Analytical Modeling of the Temperature Effect on Radial Consolidation Associated with Vertical Drains

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ABSTRACT: Construction in soft soils has been a challenging task for engineers due to the excessive time taken for dissipation of pore water pressure and post-construction settlement. Use of vertical drains has proven to be an effective and economical method for soft ground improvement and hence research has been carried out to further improve its efficiency. Effect of temperature on radial consolidation is one aspect of such research among many others that have been investigated. Although there are many empirical relationships developed to account for the effect of temperature on consolidation by Abuel-Nagaa et al. (2006), Laloui et al. (2008) and Artidteang et al. (2011), analytical relationships are scarce. Temperature certainly has a pronounced effect on the hydraulic conductivity due to the changes it causes in the viscosity of water. It also generates excess pore water pressure due to changes in the porosity of the soil matrix caused by differential expansion. University of South Florida (USF) researchers have developed a numerical methodology based on the 3D Navier-Stokes equations to model transient fluid flow through porous media. This model has broader applications and is well-formulated to take into account the above temperature effects as well. Temperature related variations of porosity and viscosity were obtained from the literature and used in the USF model to observe the effect of temperature on consolidation of normally consolidated clay. The results are presented in terms of a parametric study where the rates of consolidation under thermal treatment were compared to that without it.

1 INTRODUCTION

Pre-consolidation is used as a means of overcoming long-term settlement problems encountered when constructing on clayey soils. More recently, use of prefabricated vertical drains has been widely proven as an effective method for pre-consolidation and soft ground improvement. An innovative thermal technique to further enhance its performance has been investigated by Abuel-Naga et al. (2006) in which the soil is heated during the preloading process. Many laboratory and field scale experiments have been carried out by Abuel-Naga et al. (2006), Pothiraksanon et al. (2010) and Artidteang et al. (2011) to verify and validate the effect of temperature on soil properties, consolidation rate and settlement. This paper presents the results of an analytical study on the effects of temperature on the consolidation rate of normally consolidated clay.

2 CONSOLIDATION UNDER A TEMPERATURE INCRESE

When a saturated soil layer is subjected to an increase in overburden stress, excess pore water pressure is generated. This creates a hydraulic gradient and the excess pore water pressure is dissipated over time until the applied stress is transferred to the soil grains. The rate at which pore water pressure dissipates will depend on the instantaneous hydraulic gradient created and the hydraulic conductivity of the soil. The pore water pressure dissipation rate will determine the rate of settlement. With an increase in temperature of the soil medium, two main phenomena can be observed;

1. Increase in hydraulic conductivity due to a decrease in viscosity
2. Generation of excess pore water pressure due to the differential expansion which increases the hydraulic gradient

Therefore, it is evident how a temperature increase would have a significant effect on the rate of settlement.

2.1. *Effect of temperature on hydraulic conductivity*

The hydraulic conductivity, *k*, can be expressed as in equation 1.

$$k = \frac{K\gamma}{\mu} \tag{1}$$

where *K* is the intrinsic hydraulic conductivity, γ is the unit weight of water and μ is the viscosity of water. Research by Abuel-Naga et al. (2006) indicates an increase in hydraulic conductivity with an increase in temperature. This can be attributed to the corresponding decrease in viscosity of water. The relationship between temperature and water viscosity was given by Hillel (1980) & Pothiraksanon et al. (2010) as indicated in equation 2 with water viscosity in Pa.s and temperature in ˚C.

$$\mu(T) = -0.00046575\ln(T) + 0.00239138 \tag{2}$$

2.2. *Effect of temperature on pore water pressure*

There is a significant difference between the thermal expansion coefficients of water and soil grains with that of water being much higher. As a result, when a soil mass is heated, porosity of the soil skeleton would change generating excess pore water pressure. Considering a dry soil skeleton subject to a temperature increase ofΔT, the final vo l-ume of voids, V_v can be expressed by equation 3.

$$V_v = V_0(1 + \beta_{soil})\Delta T \tag{3}$$

where V_0 is the initial volume of voids and β_{soil} is the thermal expansion coefficient of the soil grains. If the pores are saturated with water, the volume of water after a temperature increment of ΔT would be as in equation 4.

$$V_w = V_0(1 + \beta_{water})\Delta T \tag{4}$$

where V_w is the final volume of water and β_{water} is the thermal expansion coefficient of water.

