Low strain dynamic soil properties and maximum shear modulus equation of rubber sand mixtures

Tapan Suyal[1*] and *B.K.* Maheshwari[1]

[1] Department of Earthquake Engineering, Indian Institute of Technology Roorkee, Uttarakhand, India

1 Introduction

Crumb rubber has emerged as a sustainable method for recycling non-biodegradable rubber waste. Rubber Sand Mixtures *(RSM)* can be effectively used in geotechnical engineering applications such as backfill material, embankment construction, base isolation layers, and vibration mitigation systems. This becomes particularly important in soft soil conditions, where excessive ground vibrations induced by train movement or machine operation can adversely affect ground stability and nearby structures.

The rubber content and unit weight significantly influence the shear strength of the *RSM* [1]. The small strain shear modulus decreases with increasing rubber content, while damping ratio increases due to higher energy dissipation [2-5]. The shear modulus increases with confining pressure but decreases with rubber inclusion [6]. Studies by [7-8] on *RSM* observed reduction in pore pressure under cyclic loading.

Although numerous studies have investigated the behaviour of Rubber Sand Mixtures *(RSM)*, limited research has focused on developing analytical relationships to estimate the maximum shear modulus for varying rubber contents. In this study, resonant column tests were performed to evaluate the dynamic properties of RSM. The shear modulus and damping ratio were determined for mixtures with different rubber percentages. Furthermore, a modified empirical relationship is proposed to predict the maximum shear modulus by incorporating the effect of rubber content, and the predicted values show good agreement with the experimental results. The results indicate that RSM has significant potential as a sustainable material for vibration mitigation and ground improvement applications.

2 Experimental procedure

In this study, Solani sand, from Solani River near Roorkee, Uttarakhand, India has been used for laboratory tests. Industrial crumb rubber has been procured from the local market and can be classified as Ground rubber [9]. The particle size distribution and representative photograph of Solani sand and crumb rubber are shown in Figure 1. Solani sand can be classified as poorly graded sand *(SP)* [10]. The index properties of Solani sand and crumb rubber, including specific gravity, dry unit weight, and void ratio, are summarized in Table 1. Where, $e_s$and e_r denote the void ratio, defined as the ratio of volume of voids to the volume of solids, for Solani sand and crumb rubber whose values are taken as 0.72 and 0.32, respectively.

Table 1. Index properties of material used in study.

	C_u	C_c	G_s	$\gamma_{d\,max}$ (kN/m^3)	$\gamma_{d\,min}$ (kN/m^3)	e
Solani Sand	2.1	1.05	2.64	16.76	13.71	e_s= 0.72
Crumb Rubber	3.46	0.96	0.67	5.89	4.28	e_r= 0.32

(a) (b) (c)

Figure 1. Grain Size Distribution curve of Solani Sand and Crumb Rubber.

All tested specimens have a diameter of 50 mm, height of 100 mm, and are prepared at a relative density of 50% using the dry tamping method. For a fixed sample volume, the required mass of sand and rubber for each mixture proportion is calculated following the methodology proposed by Rios et al. [8]. To ensure uniform distribution between sand and rubber particles, the required quantities of sand and crumb rubber were first mixed until a homogeneous mixture was achieved. The mixture was then placed into the mold in equal layers and was compacted using uniform tamping applied around the mold. Sand and crumb rubber are mixed in varying proportions by weight (0%, 7.5%, 15%, and 25%). The corresponding

* Corresponding author: tapan_s@eq.iitr.ac.in

percentages of rubber by weight in rubber sand mixture and specific gravity of the mixture are provided in Table 2. The letter *R* in notation refers to the rubber and digit 0, 7.5, 15, and 25 refers to the percentage of rubber.

Table 2. Ratio of rubber sand mixture used in study.

	By Weight		$G_{s\ mix}$
Notation	Sand	Rubber 1	
R0	100	0	2.64
R7.5	92.5	7.5	2.16
R15	85	15	1.83
R25	75	25	1.52

A split mould is used for sample preparation, with a rubber membrane positioned inside the mould. The sample was prepared by filling the split mould in five equal layers, to ensure uniformity. After filling the mould, the top cap was placed, and the rubber membrane was sealed with O-rings at both the top and bottom ends to prevent leakage. Saturation of the sample was achieved by incrementally applying back pressure, maintaining a constant differential of 20 kPa between the cell pressure and the back pressure. The sample was saturated until Skempton Pore Pressure Coefficient *(B-value)* became 0.95 which is defined as in Equation (1).

$$B = \frac{\Delta u}{\Delta \sigma'_3} \tag{1}$$

Where, Δu: an increment in pore pressure referred as excess pore pressure, $\Delta\sigma'_3$: an increment in minor effective principal stress.

