INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING

Simulation of Radial Consolidation Using Prefabricated Vertical Drains

C.H. Lee
Associate Principal Geotechnical Engineer, SMEC Australia and New Zealand
J.P. Hsi
Chief Technical Principal Geotechnics, SMEC Australia and New Zealand

ABSTRACT: Consolidation of soft clay involves dissipation of excess pore water pressure (EPWP) generated by additional overburden stress. To accelerate the consolidation process, prefabricated vertical drains (PVD's) are often used to assist with the dissipation of EPWP. There have been many well established analytical and numerical methods developed to simulate radial consolidation through PVD's, e.g. Hansbo (1981), Hird *et al.* (1992), Chai *et al.* (2001) and Indraratna *et al.* (2005).However, the consolidation process is often complicated by the presence of complex drainage boundaries, smear effect due to PVD installation, multiple subsurface soil layers, varying loading conditions, etc. These complexities can generally be handled with ease by the finite element method (FEM). In practice, the two-dimensional (2D) plane strain finite element analysis (FEA) is normally adopted in lieu of the more complicated three-dimensional (3D) FEA. In order to simulate the 3D condition in reality, i.e. radial drainage, the 2D finite element (FE) model needs to be adjusted. Such adjustment can be achieved via geometric and/or permeability matching, as detailed in the above mentioned publications. This paper presents and discusses the application of the various methods used in the 2D FEA of radial consolidation. A case study is also presented to demonstrate the applicability of these methods.

1 INTRODUCTION

Ground improvement using prefabricated vertical drains (PVD's) is one of the most cost effective soft ground treatment methods. PVD's shorten the drainage path for the dissipation of excess pore water pressure (EPWP),thereby accelerate the consolidation process.

There are many analytical methods developed to solve the problem of radial consolidation of soil incorporated with vertical drains. One of the closed-form solutions was proposed by Hansbo (1981). His solution is based on a unit cell approach, which assumes a single circular vertical drain surrounded by a soil column with an equivalent diameter to the drain spacing. Dewatering is achieved via radial drainage of water into the vertical drain in an axisymmetric condition.

Hansbo's (1981) method takes into account both the smear effect and well resistance, but neglects vertical water flow in the soil. It assumes that consolidation takes place in a uniform soil column with equal soil strains in both lateral and vertical directions.

However, consolidation of soil often occurs in an environment that may include complex drainage boundaries, multiple soil layers, varying loading conditions, etc. Analytical methods, e.g. Hansbo (1981), have limited ability to solve problems with complex conditions, whilst the finite element method (FEM)has greater flexibility in dealing with such problems. However, the finite element analysis (FEA) is often undertaken in a two-dimensional (2D) plane strain condition whilst the radial consolidation is a three-dimensional (3D) problem.

There have been many methods developed for the simulation of radial consolidation in a 2D plane strain condition based on geometric and/or permeability matching. Such matching methods have been proposed by Hird *et al.* (1992), Chai *et al.* (2001) and Indraratna *et al.* (2005).

In this paper, the radial consolidation is firstly analysed using Hansbo's (1981) closed-form solution and the FEM with an axisymmetric model. The various matching methods, as discussed above, are then adopted in the 2Dplane strain FEA of radial consolidation and the calculated results are compared with the axisymmetric and analytical solutions. The rate of consolidation has been assessed based on calculated settlement and EPWP dissipation. A case study is presented to demonstrate the application of these methods in predicting settlements associated with embankment filling over soft clays treated with PVD's.

