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A Constitutive Model for Lignosulfonate Treated Silty Sand

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ABSTRACT: In this paper a new constitutive model based on the Disturbed State Concept (DSC) is proposed for a lignosulfonate treated highly erodible silty sand. In this model, an elasto-plastic Relative Intact (RI) response and the lignosulfonate bond strength were used as the two reference states to define the disturbance function. In order to determine the model parameters and to validate the proposed DSC model, a series of laboratory direct shear tests were conducted under drained condition. It was observed from the laboratory tests that the lignosulfonate treatment increased the shear strength of soil irrespective of the effective normal stress. Also, the volumetric responses showed dilative behaviour with the lignosulfonate treatment. The comparisons of the model predictions with experimental data clearly showed that the proposed model can accurately capture the stress-strain and volume change behaviour of the silty sand similar to the laboratory experiments.

1 INTRODUCTION

It has recently been proved that lignosulfonate can increase the resistance of highly erodible and dispersive soils, overcoming the environmental problems caused by traditional chemical stabilisers (e.g. Indraratna et al; 2008; Athukorala et al 2013). Lignosulfonate increases the critical shear stress (shear stress required to initiate soil erosion) and decreases the coefficient of soil erosion. Also, it has been revealed that the amount of lignosulfonate required to make the soil non-erodible is less than that of cement for certain types of soils. Recent laboratory studies on silty sand have revealed that lignosulfonate increased the shear strength of soil, and the volumetric response was more dilative after treatment. However, the stress-strain behaviour of the lignosulfonate treated soil is different from that of the soils stabilised by traditional admixtures such as cement and lime. For instance, unlike the cement treated soil, there is no considerable increase in the brittleness or the stiffness in soil due to lignosulfonate treatment (Indraratna et al 2013; Indraratna et al; 2015). Therefore, the conventional modelling approach used for the chemically stabilised soils is not suitable for lignosulfonate treated soils. Based on the above laboratory observations, a new constitutive model for lignosulfonate treated silty sand is proposed in this paper, incorporating the Disturbed State Concept (DSC) (Desai, 2001). The DSC can be used to characterise the behaviour of the bonded materials such as cemented soils, as well as the untreated soil. However, to the best of the author's knowledge there is no DSC model for bonded soils reported in literature. In this model the relative intact (RI) response of lignosulfonate treated soil was modelled by incorporating the δ_0 version of the Hierarchical Single Surface (HiSS) plasticity models where non-associated yielding was considered through the disturbance function. The response of lignosulfonate bonds was modelled separately using DSC, by considering a linear elastic RI response and zero strength condition as the relative states.

2 DEVELOPMENT OF DSC MODEL FOR LIGNOSULFONATE TREATED SILTY SAND

The reference states considered for this DSC model and the disturbance for lignosulfonate treated soils are illustrated in Fig. 1.

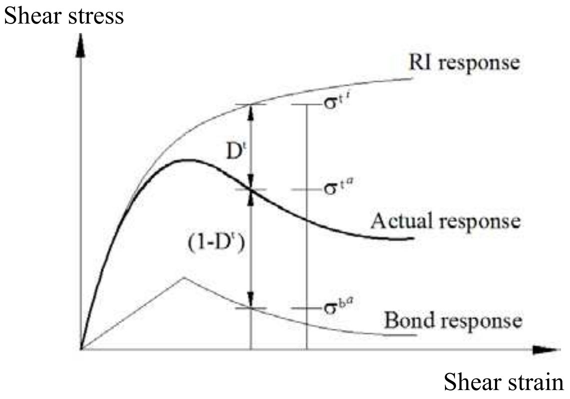

Fig. 1 Disturbance for lignosulfonate treated soil (after Athukorala et al; 2014).

For the stand alone clarity of this paper and for the benefit of the reader, the salient equations published in Athukorala et al; (2014) are listed below.

Based on the definition of the disturbance function by Desai (2001), the disturbance for lignosulfonate treated silty sand expressed in terms of stresses can be given as:

$$D^t = \frac{\sigma^{t^i} - \sigma^{t^a}}{\sigma^{t^i} - \sigma^{b^a}} \tag{1}$$

where the superscripts i, a, and b are used to indicate the RI response, the observed (actual) response, and the bond (FA) response, respectively. The superscript "t" used in Eq. (1) represents the lignosulfonate treated behaviour.

