was able to define the coefficient of consolidation, c_v, (analogous to the coefficient of thermal diffusivity): $c_v = k\,(1 + e)/\gamma_w\, a_v$, where γ_w is the unit weight of water. Because k, e, and a_v are assumed to be constant, c_v was also constant. However, as is well-known, the value of c_v, calculated by the Casagrande method is typically different from the value calculated by the Taylor method. That is because for real clays, t_{90} / t_{50} is not a constant, as discussed below.

Note that the Fourier heat flow equation is for a "thermally thick" layer, in which the temperature is variable both with time and with location inside the layer. About two centuries earlier, Newton developed a heat flow equation for a "thermally thin" layer, in which the temperature is uniform inside the layer, and is only variable with time. Newton's heat flow equation is a "simple", ordinary single-order differential equation; and the solution is an exponential decay, in which the ratio between the two reference times is also constant and equal to $t_{90} / t_{50} = 3.32$.

Gibson et al. [2] un-assumed all the Terzaghi assumptions and arrived at a rather large and formidable partial differential equation. To date, no practical, closed-form solutions have been derived from the Gibson et al. equation.

Somogyi [3] was the first to numerically solve the Gibson et al. equation. Using a finite-difference technique, Somogyi incorporated non-linear compressibility and permeability and non-linear geometry (i.e., large strain). For the latter, Somogyi used a Lagrangian framework instead of a Eulerian one. The author refers to this solution as large strain, non-linear (LSNL) consolidation theory.

2 Practical experience

Since 1980, the author, together with his colleagues (see acknowledgments), has applied LSNL to a variety of large volume clayey slurries: dredged material, phosphatic clay, oil sand fine tailings, alumina red mud, bauxite tailings, etc. These materials cover a range of plasticity index, PI, from 10 to 200. The high plasticity materials are, of course, extremely soft. In some cases, they exhibit soil-like behavior (i.e., response to effective stress) at a void ratio of 15, corresponding to a water content of 550% (Carrier et al. [4]; and Dunmola et al. [5]), and the self-weight consolidation settlement is as much as 50% of the initial thickness.

The experimental data was modelled by LSNL without imposing a constant value for c_v. Instead, the basic material properties, compressibility and permeability, were adopted as input functions of the soil state. Initially, we used a non-linear compressibility in the form $e = A\,(\sigma'^{B})$, where e is the void ratio, with a typical value of $B = -0.2$. For permeability the non-linear function $k = C\,e^{D}$, with a typical value of $D = 4$, was used. Since then, we have explored various expressions for compressibility and permeability.

In the course of our work, we found that the behavior of these very soft clayey slurries is very dissimilar to Terzaghi consolidation: (1) Double drainage did not quadruple the rate of consolidation. In fact, in many cases, double drainage only increased the rate by 10% to 15%. Instead of H^2, it was more like $H^{0.2}$. The reason is that very soft non-linear material on the bottom of the deposit would consolidate very quickly and form a low permeability "cake". This cake

* Corresponding author: wdcarrier@verizon.net

layer would effectively seal off any further double drainage. For most design cases, we conservatively assumed only single drainage. (2) Settlement occurred faster than pore pressure dissipation. An example is shown in Figure 1. The magnitude of the difference depends on how non-linear the material is. (3) Finally, we found that the settlement rate is not unique – it depends on the material properties and the geometry. In other words, the ratio of t_{90} to t_{50} is a variable. For that matter, this ratio for settlement is different from that of pore pressure dissipation.

Obviously, these results were non-intuitive, but they were born out by extensive lab and field testing that were run over many years, including instrumenting large ponds of clayey slurry, such that they were tantamount to gigantic consolidometers.

3 Relevance to conventional settlement analyses

The question naturally arose: how significant is LSNL consolidation theory with respect to "conventional" settlement analyses? In recent years, the author has systematically turned the Terzaghi assumptions "on" and "off" to measure the effect. There are four assumptions and therefore sixteen possible combinations of "on" and "off".

Of these sixteen possibilities, only two are of practical significance: (1) No self-weight, large strain, non-linear compressibility, and non-linear permeability; and (2) Self-weight, large strain, non-linear compressibility, and non-linear permeability. Both cases obviously require the use of LSNL consolidation theory. The former applies to the interpretation of lab oedometer or consolidometer tests; and the latter applies to the prediction and interpretation of field behavior. They are discussed in more detail below.

For the sixteen possibilities applied to a natural clay layer, it was found that non-linear permeability was only important if the clay layer experienced large strain. If it were a small strain case, it did not matter if the permeability were linear or non-linear as the value did not change enough to make a significant difference. The same was not true for the compressibility. Consequently, the number of possible combinations is reduced to eight.

