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Calibration and Performance of a Newly Developed Cyclic Model to Describe the Liquefaction Behaviour of Loose and Dense Sand Under Torsional Simple Shear Conditions

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ABSTRACT: A newly developed cyclic model for simulating the undrained simple torsional shear behaviour of loose and dense saturated sand is presented. The model is developed under a generalised hyperbolic equation (GHE) approach, where the void ratio and stress level dependence upon stress-strain response of sand is incorporated. The cyclic stress-strain response is modelled by employing extended Masing's rules considering damage of plastic shear modulus at large shear strains and hardening of the material during cyclic loading. An empirical state-dependent stress-dilatancy relationship is introduced to account for the effect of density on the stress ratio and to model the excess pore water pressure generation in undrained shear loadings as the mirror effect of volumetric change in drained shear conditions. It is shown that the overall mechanical behaviour of loose and dense Toyoura sand specimens, described in terms of stress-strain relationship and effective stress path, can be satisfactorily captured by the model.

1 INTRODUCTION

The generalised hyperbolic equation (GHE) proposed by Tatsuoka and Shibuya (1992) can properly describe the highly non-linear stress-strain relations from very small to large strain levels for a wide range of geomaterials under general loading conditions (Tatsuoka et al., 2003).

De Silva and Koseki (2011) and Chiaro et al. (2011) effectively used a GHE combined with an empirical stress-dilatancy equation to simulate the overall behaviour of Toyoura sand undergoing drained/undrained cyclic torsional shear loading conditions. However, the combined influence of density and stress level was considered as a variable. That is, sand with different densities was regarded as different material and the effects of stress level were considered to be independent from the density state. Thus, a number of soil parameters were needed for simulating different density and stress level conditions. Nevertheless, more recently, using an extended GHE combined with a state-dependent empirical stress-dilatancy equation, Chiaro et al. (2013) proposed a monotonic model that deals simultaneously with density and stress level dependency upon undrained behaviour of sand.

In this paper, an attempt is made to develop a robust cyclic model by combining former GHE-based monotonic and cyclic models. After calibrating the model for the case of Toyoura sand, numerical simulations of cyclic undrained torsional shear tests were carried out. By comparing experimental and simulation results, model validity is proven for specimens isotropically consolidated at various void ratios and effective mean principal stress levels.

2 MODEL FORMULATION

2.1 *Stress-strain relationship*

The non-linear stress-strain behavior of sand subjected to shear loading was modeled by using the GHE (Tatsuoka and Shibuya, 1992):

$$y = \frac{x}{1/C_{1(x)} + x/C_{2(x)}} \quad (1)$$

where x and y are two functions representing normalized plastic shear strain and shear stress ratio, respectively. Importantly, $C_{1(x)}$ and $C_{2(x)}$ are two fitting parameters that vary with the strain level:

$$C_{1(x)} = \frac{C_{1(0)} + C_{1(\infty)}}{2} + \frac{C_{1(0)} - C_{1(\infty)}}{2}\cos\left\{\frac{\pi}{(\alpha/x)^{a} + 1}\right\} \quad (2)$$

$$C_{2(x)} = \frac{C_{2(0)} + C_{2(\infty)}}{2} + \frac{C_{2(0)} - C_{2(\infty)}}{2}\cos\left\{\frac{\pi}{(\beta/x)^{b} + 1}\right\} \quad (3)$$

in which $C_{1(0)}$, $C_{1(\infty)}$, α, a, $C_{2(0)}$, $C_{2(\infty)}$, β and b are GHE curve fitting parameters.

In the original GHE form x and y are defined as

$$\text{(original GHE)}\ y = \frac{\tau}{\tau_{max}} \quad \text{and} \quad x = \gamma^{p}\frac{G_{max}}{\tau_{max}} \quad (4)$$

where γ^{p} is the plastic shear strain; τ is the shear stress; τ_{max} is the peak shear stress; and G_{max} is the small strain stiffness. However, Chiaro et al. (2013) showed that by properly choosing *y* and *x* functions, the normalized stress-strain relationship of sand can be represented by a unique curve irrespective of density level and drainage conditions. Accordingly, by using an extended GHE, both the void ratio and the effective mean principal stress dependence of stress-strain behavior of sand can be incorporated into the original GHE, as follows:

(extend. GHE) $y = \dfrac{\tau / p'}{(\tau / p')_{max}}$ and $x = \gamma^{p} \dfrac{G_0 / p_0'}{(\tau / p')_{max}}$ (5)

where γ^p is the plastic shear strain; τ is the shear stress; p' and p_0' are the current and initial effective mean principal stress, respectively; $(\tau / p')_{max}$ is the peak shear stress ratio in the plot τ/p' vs. γ^p; and G_0 is the initial small strain shear modulus. Note that G_0 and $(\tau/p')_{max}$ are two parameters with clear physical meaning.

