Non-Linear Analysis of Displacements of GPA in Homogenous Ground (Undrained Strength Constant with Depth)

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ABSTRACT: Granular Piles (GPs) improve the response of the ground by increasing bearing capacity, reducing settlements and mitigating liquefaction related damages by reinforcement and densification effects. GPs can be made to resist pullout or uplift by a simple modification of placing an anchor at the base with a cable or rod attached to the footing to transfer the applied pullout forces to the bottom of the GP. Such an assembly is termed a Granular Pile Anchor (GPA). Undrained strength and deformation modulus of the soil increase linearly with depth in normally consolidated soils. Deformation modulus of granular pile material can be considered to increase with depth (non-homogenous GPA) due to increasing confining pressure with depth. An analysis of displacements of GPAs, whose modulus of deformation increases linearly with depth in homogenous ground (undrained strength constant with depth) is presented in this paper. A parametric study was carried out to quantify the effects of length to diameter (L/d) ratio and relative stiffness of GP material with respect to that of in situ soil on the variation of tip and head displacements of GPAs. Results are presented as variation of normalized load-displacements as a function of non-homogenous stiffness modulus of GPA.

1 INTRODUCTION

Granular piles/stone columns improve the performance of foundations on soft ground by reducing the settlements to an acceptable level and by increasing the load carrying capacity. In addition, they increase the rate of consolidation, improve the stability and resistance to liquefaction by (a) preventing build-up of high pore pressures and (b) increasing the strength and stiffness of ground. GPs are ideally suited for improvement of soft clays, silts and loose granular deposits by forming reinforcing elements of low compressibility and high shear strength (Madhav & Nagpure 1995).

Granular piles can be made to resist pullout or uplift forces by a simple modification of placing an anchor at the base and attaching the same by a cable or rod to the footing to transfer the applied force to the bottom of the GP. Such an assembly is termed a Granular Pile Anchor (GPA). Phani Kumar (1997) reported tests on models of granular pile anchors to control heave in expansive soils. Granular pile treated expansive soil adjusts itself to changes in moisture better than an untreated soil (Phani Kumar *et al.* 2008, Sharma *et al.,* 2004). White *et al.* (2001) studied the application of reinforced geopiers for resisting tensile loads and settlement control. Lillis *et al.* (2004) reported results from in situ tests on pullout response of GPAs. Kumar *et al.* (2004) presented results from laboratory and field tests on pullout response of GPAs in cohesive and cohesionless soils. A linear analysis of displacements of GPAs is presented by Madhav *et al.* (2008). Hsu (2000), Wissmann *et al.* (2001) and Caskey (2001) studied the uplift capacity of rammed aggregate piers in uplift by conducting in-situ tests. Krishna and Murty (2013) carried out laboratory tests to study the effect of granular anchor pile in resisting the heave in soils. This paper presents the effect of deformation modulus of granular pile increasing with depth on displacements of GPA in homogenous ground.

2 PROBLEM DEFINITION

2.1 *Definition of GPA*

A granular pile of length, L, and diameter, d, with the soil and pile material characterized by moduli of deformation E_s and E_{gp}, and unit weights of γ_s and γ_{gp}, respectively is considered (Fig. 1). The undrained shear strength of the soil is considered as a constant with depth (Fig. 2). Poisson's ratio of the soil is ν_s. A force, P_o, applied at the base of the GPA is resisted by the shear stress, τ, acting along the periphery of the pile. The force and the stresses acting on and the displacements (upward movements) of the GPA are depicted in Fig. 3a. The shear stresses vary with depth, z. The displacements for a compressible pile increase with depth from a value of ρ_{u0} at the top to ρ_{uL} at the tip. The stresses transferred by GP to the in situ soil are shown in Fig. 3b. To evaluate the upward displacements of the elements of the soil adjacent to the GPA due to the boundary stresses, τ, the GPA surface is divided in to ‘n’ elements of length, L (=L/n). The stress acting on a typical element, j, is τ_j. The displacement at the centre of an element, i, due to stresses acting on element, j, are obtained by the method described by Poulos & Davis (1980).

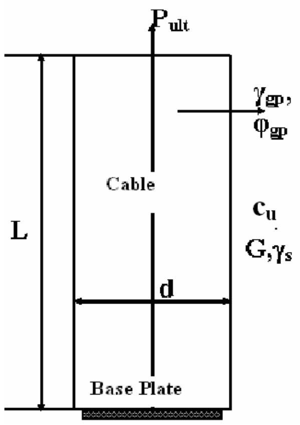

Fig. 1 GPA under pullout.