Furthermore, there are two cases that are non-sensical: large strain with linear compressibility, with and without self-weight. These two cases would never be encountered in practice; and thus, the number of possible combinations is further reduced to six: three without and three with self-weight. Of the three cases without self-weight, one of them is the Terzaghi case with all four assumptions turned "on". The results of that case are already known. For the remaining two cases (large strain, non-linear compressibility, non-linear permeability; or small strain, non-linear compressibility, constant permeability), it was found that the value of "n" in H^n is equal to 2. Furthermore, it was found that the value of n is dependent on the distribution of vertical strain at the end of consolidation: If the strain is uniform throughout the clay layer, then $n = 2$; if it is not uniform, then $n < 2$. Either of these two cases without self-weight correspond to a standard oedometer or consolidometer test, because the applied stress is much greater than the "in situ" stress, such that the final vertical strain is uniform throughout the sample.

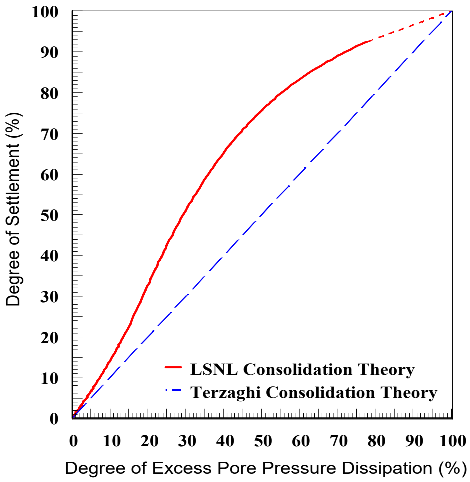

Figure 1. Typical self-weight consolidation of a clayey slurry. Settlement occurs faster than pore pressure dissipation. For example, whereas Terzaghi consolidation theory predicted 50% settlement at 50% pore pressure dissipation, LSNL predicted more than 70% settlement at 50% pore pressure dissipation. The magnitude of the "bulge" in the LSNL solution depends on how non-linear the material is.

Even though $n = 2$ in these two cases without self-weight, settlement precedes pore pressure and t_{90} / t_{50} is variable, and so, a conventional oedometer test requires the use of LSNL consolidation theory. The large strain case obviously incorporates the small strain case, and thus, the former is recommended for the interpretation of an oedometer test.

Of the three cases with self-weight, one of them would never occur in practice but is of interest from a research perspective. It is the case with small strain, constant a_v, and constant permeability. That is, the same as the Terzaghi assumptions except for self-weight. For this case, the Terzaghi results are also obtained: $n = 2$; settlement and pore pressure are identical; and $t_{90} / t_{50} = 4.33$. In other words, Terzaghi originally had to assume no self-weight to obtain a mathematical solution (i.e., the Fourier heat flow equation), but that assumption turns out to be superfluous.

Finally, the remaining two cases with self-weight apply to all practical, field cases. These two cases are: small and large strain, non-linear compressibility, and non-linear permeability in the case of large strain. Small strain might correspond to an over-consolidated clay layer, while large strain might correspond to a normally consolidated clay layer. It was found that $n < 2$ (and n is not even constant – it is variable with time; see Figure 2); settlement precedes pore pressure; and t_{90} / t_{50} is variable.

For purposes of numerical comparison, the simplest possible geometry was considered: A layer of homogeneous Boston Blue Clay (BBC; PI = 20), overlain by a layer of overburden, onto which a surcharge is imposed with an infinite extent, replicating one-dimensional consolidation.

In the case of a layer of homogeneous normally consolidated BBC under its own weight, the initial void ratio (prior to imposition of the surcharge) will decrease with depth in the stratum. If the compressibility is non-linear, possibly described by a constant compression index, C_c, the strain due to the surcharge is non-uniform and $n < 2$. A "typical" value for n is 1.5 at t_{50} (with respect to settlement) – meaning that double drainage is about 2.8 times as fast as single drainage, not 4 times. And, of course, settlement precedes pore pressure and t_{90} / t_{50} is variable.

In the case of a layer of homogeneous over-consolidated BBC under its own weight, the initial and final void ratios are more complicated due to the stress history, and the final strain distribution is not uniform. In any case, the change in compressibility over depth is less than in the previous case, because C_r, the recompression index, is lower than C_c. A "typical" value for n is 1.8 at t_{50} – meaning that double drainage is about 3.5 times as fast as single drainage. And, of course, settlement precedes pore pressure and t_{90} / t_{50} is variable. Again, the large strain case is more general than the small strain case and is recommended for the prediction and interpretation of field behavior.