2.1 *Relative Intact (RI) response*

The RI behaviour of this model was simulated using the δ_0-version of HiSS plasticity models (Desai 2001) where the yield function (F) is expressed by:

$$F = \left(\frac{\tau^t}{P_a}\right)^2 + \alpha^t \left(\frac{\sigma_n'}{P_a}\right)^{n^t} - \gamma^t \left(\frac{\sigma_n'}{P_a}\right)^{q^t} = 0 \tag{2}$$

and where σ'_n is the effective normal stress, τ^t is the shear stress, P_a is the atmospheric pressure, n^t is the phase change parameter, γ^t is the ultimate parameter, q^t is a parameter depends on the linearity of the ultimate envelope, and α^t is the hardening function given by:

$$\alpha^t = \frac{a^t}{{\xi_D}^{b^t}} \tag{3}$$

In Eq. (3), a^t and b^t are the hardening parameters and ξ_D is the trajectory of the deviatoric plastic strains.

2.2 *Fully Adjusted (FA) response*

In this model the observed (actual) response of lignosulfonate bonds was considered as the FA response for lignosulfonate treated behaviour. The bond response was modelled using the DSC concept with a linear elastic RI response and a zero strength FA response. The incremental stress-strain relationships for the observed response of the lignosulfonate bonds can be given as:

$$d\underset{\sim}{\sigma}^{b^a} = \underset{\sim}{C}^{b^{DSC}} d\underset{\sim}{\varepsilon}^{b^i} \tag{4}$$

where $\underset{\sim}{C}^{b^{DSC}}$ is the disturbed state concept constitutive matrix for lignosulfonate bonds. More details on modelling the bond response are given in Athukorala et al. (2014).

2.3 *Actual response*

Based on the definition of the disturbance given by Eq. (1), the incremental stress-strain relationship for the actual response of lignosulfonate treated soil can be evaluated as:

$$d\underset{\sim}{\sigma}^{t^a} = \underset{\sim}{C}^{t^{DSC}} d\underset{\sim}{\varepsilon}^{t^i} \tag{5}$$

where $d\underset{\sim}{\varepsilon}^{t^i}$ is the vector of incremental strains of lignosulfonate treated soil in the RI state, and $\underset{\sim}{C}^{t^{DSC}}$ is the disturbed state concept constitutive matrix for lignosulfonate treated soil as given by:

$$\underset{\sim}{C}^{t^{DSC}} = (1 - D^t)\underset{\sim}{C}^{t^{(ep)i}} + D^t \underset{\sim}{C}^{b^{DSC}} + \underset{\sim}{\sigma}_{\underset{\sim}{R}}^{t} \underset{\sim}{C}^{t^R} \tag{6}$$

where the disturbance function D^t, can be expressed in terms of the plastic strain trajectory ξ, as:

$$D^t = D_u^t \left[1 - e^{-A^t \left(\xi_D^t - \xi_D^{t*}\right)^{Z^t}}\right] \tag{7}$$

where D_u^t is the ultimate value of D^t at the residual, ξ_D^{t*} is the deviatoric plastic strain trajectory below which the disturbance is zero, and A^t and Z^t are the model parameters for lignosulfonate treated soil. In Eq. (6), $\underset{\sim}{\sigma}_R^t = \left(\underset{\sim}{\sigma}^{b^a} - \underset{\sim}{\sigma}^{t^i}\right)$, $\underset{\sim}{C}^{t^{(ep)i}}$ is the elasto-plastic constitutive matrix for the RI behaviour of lignosulfonate treated soil, and $\underset{\sim}{C}^{t^R}$ is given by:

$$\underset{\sim}{C}^{t^R} = \frac{\left[D_u^t A^t Z^t \left(\xi_D^t - \xi_D^{t*}\right)^{(Z^t - 1)} e^{-A^t \left(\xi_D^t - \xi_D^{t*}\right)^{Z^t}}\right] \left[\left(\frac{\partial F^t}{\partial \underset{\sim}{\sigma}}\right)^T \cdot \frac{\partial F^t}{\partial \underset{\sim}{\sigma}}\right]^{1/2}}{\left(\frac{\partial F^t}{\partial \underset{\sim}{\sigma}}\right)^T \cdot \underset{\sim}{C}^{t^{(e)i}} \cdot \frac{\partial F^t}{\partial \underset{\sim}{\sigma}} - \frac{\partial F^t}{\partial \xi_D^t} \cdot \left[\left(\frac{\partial F^t}{\partial \underset{\sim}{\sigma}}\right)^T \cdot \frac{\partial F^t}{\partial \underset{\sim}{\sigma}}\right]^{1/2}} \left(\frac{\partial F^t}{\partial \underset{\sim}{\sigma}}\right)^T \cdot \underset{\sim}{C}^{t^{(e)i}} \tag{8}$$

3 DETERMINATION OF THE MODEL PARAMETERS

In order to determine the model parameters, a laboratory experimental investigation was carried out on direct shear box. The stress-strain and volume change behaviour of a lignosulfonate treated silty sand were observed in the drained condition. The properties of the silty sand and lignosulfonate used in this study are given in Table 1 while Fig. 2 shows the particle size distribution of the silty sand.