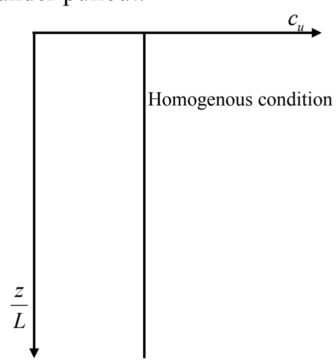

Fig. 2 Homogenous ground: Undrained Shear Strength, c_u constant with depth.

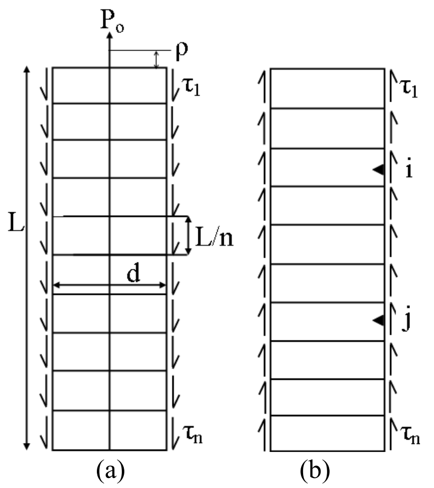

Fig. 3 Forces and stresses acting on GPA and Soil.

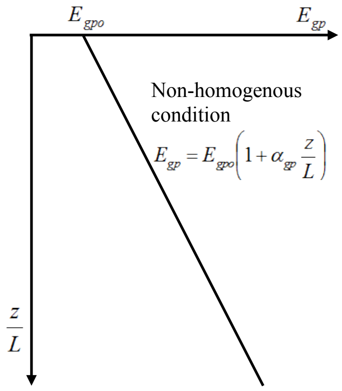

Fig. 4 Variation of deformation modulus of granular material with depth.

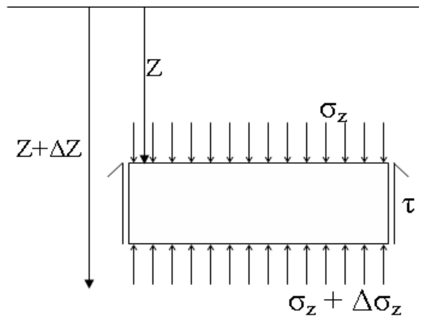

Fig. 5 Stresses acting on an infinitesimal Element.

Integrating numerically, Mindlin’s equation (1936) for a point load in the interior of a semi-infinite elastic continuum over the cylindrical periphery of the element, the displacement, $\rho_{s,ij}$, of the soil adjacent to the centre of the i^{th} element due to stress, τ_j, acting on the element, j, is obtained as

$$\rho_{s,ij} = \frac{d.I_{s,ij}.\tau_j}{E_s} \qquad (1)$$

where $I_{s,ij}$ – is the soil displacement influence coefficient. The total soil displacement, $\rho_{s,i}$, adjacent to node ‘i’ due to stresses on all the elements of the GPA, is obtained by summing up all the displacements at node ‘i’, as

$$\rho_{s,i} = \frac{d}{E_s}\sum_{i=1}^{n} I_{s,ij}.\tau_j \qquad (2)$$

Displacements at all the nodes are collected to arrive at

$$\{\rho_S\} = \frac{d}{E_s}[I_S].\{\tau\} \qquad (3)$$

where $\{\rho_s\}$ & $\{\tau\}$ are the soil displacement and shear stress vectors of size, n, respectively and $[I_s]$

is the soil displacement influence coefficient matrix of size nxn.

2.2 *Displacements of GPA*

The displacements of GPA are obtained considering it to be a compressible pile. The deformation modulus, E_{gp}, of GP increases linearly with depth (Fig. 4) as,

$$E_{gp} = E_{gpo}\cdot\left(1+\alpha_{gp}\cdot\left(z/L\right)\right) \quad (4)$$

where α_{gp} – non-homogeneity parameter signifying the rate of increase of modulus with depth. Considering stresses on an infinitesimal element (Fig. 5) of GPA of thickness, Δz, the equilibrium of forces in the vertical direction reduces to,

$$\frac{\sigma_z.\pi.d^2}{4} - \frac{(\sigma_z+\Delta\sigma_z)\pi.d^2}{4} - \tau.\pi.d.\Delta z = 0 \quad (5)$$

where σ_z and ($\sigma_z+\Delta\sigma_z$) are respectively the normal stresses on the top and bottom faces of the infinitesimal element. Eq. 5 is reduces to

$$\frac{d\sigma_z}{dz} - \frac{4}{d}\tau = 0 \quad (6)$$

The stress-strain relationship for GPA material is,

$$\sigma_z = E_{gp}.\varepsilon_z = E_{gp}.\frac{d\rho_{gp}}{dz} \quad (7)$$

where ε_z and ρ_{gp} are respectively the axial strain and displacement in GPA and E_{gp} is the modulus of deformation of the granular pile. Substituting for E_{gp} from Eq. 4 and differentiating with depth z, Eq. 7 reduces to,

$$\frac{d\sigma_z}{dz} = -\frac{d}{dz}\left[E_{gpo}\cdot\left(1+\alpha_{gp}\cdot\left(z/L\right)\right)\frac{d\rho_{gp}}{dz}\right] \quad (8)$$