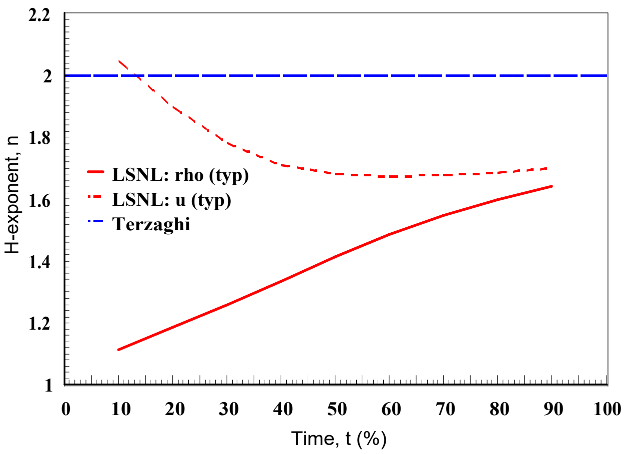

Figure 2. "Typical" results for a normally consolidated clay layer: With LSNL consolidation theory, n is a variable with time. For settlement, n increases from nearly 1 to about 1.6 at a time corresponding to about 90% degree of settlement. Whereas, for pore pressure dissipation, LSNL decreases from more than 2 (not an artifact!) to about 1.7 at a time corresponding to about 90% degree of excess pore pressure dissipation. The two LSNL curves converge to a common value at 100% consolidation. Terzaghi consolidation theory predicts n = 2 for both settlement and pore pressure, constant with time.

The previous analysis demonstrates that there are two cases of practical significance: (1) Without self-weight, large strain, non-linear compressibility, and non-linear permeability; and (2) With self-weight, large strain, non-linear compressibility, and non-linear permeability. The former applies to lab-scale tests; and the latter applies to field-scale behavior. In both cases, the use of LSNL consolidation theory is required for reliable analysis of the process.

Most practical field cases involve a two-dimensional load imposed on an over-consolidated layer of clay, underlain by normally consolidated clay. The resulting strain distribution is even more complicated than the cases considered herein. However, the results would be the same: $n < 2$ (and variable with time), the settlement precedes pore pressure and the ratio t_{90} / t_{50} is variable.

In the course of this present investigation, it was found that Taylor [6], Davis and Raymond [7], Janbu [8], and Raymond [9] obtained valuable, related results, long before the current numerical models were available.

4 Conclusions

The advantage of using Terzaghi consolidation formulation is the availability of fairly simple chart solutions. Nonetheless, the author recommends LSNL consolidation theory for stiff soils, soft soils and especially very soft soils.

The author has had the good fortune to work with, and learn from, many colleagues regarding large strain, non-linear consolidation theory and practice, including: Les Bromwell, Frank Somogyi, Berg Keshian, Bob Martin, Bob Schiffman, Bill Shaw, Don Scott, Rick Lahaie, Ed McRoberts, Robert Mahood, Dobroslav Znidarcic, Deji Dunmola, Song Jeeravipoolvarn, et al.

References

1. K. Terzaghi, *Erdbaumechanik* (Franz Deuticke, Vienna, 1925).
2. R. E. Gibson, G. L. England, M. J. L. Hussey, The theory of one-dimensional consolidation of saturated clays, Géotechnique, **17** (3), 261-273, 1967.
3. F. Somogyi, Large-strain consolidation of fine- grained slurries, Canadian Society of Civil Engineering 1980 Annual Conference, Winnipeg
4. W. D. Carrier, L. G. Bromwell, F. Somogyi, Design capacity of slurried mineral waste ponds, J. Geotech. Eng., ASCE, **109** (5), 699-716, May 1983.
5. A. Dunmola, R. A. Werneiwski, D. McGowan, W. Shaw, D. Carrier, Geotechnical performance of fine tailings in an oil sands pit lake, Can. Geotech. J., **60** (5), 595-610, May 2022.
6. D. W. Taylor, Research on consolidation of clays, MIT, Aug 1942.
7. E. H. Davis, G. P. Raymond, A non-linear theory of consolidation, Géotechnique, **15** (2), 161-173, 1965.
8. N. Janbu, Consolidation of clay layers based on non-linear stress-strain, Proc., 6th ICSMFE, Montreal, Sep 8-15, Vol. **II**, 83-87 (U of Toronto Press, 1965).
9. G. P. Raymond, Consolidation of deep deposits of homogeneous clay, Géotechnique, **19** (4), 478-494, 1969.