2 AXISYMMETRIC ANALYSIS

Axisymmetric analysis can be used to solve a radial drainage problem, such as consolidation of soil around an individual vertical drain. Fig. 1 shows the axisymmetric finite element (FE) mesh created by PLAXIS using 15-node triangular elements. The analysis of radial consolidation includes a single 5m long circular vertical drain surrounded by a soil column with an equivalent external radius R of 1.05m to simulate the consolidation behaviour with PVD's installed at 2m spacing in a triangular pattern. The equivalent radius of the vertical drain $r_w$is assumed to be0.033m. The soil within the cylindrical column is modelled as a linear elastic material with a Young Modulus E'_{ref} of 15MPa. The horizontal coefficient of consolidation c_h is assumed to be 30m^2/year based on the typical field value presented in Section 5. The horizontal permeability $k_h$can be calculated from$k_h= c_h m_v \gamma_w$ (e.g. Craig 2004), where, m_v is the coefficient of volume compressibility, which can be derived from the $E'_{ref}$and drained Poisson's ratio ν'(assumed to be 0.3), and γ_w is the unit weight of water.

Drainage within the soil column only takes place in the radial direction where the zero excess pore water pressure boundary is defined by the drain elements located along the centre line of the axisymmetric model. The smear zone adjacent to the vertical drain, resulting from soil disturbance due to PVD installation, is simulated with a reduced soil permeability k_s within a radius r_s.In the following parametric studies, it is assumed that r_s/r_w = 4 and k_s/k_h = 4which are in-line with the typical values reported by Indraratna *et al*. (2005a).

A 50kPa pressure is initially applied under an undrained condition and then consolidation is allowed to take place. The undrained Poisson's ratio ν_u, which governs the compressibility of pore water is manually increased to a maximum permissible value of = 0.496 (PLAXIS Scientific Manual) by modifying the Skemption's B-parameters to 0.995.

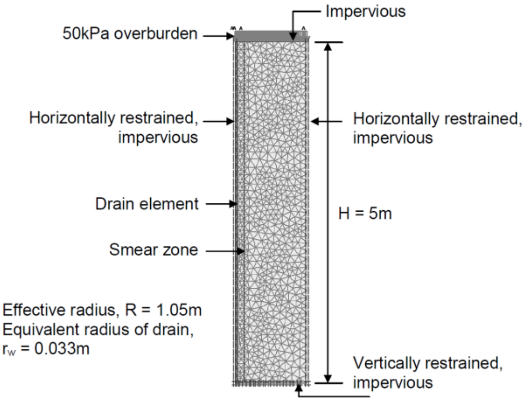

Fig.1Finite element mesh for axisymmetric analysis

Fig. 2 shows the average radial consolidation ratios obtained from the axisymmetric and analytical analyses. The analytical solution is based on the following radial consolidation equation,

$$u_h=1-e^{-8\cdot T_h/\mu_s} \qquad (1)$$

where,u_h is the rate of radial consolidation, $T_h=c_h\, t/D^2$,t is the elapsed consolidation time, D = 2R and μ_s is a parameter based on Hansbo (1981), which accounts for PVD arrangement and smear effect, and can be calculated as follows,

$$\mu_s = \ln\left(\frac{n}{s}\right)+\frac{k_h}{k_s}\ln(s)-\frac{3}{4} \qquad (2)$$

where, $n = R/r_w$ and $s = r_s/r_w$.

The average consolidation ratio is calculated as the surface settlement at the location with the longest drainage path(R = 1.05m) over the total consolidation settlement. As shown in Fig. 2, good agreement is achieved between the results of analytical and FE axisymmetric analyses.

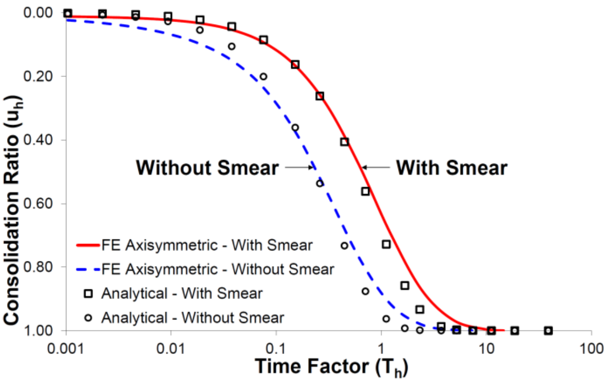

Fig.2 Comparison betweenanalytical and FE axisymmetric analysis results

3 MATCHING METHODS

Although the3DFEA would simulate the reality better, it is often cumbersome and time consuming to set up a3D model. As a result, the 2Dplane strain FEA is often adopted to solve geotechnical problems due to its simplicity and computational efficiency. For PVD's used in the 2D plane strain model, they are simulated as parallel free draining "walls", and the drainage path and behaviour are dissimilar to the actual drainage (axisymmetric) conditions. As such, the 2D plane strain model needs to be adjusted to better simulate the actual vertical drain behaviour.