On simplification

$$\frac{d\sigma_z}{dz} = -E_{gpo}\left[\left(\alpha_{gp}/L\right)\frac{d\rho_{gp}}{dz} + \left(1+\alpha_{gp}\cdot\left(z/L\right)\right)\frac{d^2\rho_{gp}}{dz}\right]. \quad (9)$$

Combining EqS. 6 & 9 and simplifying

$$-E_{gpo}\left[\left(\alpha_{gp}/L\right)\frac{d\rho_{gp}}{dz} + \left(1+\alpha_{gp}\cdot\left(z/L\right)\right)\frac{d^2\rho_{gp}}{dz}\right] + \frac{4}{d}\tau = 0 \quad (10)$$

Eq. 10 is solved along with the boundary conditions: at z =0 (i.e. at the top of GPA) P=0 (Free boundary) and at z=L (tip of the GPA), P=P_0 (the applied load). Eq. 10 in Finite Difference form becomes,

$$\left\{\frac{\alpha_{gp}}{L}\frac{(\rho_{gp,i-1}-\rho_{gp,i+1})}{2.(\Delta z)} + (1+\alpha_{gp}.z_i^{'})\frac{(\rho_{gp,i-1}-2\rho_{gp,i}+\rho_{gp,i+1})}{(\Delta z)^2}\right\} - \frac{4}{E_{gpo}.d}\tau_i = 0 \quad (11)$$

where Δz = (L/n) – differential length of GP. Rewriting Eq. 11

$$\left\{a_i.\rho_{gp,i-1} - 2.b_i.\rho_{gp,i} + c_i.\rho_{gp,i+1}\right\} - \frac{4L^2}{n^2.E_{gpo}.d}\tau_i = 0 \quad (12)$$

where a_i, b_i and c_i are coefficients.

$$a_i = \left(1+\alpha_{gp}.z_i^{'} - \frac{\alpha_{gp}}{2.n}\right),\ b_i = \left(1+\alpha_{gp}.z_i^{'}\right),\ c_i = \left(1+\alpha_{gp}.z_i^{'} + \frac{\alpha_{gp}}{2.n}\right),$$

$\rho_{gp,i}$ and τ_i are respectively the displacement at the centre of node 'i' and the shear stress on the interface of element, 'i', of the GPA. Eq. 12 is written for nodes i = 2 to (n-1). Invoking the first boundary condition, P=0 implies stress, σ_z=0 and hence strain, ε_z= 0 which leads to,

$$\rho_{gp,1} = \rho_{gp,1'} \quad (13)$$

where $\rho_{gp,1'}$ is the displacement at imaginary node 1' above the GPA. Eq. 13 for node 1 becomes,

$$a_{1'}.\rho_{gp,1'} = 2.b_1.\rho_{gp,1} \quad (14)$$

All the equations for nodes 1 to (n-1) are collated as,

$$\left[I_{gp}\right]\left\{\rho_{gp}\right\} - \frac{4.L^2}{E_{gp}.n^2.d}.\{\tau\} = 0 \quad (15)$$

The pile displacement influence co-efficient matrix of size n x (n-1) is,

$$\begin{bmatrix} -c_1 & c_1 & 0 & . & . & . & . & . & . & 0 \\ a_2 & -2b_2 & c_2 & 0 & . & . & . & . & . & 0 \\ 0 & a_3 & -2b_3 & c_3 & 0 & .. & . & . & . & 0 \\ 0 & 0 & a_4 & -2b_4 & c_4 & 0 & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & .. \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & a_{n-1} & -2b_{n-1} & c_{n-1} \end{bmatrix} \quad (16)$$

Eqs. 3 and 16 are normalized with diameter, d, and undrained shear strength c_u to get

$$\left\{\rho_s^*\right\} = \frac{c_{uo}}{E_s}\left[I_s\right]\left\{\tau^*\right\} \quad (17)$$

Pile displacement equation

$$\left[I_p\right]\left\{\rho_p^*\right\} - \frac{4.\left(L/d\right)^2.c_{uo}}{E_{gp}.n^2}[1]\left\{\tau^*\right\} = 0 \quad (18)$$