Followed by saturation the sample was consolidated at an effective confining pressure of 100 kPa. To evaluate dynamic soil properties at low shear strain levels (0.0001–0.1 %), resonant column tests *(RCT)* are conducted in torsional mode as per [11]. The *RCT* apparatus available in Soil Dynamics Laboratory of IIT Roorkee is used for tests shown in Figure 2. Harmonic vibrations of various frequencies are applied by the electromagnetic system and the output amplitude of acceleration in terms of voltage *(mV)* at the top of sample has been recorded by accelerometer. The frequency at which maximum amplitude occurs is recorded as resonating frequency (f). The shear modulus of the sample is then calculated using Equations (2-3).

$$V_s = \frac{2\pi f l}{\beta} \tag{2}$$

$$G = \rho V_s^2 \tag{3}$$

Where, G= Shear modulus, V_s= Shear wave velocity (m/s), f =Natural frequency of the sample as found from the resonant column test (Hz), l=Sample length (m), β= RCA constant, ρ =bulk density

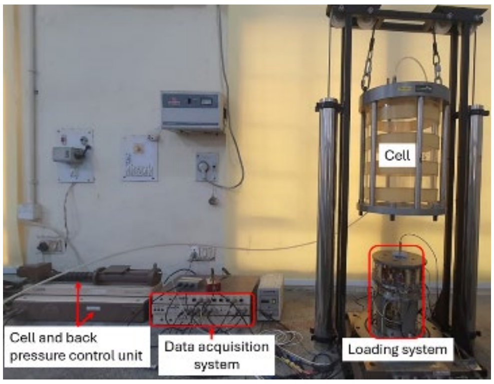
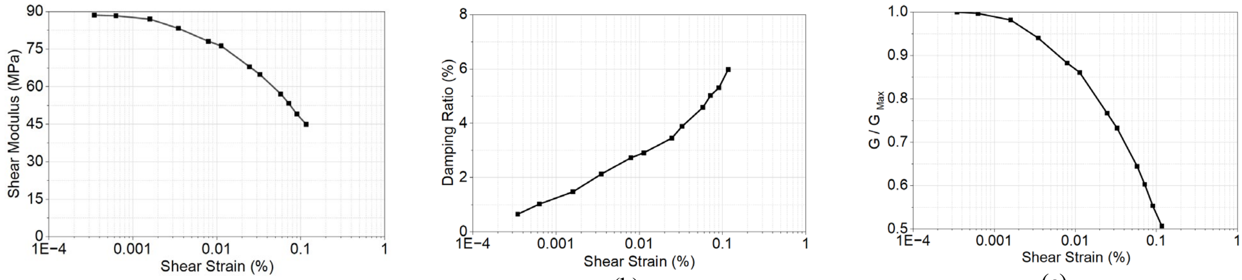


Figure 2. Resonant Column Apparatus in Soil Dynamics Laboratory, IIT Roorkee.

3 Effect of rubber inclusion on dynamic behaviour of sand

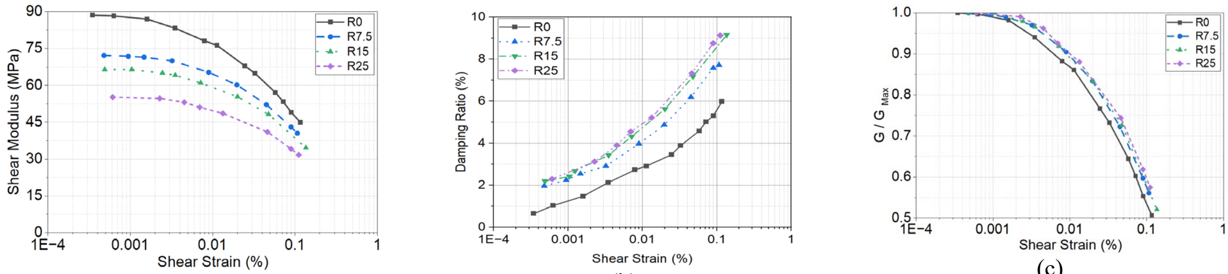
Figure 3a and 3b shows the shear modulus and damping ratio in the low strain range for pure sand. It is observed that the shear modulus decreases and the damping ratio increases progressively with increasing shear strain, which reflects the typical nonlinear behaviour of granular soils. The increase in damping ratio indicates higher energy dissipation capacity of the soil as it transitions from nearly elastic to nonlinear behaviour. At very low strains, the soil exhibits its maximum stiffness *(G_{max})*, corresponding to the elastic response of the soil. As the shear strain increases, particle rearrangement and inter-particle sliding lead to a reduction in stiffness. The shear modulus has been normalized with maximum shear modulus in the shear strain range of (10^{-4} % – 10^{-3} %) [12]. Figure 3c shows the modulus reduction curve for pure sand.

Figure 4a and 4b shows the shear modulus and damping ratio in the low strain range for different rubber sand mixture. Inclusion of rubber content reduced the shear modulus and increased the damping ratio. Increasing the rubber content up to 25 % results in further reduction in shear modulus. Damping ratio increases on increasing the rubber content but its effect becomes saturated. The damping is due to friction between the particles and the deformation of particle when the loading is applied. Rubber is more compressible as compared to sand due to which the amount of energy dissipated by deformation of particles increases by increasing the rubber content. Similar results have also been reported by [13] which finds that at low strain, strength reduces while damping increases with increasing rubber content. Figure 4c shows the modulus reduction curve for different rubber sand mixture. At any strain increasing the rubber content increases the G/G_{max} value. The elastic threshold strain (i.e., the shear strain level up to which the shear modulus remains approximately equal to G_{max} and the material exhibits linear elastic behavior) increases with increasing rubber content. For *R0* at any strain, stiffness (i.e., shear modulus) degrades more as compared to *RSM* mixture.