Conversion between the 2D plane strain and axisymmetric models can be achieved via the following methods: (A) geometric matching (e.g. Hird *et al.,* 1992) –the drain spacing in the 2D plane strain model is adjusted whilst the permeability remains unchanged; (B) permeability matching (e.g. Hird *et al.,* 1992, Indraratna *et al.,* 2005) – the permeability is adjusted whilst the drain spacing remains unchanged in the 2D plane strain model; (C) equivalent vertical permeability (e.g. Chai *et al.,* 2001) – the vertical permeability is adjusted to simulate radial consolidation without incorporating vertical drains in the 2D plane strain model; and (D) combination of permeability and geometric matching – the permeability and drain spacing are both adjusted in the 2D plane strain model.

Parametric studies have been undertaken for items (A) to (C) and the results are presented in Section 4 below.

3.1 *Hird et al. (1992)*

The Hird*et al.*(1992)method involves adjusting the spacing of vertical drains and/or the permeability of soil in the 2D plane strain FEA. The smear effect can be considered in the 2D plane strain model without the need to explicitly incorporate a smear zone.

The horizontal permeability (k_{hp}) for permeability matching is adjusted as follows,

$$k_{hp} = 2k_h \Big/ \left(3\left[\ln\left(\frac{n}{s}\right)+\left(\frac{k_h}{k_s}\right)\ln(s)-\frac{3}{4}\right]\right) \tag{3}$$

where, n = R/r_w and s = r_s/r_w. The drain spacing in the 2D plane strain model is equal to 2 times the equivalent radius R of the soil cylinder. The vertical permeability remains unchanged.

Alternatively, the drain spacing can be adjusted via geometric matching. In this instance, the drain spacing (2B) is adjusted as follows,

$$2B = 2R\sqrt{\frac{3}{2}\left[\ln\left(\frac{n}{s}\right)+\left(\frac{k_h}{k_s}\right)\ln(s)-\frac{3}{4}\right]} \tag{4}$$

whilst, the vertical and horizontal permeability values remain unchanged.

3.2 *Indraratna et al. (2005)*

The Indraratna *et al.* (2005) method involves adjusting permeability values in the undisturbed soil and the smear zone. The half drain spacing B in the 2D plane strain model is equal to the equivalent radius of the soil column R, the half width of the vertical drain b_w is equal to the equivalent radius of the drain r_w and the half width of the smear zone b_s is equal to the equivalent radius of the smear zone r_s.

The horizontal permeability $k_{hp}$of the undisturbed soilis adjusted as follows,

$$k_{hp}=k_h\,[\alpha+\beta]\Big/\left[\ln(n)-\frac{3}{4}\right] \tag{5}$$

$$\alpha=\left[2(s-n)^3\right]\Big/\left[3n^2(n-1)\right] \tag{6}$$

$$\beta=2(s-1)/n^2(n-1)\left[n(n-s-1)+\frac{1}{3}\left(s^2+s+1\right)\right] \tag{7}$$

whilst, the horizontal permeability of the smear zone k'_{hp} is calculated as

$$k'_{hp}=k_{hp}\beta\Big/\left\{\frac{k_{hp}}{k_h}\left[\ln\left(\frac{n}{s}\right)+\frac{k_h}{k_s}\ln(s)-\frac{3}{4}\right]-\alpha\right\} \tag{8}$$

3.3 *Chai et al. (2001)*

The Chai *et al.*(2005) method involves adjusting the vertical permeability of soil to match the rate of settlement from radial consolidation without incorporating vertical drains in the FE model.