Therefore, the differential expansion can be expressed by equation 5.

$$\Delta V = V_w - V_v = V_0(\beta_{water} - \beta_{soil})\Delta T \tag{5}$$

The expansion of coefficient of soil grains (β_{soil}) can be considered negligible compared to that of water. The Navier-Stokes equations used in the USF model were modified as follows to account for the temperature effect. The continuity equation and the momentum equations are given in equations 6 and 7.

$$\frac{\partial(nV_x)}{\partial x} + \frac{\partial(nV_y)}{\partial y} - n\beta_{water}\Delta T = \frac{\partial n}{\partial t} \tag{6}$$

$$\frac{\partial(nV_x)}{\partial t} = \frac{\mu}{\rho_w}\frac{\partial^2(nV_x)}{\partial^2 x} + \frac{D_x}{\rho_w} - \frac{n}{\rho_w}\frac{\partial p}{\partial x} \tag{7}$$

where V_x and V_y are the water velocities in x and y directions respectively, n is the porosity of the soil, ρ_w is the density of water and D_x is the drag force. A momentum equation similar to equation 7 can be written for the y direction as well. The drag force can be obtained from equation 8.

$$D_i = -c\mu\left(\frac{1-n}{n}\right)^2\left(\frac{1}{d_p^2}\right)V_i \tag{8}$$

where V_i is the velocity, d_p is the average particle size and c is a particle shape dependent constant which is 180 for a spherical particle.

Equation 9 was used to quantify the compressibility characteristics of the soil skeleton.

$$(1+e) = N - \lambda\ln\left(\frac{1+2K_0}{3}\sigma_v'\right) \tag{9}$$

where σ_v' is the vertical effective stress, K_0 is the coefficient of lateral earth pressure at rest, e is the void ratio and N and λ are Cam-Clay model parameters.

3 MODELING THE STEADY STATE TEMPERATURE DISTRIBUTION IN A HEATED SOIL MASS

When a soil mass is heated by a heat source in a vertical drain, a temperature regime will be created in the surrounding area. This temperature distribution will determine the ΔT to be used in equation 6 to obtain the corresponding thermal expansion. The authors propose a simple model to determine the temperature distribution around a heat source.

Consider a cylindrical soil mass with a heat source along its central axis. Using an analogy from fluid flow the steady state temperature distribution along a radius can be obtained as in equation 10.

$$T(r) = \frac{T_2 \ln\left(\frac{R}{r}\right) - T_1 \ln\left(\frac{r_w}{r}\right)}{\ln\left(\frac{R}{r_w}\right)} \qquad (10)$$

where T_1 is the initial ambient temperature, T_2 is the temperature at the heat source, r_w is the radius of the drain (heated area) and R is the radius of the influence zone.

Assuming typical experimental results published in the literature, e.g., T_1=20˚C, T_2= 90˚C, r_w= 0.05m and R = 1m, the distribution of temperature along a radius is shown in Fig. 1.

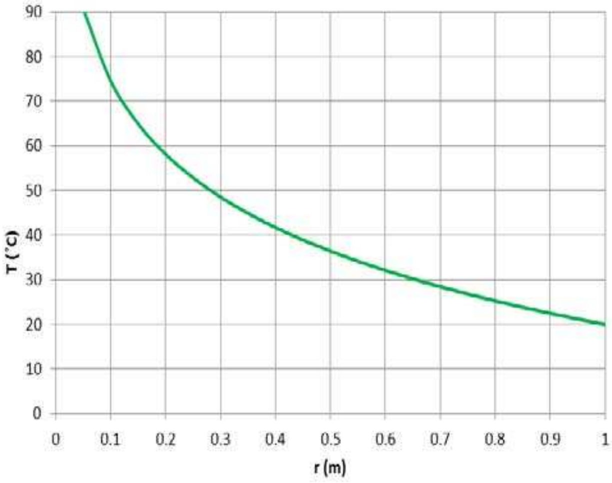

Fig. 1 Steady state temperature distribution along a radius

4 MODELING THE TRANSITIONAL TEMPERATURE VARIATION IN A HEATED SOIL MASS

The temperature at different radial locations will gradually increase with time until the steady state temperatures shown in Fig.1 are reached. Heat from the source will be transferred through the soil medium by conduction. The authors used an empirical method to model this transitional temperature variation. The variation of temperature for a particular radial location can be obtained from the following relationship.

$$T(t) = \frac{t}{\frac{1}{m} + \frac{t}{T_{ult} - T_1}} + T_1 \qquad (11)$$

where T_{ult} is the steady state temperature at a particular r and m is the initial gradient of the curve. For example, the temperature variation for a location at r_w with T_{ult} = 90˚C is shown in Fig. 2.