Compatibility of displacements requires displacements in soil and GPA to be the same as, $\left\{\rho_p^*\right\} = \left\{\rho_s^*\right\}$

Pile displacement equation written as,

$$\left[I_p\right]\left\{\rho_s^*\right\} - \frac{4.\left(L/d\right)^2}{\left(E_{gp}/c_{uo}\right).n^2}[1]\left\{\tau^*\right\} = 0 \quad (19)$$

Considering the displacement at an imaginary node, ρ_{n+1}, below the node 'n' normalized with diameter 'd', the boundary condition at z= L is written as

$$\left\{\rho_{n+1}^* - \rho_n^*\right\} = \frac{4.P_0^*.\left(L/d\right)^2}{\left(E_{gpo}/c_{uo}\right).n.\left(1+\alpha_{gp}\right)} \quad (20)$$

Eq. 20 can be simplified as,

$$\left\{\rho_{n+1}^{*}-\rho_{n}^{*}\right\}=\frac{n.\mu.P_{0}^{*}}{\left(1+\alpha_{gp}\right)} \quad (21)$$

Eqs. 17, 18 and 21 are combined and written in matrix form as,

$$\begin{bmatrix} [1] & \frac{-1}{\left(\frac{E_s}{c_{uo}}\right)}[I_s] \\ [I_p] & -\mu.[1] \end{bmatrix} \begin{Bmatrix} \rho_s^{*} \\ \tau^{*} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ n.\mu.P^{*} / \left(1+\alpha_{gp}\right) \end{Bmatrix} \quad (22)$$

If the shear stress, τ_i, on any element, 'i' reaches or exceeds the undrained strength, c_u, the i^{th} row in Equations 23 is replaced with,

$$\{0 \quad 0 \quad . \quad . \quad 1 \quad 0 \quad 0\}\{\tau\}=c_u \quad (23)$$

Yielding generally starts from the tip, i.e. at n^{th} element and progresses towards the top.

3 RESULTS AND DISCUSSION

The load-displacement responses as a function of the non-homogenous granular material deformation modulus parameter, α_{gp} for L/d=10, ν_s = 0.5, E_{gpo}/E_s=50 and E_s/c_u=200 are presented in Figure 6. The load, P^*_N is normalized with $\pi d^2 c_u/4$ tip displacement, δ^* with diameter (ρ/d) for compressible GPA. The maximum normalized load, P^*_N = 40 for L/d=10. The slopes of the pullout load versus tip displacement plots increase with increasing α_{gp} as is to be expected. The decrease in displacements is mainly due to the increasing stiffness of the granular material considered. The displacements decrease from 0.6 to 0.4 for P^*_N value of 30 for α_{gp} increasing from 0 to 2. The variations of load-displacement responses as a function of α_{gp} are small at smaller loads up to P^*_N=20 and become significant at higher loads.

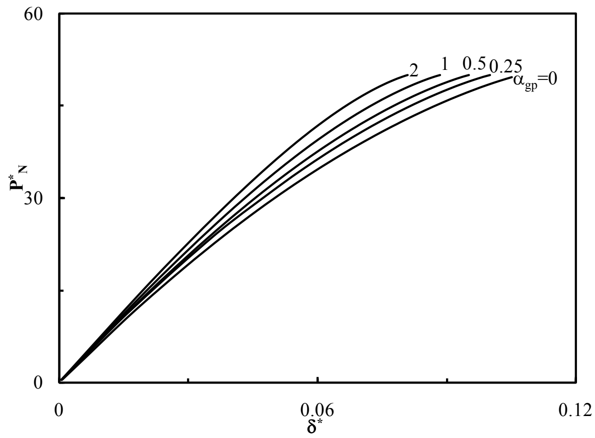

Fig. 6 P^* vs. δ^* for E_{gpo}/E_s=100, ν_s=0.5, E_s/c_u=200, & L/d=10 - Effect of α_{gp}

4 CONCLUSIONS

Analysis of GPA considering non-homogenous GPA (modulus of GPA increases linearly with depth) and the in situ soil - interface response to be elasto-plastic is presented. The deformation modulus is considered to increase linearly with depth. The elastic continuum approach of Poulos and Davis (1980) is extended to predict the pullout load – displacement response of GPA. A parametric study has been carried and results in the form of load – displacement responses with relative stiffness factor, K (=E_{gpo}/E_s) and E_s/c_{uo} are presented for compressible GPA. The normalized displacements at given load decrease with increasing deformation modulus of GPA. The load - displacement response is not sensitive to α_{gp} at working loads.

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