For clean sands, a number of empirical relationships have been proposed to relate G_0 to p_0' and void ratio (e_0) (e.g. Hardin and Richart, 1963). Above all, for the case of sand subjected to torsional shear loading, the following expression is valid:

$$G_0 = G_n \, f(e_0) \left(p_0' / p'_{ref} \right)^n \quad (6)$$

where G_n is a small strain shear stiffness parameter; p'_{ref} is a reference stress (=100 kPa); *n* is a soil parameter to express the stress-level dependence of G_0; and $f(e_0) = (2.17 - e_0)^2 / (1 + e_0)$ is the void ratio function proposed by Hardin and Richart (1963) for sand with round particles.

As suggested by Chiaro et al. (2013), for undrained torsional shear loading, a linear empirical correlation exists between $(\tau/p')_{max}$ and e_0:

$$(\tau / p')_{max} = r_1 + r_2 \, e_0 \quad (7)$$

where r_1 and r_2 are two soil strength parameters.

The cyclic behavior of soil was modeled by employing the well-known 2nd Masing's rule. However, due to rearrangement of particles, soil behavior does not necessarily follow the original Masing's rule during cyclic loadings (Tatsuoka et al., 1997). This feature can be taken into account by dragging the corresponding skeleton curve in the opposite direction to the loading path by an amount β while applying the Masing's rule (Tatsuoka et al., 2003). In this study the following drag function proposed by HongNam (2004) was used:

$$\beta = x' / \left(1/F_1 + x'/F_2 \right) \quad (8)$$

where F_1 is the maximum amount of drag; F_2 is a fitting parameter, which is equivalent to the initial gradient of the drag function; and $x' = \Sigma\Delta x$, where Δx denotes the increment of normalized plastic shear strain.

De Silva et al. (2014) introduced two conceptual factors to take into account the damage (*D*) of plastic shear modulus at large stress level and the hardening (*H*) of the material during cyclic loading:

$$D = \frac{G^p}{G^p_{ini}} = \frac{1.45(1 - D_{ult})}{1 + \exp\left(\Sigma \left| \Delta\gamma^p \right|_p - 0.8 \right)} + D_{ult} \quad (9)$$

where D_{ult} is the minimum value of *D*; $\Sigma|\Delta\gamma^p|_p$ is the torsional plastic shear strain accumulated between the current and the previous turning points;

$$H = 1 + H_x / \{ F_2 / F_1 + H_x / (H_{ult} - 1) \} \quad (10)$$

where H_x is the $\Sigma|\Delta x|$ up to current turning point; H_{ult} is the maximum value of *H* after applying an infinite number of cycles; F_1 and F_2 are the same parameters used in the drag function.

To summarize, after introducing drag, damage and hardening effects, the skeleton curve during cyclic loading was modeled as follows:

$$y = \frac{x - \beta}{\dfrac{1}{D \, C_1(x)} + \dfrac{|x - \beta|}{H \, C_2(x)}} \quad (11)$$

2.2 *Stress-dilatancy relationship*

Stress-dilatancy relationship relates the dilatancy ratio ($-d\varepsilon^p_{vol}/d\gamma^p$) to the stress ratio ($\tau/p'$). For sands undergoing cyclic torsional shear loading, Nishimura and Towhata (2004) recommended an empirical bi-linear stress-dilatancy relationship, which De Silva et al. (2014) amended to account for the damage (*D*) of plastic shear modulus:

$$-\frac{d\varepsilon^p_{vol}}{d\gamma^p} = \frac{1}{D \, N_d} \left[\frac{\tau}{p'} \pm \frac{(\tau / p')_{PTL}}{D} \right] \quad (12)$$

In the above, $(\tau/p')_{PTL}$ is the stress ratio at the phase transformation (i.e. zero dilatancy state; Ishihara et al., 1975); N_d is a density dependent parameter (Chiaro et al., 2013), such as the denser the soil, the greater the N_d:

$$N_d = d_1 - d_2 e_0 \quad (13)$$

where d_1 and d_2 are two soil parameters to express the dependence of N_d on density.