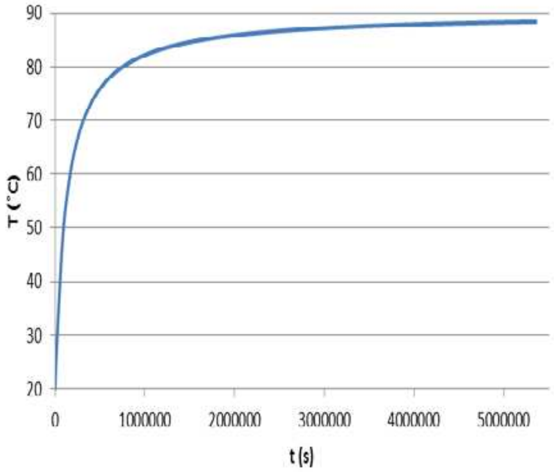

Fig. 2 Temperature variation at r_w

The initial gradient m can be obtained from the following approximate heat transfer formulation given in equation 12. Consider a point very close to the heat source at a Δr distance. Assuming that within an initial time interval of Δt, the soil mass element from r_w to r_w + Δr is heated with no heat output from that element, we can establish a relationship between the heat input and the increase in enthalpy for a unit height of the soil mass.

$$Q_{in} = 2\rho\pi r_w(\Delta r)C\left(\frac{\Delta T}{\Delta t}\right) \qquad (12)$$

where Q_{in} is the initial heat input, ρ is the density of the soil and C is the specific heat capacity of the soil. If heat is supplied by water circulation, Q_{in} can be obtained from the inlet and outlet temperatures, mass flow rate and the specific heat capacity of water. The above distance Δr will depend on the thermal conductivity of the soil.

Δr was taken as αk where k is the thermal conductivity of the soil and α is a calibration factor obtained based on previous research (Pothiraksanon, Bergado, & Abuel-Naga, 2010). In this research, the values for Q_{in}, ρ, C, k and α were taken as 2360W/m, 2180kg/m^3, 680J/kg˚C, 1.3W/m/˚C and 7 respectively to obtain the initial gradient m which is $\Delta T/\Delta t$.

Temperature variation curves for each radial location can be obtained by offsetting the curve at

r_w along the T axis by a distance equal to the difference between the steady state temperatures given in Fig.1.

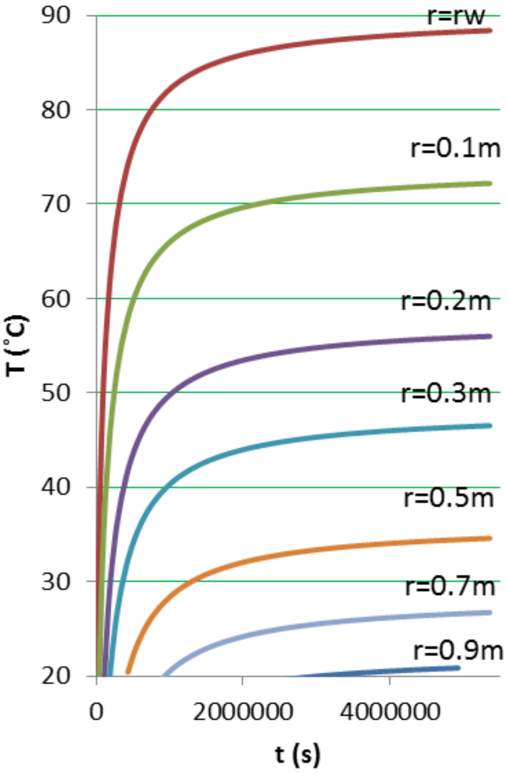

Fig. 3 Temperature variation for different radial locations

The effect of temperature on the viscosity was incorporated by modifying the μ term in equation 8 and using it in equation 7.

5 RESULTS

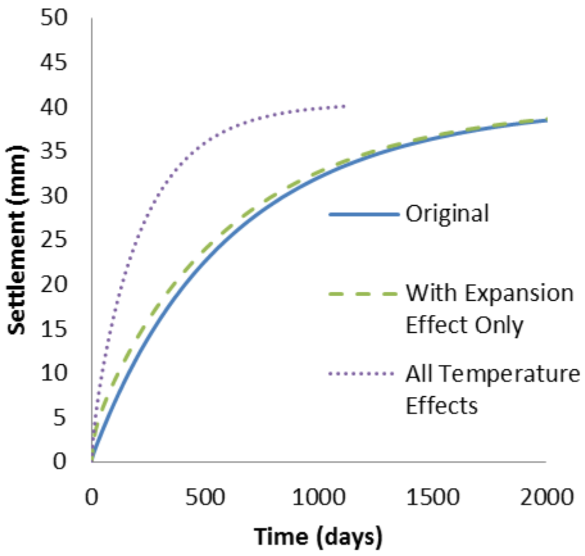

Fig. 4 Comparison of consolidation rates with and without the thermal treatment

The rates of consolidation with and without the thermal treatment are shown in Fig.4. The results show that the consolidation rate increased significantly when both the thermal expansion and the temperature effects on viscosity were included. However, the effect of temperature on excess pore pressure under constant viscosity showed a consolidation rate which is only marginally increased compared to that without thermal treatment.

6 CONCLUSIONS

The authors' analytical model which uses Navier-Stokes flow equations incorporating a modification for the temperature effect was shown to model the enhanced consolidation effect effectively.

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