The contribution of radial consolidation is modelled through the adjustment of the vertical permeability$k_v$to an equivalent valuek_{ve}, as follows,

$$k_{ve}=\left(1+\frac{2.5H^2k_h}{\mu D^2k_v}\right)k_v \tag{9}$$

$$\mu=\ln\left(\frac{n}{s}\right)+\left[\frac{k_h}{k_s}\right]\ln(s)-\frac{3}{4}+\pi\frac{2H^2k_h}{3q_w}, \tag{10}$$

q_w is discharge capacity of the vertical drain, H is thickness (height) of the soil column, and D is equivalent diameter of the soil column (2R).

4 2D PLANESTRAIN ANALYSIS

4.1 *Consolidation ratio based on settlement*

Fig. 3shows the calculated rates of radial consolidation, with the smear effect considered ,from the 2D plane strain FEA using the various matching methods. Also included in Fig. 3 are the results of the axisymmetric and analytical (Hansbo, 1981) analyses. The consolidation ratio is calculated as the surface settlement at the location with the longest drainage path over the total consolidation settlement. It is noted that for the purpose of this study, the vertical permeability of soil is assumed to be negligible so that settlement is governed by radial consolidation. As shown in Fig. 3, the rates of settlement based on the 2D plane strain analyses match reasonably closely with, although slightly faster at a small T_h than, those from the axisymmetric and analytical analyses.

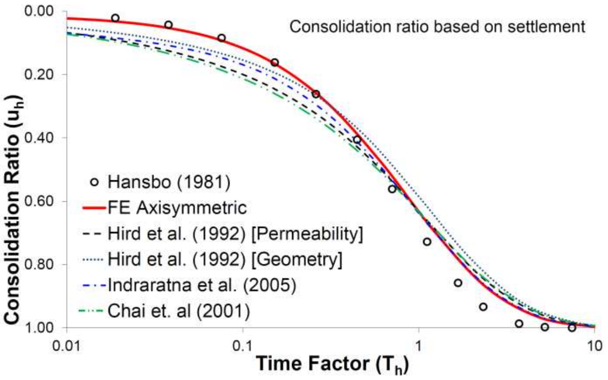

Fig. 3Consolidation ratios based on settlement

4.2 *Consolidation ratio based on EPWP dissipation*

Fig. 4 shows the calculated consolidation ratios, based on the rate of excess pore water pressure (EPWP) dissipation, from the various matching methods. The consolidation ratio is calculated as the dissipated EPWP at the location with the longest drainage path to the PVD.As shown in Fig. 4, the rates of EPWP dissipationfromthe2D plane strain analyses are slower than that of the axisymmetric analysis. Among the various methods compared, the Indraratna *et al.* (2005)method yields the best match with the axisymmetric results, whilst the Chai *et al.* (2001)method predicts the slowest rate of consolidation.

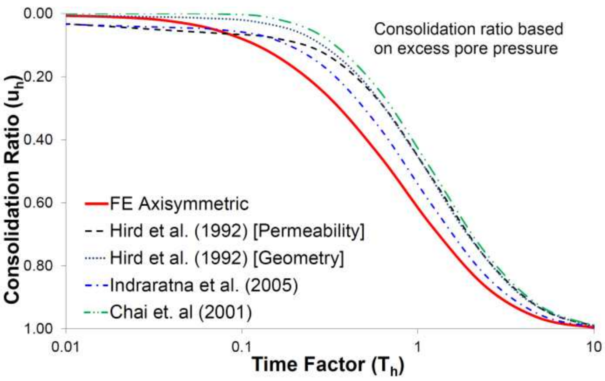

Fig. 4Consolidation ratios based on EPWP dissipation

5 CASE STUDY

The above matching methods are applied in a case study where settlements were measured corresponding to fill thickness. The geotechnical model and fill thickness are shown in Tables 1 and 2.