During cyclic loadings, the effective mean stress (p') decreases with number of cycles due to two possible mechanisms: (i) the soil is subjected to significant effects of over-consolidation until the stress state exceeds for the first time the phase transformation stress state (i.e., the first time where the volumetric behavior changes from contractive to dilative, $dp' > 0$); and (ii) soil enters into the stage of cyclic mobility. In particular, the over-consolidation significantly alters the stress-dilatancy behavior of sand during the virgin loading and its effect evanishes with the subsequent cyclic loading. Oka et al. (1999) suggested a distinct stress-dilatancy equation to reproduce the effect of over-consolidation (OC) within certain boundaries. Following the same approach, later De

Silva et al. (2014) proposed the following stress-dilatancy relationship for the case of torsional shear loading as follows:

$$-\frac{d\varepsilon^{p}{}_{vol}}{d\gamma^{p}}=\frac{1}{N_{d}}\left(\frac{\tau}{p'}-\frac{\tau/p'}{\ln(p_{0}'/p')}\right)\times \left[\frac{\tau/p'}{(\tau/p')_{PTL}\ln(p_{0}'/p')}\right]^{3/2} \qquad (14)$$

Note that, similarly to De Silva et al. (2014), in the proposed model, the stress path during undrained loading is divided into four sections namely: (A) monotonic (virgin) stress path; (B) stress path within the limits of phase transformation stress state, but outside the OC boundary surface; (C) stress path within the limits of OC boundary surface, but before exceeding the phase transformation line (PTL) for the first time; and (D) stress path after exceeding the PTL for the first time. Schematic illustration of employed four-phase dilatancy-relationship is shown in Fig. 1.

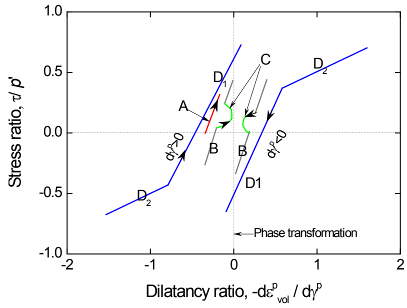

Fig. 1 Four-phase stress-dilatancy relationship

2.3 *Excess pore water pressure generation*

The concept of zero plastic volumetric increment during undrained loading as a way of capturing shear-induced pore water pressure response of sand was first introduced by Martin et al. (1975). Accordingly, in modeling the undrained shear behavior, it was assumed that the plastic volumetric strain increment ($d\varepsilon^{p}{}_{vol}$) during undrained loading, which consists of dilatancy ($d\varepsilon^{p(d)}{}_{vol}$) and consolidation/swelling ($d\varepsilon^{e(c)}{}_{vol}$) components, is equal to zero. That is, a change of effective mean principal stress (p') during undrained loading causes recompression/swelling of the specimen. Instead, a change of shear stress (τ) causes dilatation of the specimen. Thus, the following equation is valid during undrained loading:

$$d\varepsilon_{vol}=d\varepsilon^{p(c)}_{vol}+d\varepsilon^{p(d)}_{vol}=0 \quad \Rightarrow d\varepsilon^{p(c)}_{vol}=-d\varepsilon^{p(d)}_{vol} \qquad (15)$$

Experimental evidence suggests that the bulk modulus K (= $dp'/d\varepsilon^{p(c)}{}_{vol}$) can be expressed as a unique function of p':

$$K=dp'/d\varepsilon^{p(c)}_{vol}=K_{0}f(e)/f(e_{0})(p'/p_{0}')^{m} \qquad (16)$$

where K_0 is the bulk modulus at initial effective mean stress (p_0'); f(e) and f(e_0) are the void ratio function at current and initial stress state, respectively; and m is a coefficient to model the stress-state dependency of K. Considering that f(e)=f(e_0) in undrained tests, by combining Eqns. (10) and (11), the change of effective mean principal stress (i.e. generation of excess pore water pressure, Δu) during undrained shearing is evaluated as follows:

$$dp'=\Delta u=K_{0}(p'/p_{0}')^{m}(-d\varepsilon^{p(d)}_{vol}) \qquad (17)$$

The initial bulk modulus K_0 depends on initial void ratio and effective mean principal stress as follows:

$$K_{0}=K_{m}f(e_{0})(p_{0}'/p_{ref}')^{m} \qquad (18)$$

where K_m is a soil compressibility parameter; p'_{ref} is a reference stress (=100 kPa) and m is a soil parameter to express the stress-level dependence of K_0.

3 MODEL PARAMETERS

The proposed model requires a unique set of parameters (Table 1) for simulating cyclic undrained torsional shear behavior of saturated sand over a wide range of void ratios and confining pressures. As shown in Fig. 2, all the GHE parameters in Eqns. (2) and (3) can be determined by fitting the experimental data plotted in terms of y/x vs. y relationship (Fig. 2). Shear modulus, shear strength and dilatancy parameters were experimentally determined as reported in Chiaro et al. (2013) and De Silva et al. (2014).