(a) (b) (c)

Figure 3. For pure sand (a) shear modulus, (b) damping ratio, and (c) modulus reduction curve.



(a) (b) (c)

Figure 4. For different rubber sand mixture (a) shear modulus, (b) damping ratio, and (c) modulus reduction curve.

4 Modified maximum shear modulus relationship for rubber–sand mixtures

The maximum shear modulus (G_{max}) of granular soils can be estimated using the empirical relationship proposed by Hardin and Richart [14] which relates G_{max} to void ratio and effective confining pressure as in Equation (4). However, this relationship was developed for clean sands and does not consider the influence of soft and compressible inclusions such as rubber particles. In *RSM*, the presence of rubber significantly alters the stiffness characteristics due to its lower rigidity and higher deformability. To incorporate the influence of rubber inclusion on the packing characteristics of the mixture, the concept of equivalent void ratio was adopted. Furthermore, to account for the stiffness reduction, an exponential rubber influence factor was incorporated. Based on regression analysis of the experimental data, a modified Hardin-type relationship was developed and is expressed as:

$$G_{\max(RSM)} = \mathrm{A} \times \frac{(2.973-e_{eq})^2\sqrt{\sigma'_m}\, e^{-\beta R}}{1+e_{eq}} \tag{4}$$

$$e_{eq} = e_s + R(e_r - e_s) \tag{5}$$

Where e_{eq} is equivalent void ratio, e_s and e_r are void ratio of sand and rubber respectively and R is the rubber content $G_{max(RSM)}$ is maximum shear modulus of *RSM*, σ'_m is effective confining pressure, and A and β are regression constants. Regression analysis of the experimental data yielded values of $A = 3000$ and $\beta = 0.022$, respectively.

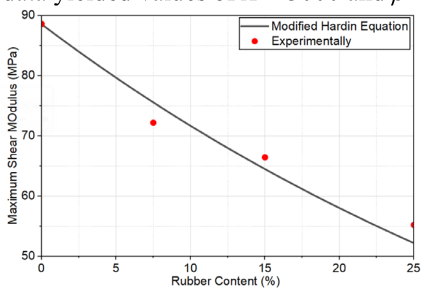


Figure 5. Comparison of maximum shear modulus calculated from the Modified Hardin equation and experimental results.

The proposed model shows good agreement with the experimental results, with a coefficient of determination *(R^2 = 0.95)*, indicating that the model reasonably captures the reduction in small-strain stiffness with increasing rubber content. The exponential term effectively represents the nonlinear degradation of stiffness, while the equivalent void ratio accounts for the influence of mixture packing characteristics. A comparison between the predicted and experimental G_{max} values is shown in Figure 5. Although the proposed modified Hardin relationship shows good agreement with the experimental results, it should be noted that the model is currently based on a limited number of resonant column tests and a narrow range of rubber contents. Therefore, the applicability of the proposed equation may be limited to the range of materials and testing conditions considered in this study. The present formulation considers only the effect of rubber content, while other factors such as confining pressure, rubber particle size, and gradation effects were not explicitly investigated. Future work can focus on validating and refining the proposed relationship by conducting additional tests with a wider range of rubber contents, different types of rubber, and varying confining pressures.

5 Conclusion

In this study, resonant column tests were conducted to investigate the small-strain dynamic properties of Solani sand and rubber–sand mixtures *(RSM)* with varying rubber contents. The influence of rubber inclusion on shear modulus degradation, damping characteristics, and modulus reduction behaviour was evaluated for rubber contents ranging from 0–25 % and confining pressures of 100 kPa. Furthermore, a modified empirical relationship based on the Hardin equation was developed to predict the maximum shear modulus of rubber–sand mixtures by incorporating the effect of rubber content. Based on the experimental investigations, the following conclusions can be drawn:

- Pure sand showed a typical nonlinear response of granular soils, where the shear modulus decreased and damping ratio increased with increasing shear strain. The maximum shear modulus was observed at very small strain levels, while stiffness degradation occurred due to particle rearrangement and inter-particle sliding. The modulus reduction behaviour was found to be consistent with established trends reported in the literature.
- The inclusion of rubber particles resulted in reduction of shear modulus and an increase in damping ratio due to the higher compressibility and energy absorption capacity of rubber. Increasing rubber content further reduced the stiffness while increasing the elastic threshold strain, indicating increased linear range. The results suggest that *RSM* can be effective materials for vibration mitigation due to their enhanced damping properties.
- A modified Hardin equation was proposed by introducing a rubber influence parameter to account for the reduction in stiffness due to rubber inclusion. The proposed relationship showed good agreement with experimental results, demonstrating its capability to predict the maximum shear modulus of rubber–sand mixtures.

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