Table 1. Geotechnical model

Soil	Depth (m)	γ_{sat} (kN/m^3)	E' (MPa)	OCR	$C_{c\varepsilon}$	$C_{r\varepsilon}$
CL-Fill	1.25	16.5	15.0	-	-	-
CH-S	2.75	17.0	-	3	0.12	0.03
SP-VL	3.25	17.0	10.0	-	-	-
CH-S	5.5	17.0	-	1.5	0.12	0.03
SP-MD	12.5	19.0	18.0	-	-	-
SP-D	19.0	19.0	40.0	-	-	-

Note: γ_{sat} =unit weight of soil; E' = Young's modulus; OCR = over consolidation ratio;$C_{c\varepsilon}$= modified compression index; $C_{r\varepsilon}$= modified recompression index

Table 2. Measured fill thickness

Time (day)	Thickness (m)	Time (day)	Thickness (m)	Time (day)	Thickness (m)
1	1.35	235	5	294	11.1
90	1.80	237	6.7	300	11.8
108	2.2	244	7.2	304	12.2
161	2.4	250	7.7	313	13.4
175	2.8	272	9.1	403	13.4
223	3.3	285	10.5		

The PVD spacing is 1.4m centre to centre in a triangular pattern. The equivalent radius r_w of the PVD is 0.033m. The horizontal coefficient of consolidation c_h is 30m^2/year. The smear effect is assumed to be $r_s/r_w = 2$ and $k_s/k_h = 2$.

Fig. 5 shows the field measurements compared with the predictions based on the various methods. As is seen, the measurements and predictions are in reasonable agreement.

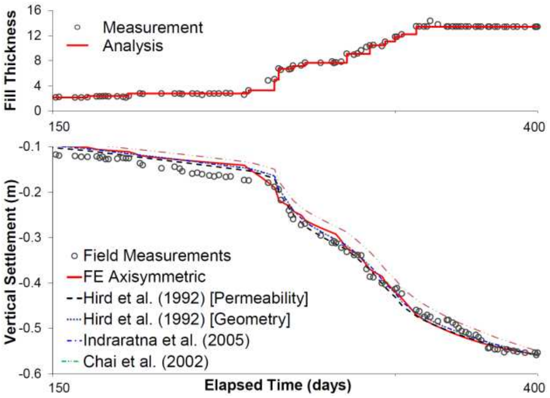

Fig.5Field measurements and predictions

6 CONCLUSIONS

This paper discusses various methods for simulation of radial consolidation, including the axisymmetric finite element analysis, an analytical solution published by Hansbo (1981) and the 2D plane strain finite element analysis based on geometric and/or permeability matching, as presented by Hird *et al.* (1992), Chai *et al.* (2001) and Indraratna *et al.* (2005). The parametric studies show the consolidation ratio based on settlement can be reasonably predicted by all methods, whilst the calculated rates of consolidation are more scattered based on EPWP dissipation. A case study is presented to demonstrate the methods discussed.

REFERENCES

Craig, R.F. (2004)., Soil Mechanics. Taylor & Francis Group.

Chai, J.C., Shen, S.L., Miura, N. & Bergado, D.T. (2001)., Simple method of modelling PVD improved subsoil. J. Geotech. Eng., ASCE, 127(11), 965-972.

Hansbo, S. (1981)., Consolidation of fine-grained soils by prefabricated drains, Proceedings of 10th Int. Conf. on SMFE., Stokholm, 3, 677-682.

Hird, C.C., Pyrah, I.C. & Russell, D. (1992)., Finite element modelling of vertical drains beneath embankments on soft ground. Geotechnique, 42(3), 499-511.

Indraratna, B., Sathananthan, I., Bamunawita, C.& Balasubramaniam, A.S. (2005a)., Theoreticaland numerical perspectives and field observations for the design and performance evaluation of embankments constructed on softmarine clay, Ground Improvement, Elsevier, 199-230.

Indraratna, B., Sathananthan, I., Rujikiatkamjorn C. & Balasubramaniam, A.S. (2005)., Analytical and numerical modelling of soft soil stabilized by PVD incorporating vacuum preloading. Int. J of Geomechanics, 5(2), 114-124.