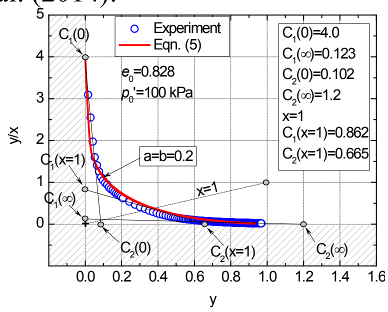

Fig. 2 Evaluation of GHE curve fitting parameters

4 MODEL PERFORMANCE

Figs. 3 and 4 compare model predictions and experimental observed behaviour of two specimens consolidated to p_0' = 100 kPa and different void ratios of e_0 = 0.828 (loose specimen) and 0.706 (dense specimen). All predictions are obtained using the set of model parameters listed in Table1. It can be seen that the overall soil response of both Toyoura sand specimens is satisfactorily predicted by the model.

5 CONCLUSIONS

In this paper, by using an extended generalized hyperbolic equation (GHE) combined with an empirical stress-dilatancy relation, a cyclic model that deals with state-dependency upon undrained torsional shear behaviour of sand was presented. The proposed model can satisfactorily predict the cyclic torsional shear response of loose and dense Toyoura sand specimens using a unique set of soil parameters, which can be straightforwardly determined by a limited number of laboratory tests.

Table 1. Model parameters for Toyoura sand

GHE				
$C_{1(0)}$	$C_{1(\infty)}$	α	A	
4.0	0.123	0.01073	0.2	
$C_{2(0)}$	$C_{2(\infty)}$	β	b	
0.102	1.2	0.85012	0.2	
Shear modulus and peak shear strength				
G_n (kPa)	n	r_1	r_2	
81969	0.51	1.828	-1.406	
Dilatancy and bulk modulus				
d_1	d_2	$(\tau/p')_{\mathrm{PTL}}$	K_m (kPa)	m
5.793	-5.0	0.6	47710	0.5
Drag, damage and hardening				
F_1	F_2	D_{ult}	H_{ult}	
0.5	12	0.6	1.15	
Four-phase dilatancy				
Phase	*Eqn.*	τ_{PTL}	N_d	*D*
A	12	0.6	Eqn. (13)	1
B	12	0.45	2.2	1
C	14	0.45	2.2	1
D1	12	0.36	2.2	Eqn.(9)
D2	12	0.18	0.33	Eqn.(9)

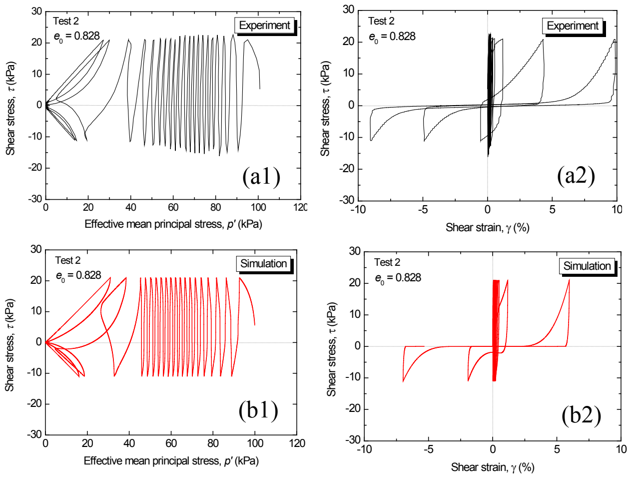

Fig. 3 Comparison between (a) experimental data and (b) simulation results for loose Toyoura sand

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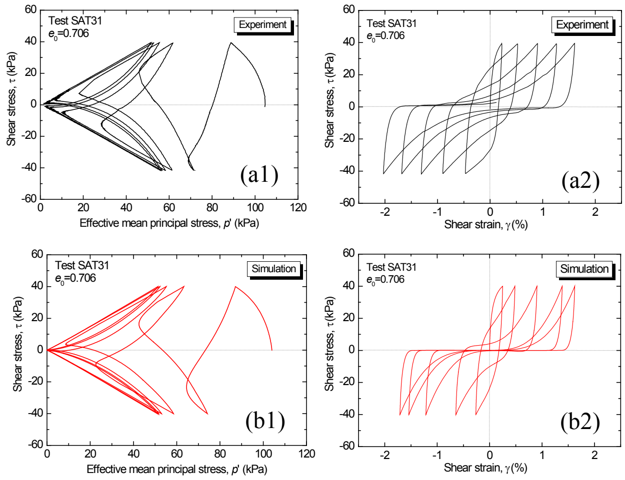

Fig. 4 Comparison between (a) experimental data and (b) simulation results for dense Toyoura sand

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