The shear tests were conducted for 0.2%, 0.6%, 1.2% lignosulfonate treated and untreated soil under five different effective normal stresses (5kPa, 10kPa, 15kPa, 22kPa and 42kPa). Only the test results corresponding to 5kPa, 15kPa, and 42kPa were used to determine the model parameters.

Table 1. Properties of the tested soil and lignosulfonate (after Athukorala et al 2014)

Silty sand		Lignosulfonate	
Property	Value	Property	Value
G_s	2.67	G_s	1.2 approx.
MDD	18.03 kN/m^3	pH	3.8 approx.
OMC	11.6%	Appearance	Dark brown liquid
Liquid limit	22.5%	Solubility in water	Completely soluble
Plasticity Index	Non-plastic	Other	Non-flammable
Classification*	SM		Non-toxic
PD†	44%		
Erodibility‡	D1		

*according to USCS, †Percent dispersion value from percent dispersion test, ‡from standard pinhole test

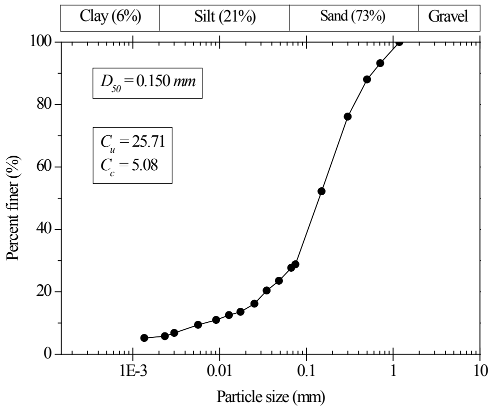

Fig. 2 Particle size distribution of silty sand used in this study (after Athukorala et al; 2014)

3.1 *RI parameters*

The experimental results of lignosulfonate treated soil were fitted to hyperbolic curves to obtain the corresponding RI responses. The hyperbolic relationship between the shear stress and the shear strain can be expressed by (Clough and Duncan 1971):

$$\tau = \frac{\gamma}{\frac{1}{E_S} + \frac{\gamma}{\tau_{ult}}} \tag{9}$$

where τ is the shear resistance, γ is the shear strain, and τ_{ult} is the asymptotic value of shear stress at infinite strain of the hyperbolic curve. The model parameters corresponding to those hyperbolic curves were then calculated.

The procedure for calculating the RI parameters for lignosulfonate treated soil is given in Athukorala et al (2014).

3.2 *FA parameters*

To determine the FA parameters for lignosulfonate treated soil, the experimental bond strengths were calculated from the laboratory shear test results as the difference between the shear stresses of treated and untreated soil. Using a linear elastic RI response and the zero strength FA response, the lignosulfonate bond strength was modelled by DSC (Fig. 3).

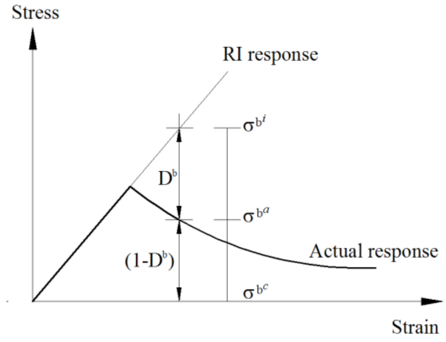

Fig. 3 DSC for lignosulfonate bonds (after Athukorala et al 2014)

The RI and disturbance parameters for lignosulfonate bonds became the FA parameters for lignosulfonate treated behaviour, as explained in section 2.2. More details of the determination of model parameters for lignosulfonate bonds are available in Athukorala et al (2014).

3.3 *Disturbance function parameters*

The values of actual disturbances for lignosulfonate treated silty sand were determined using Eq. (1), and then these disturbances were fitted to Eq. (7) to obtain the disturbance function parameters, D_u^t, A^t and Z^t.

4 MODEL PREDICTIONS

The DSC model parameters determined for lignosulfonate treated soil are summarised in Table 2.

Table 2. Calculated model parameters for lignosulfonate treated silty sand (after Athukorala et al 2014)

	Parameter	Value
RI parameters	γ^t	11.71
	q^t	1.36
	n^t	1.70
	a^t	20.25
	b^t	0.037
FA parameters	D_u^b	1
	γ^*	$\gamma^* = 0.24\sigma_n' + 2.35$
	A^b	$A^b = -0.3 \ln \sigma_n' + 1.12$
	Z^b	$Z^b = 0.24 \ln \sigma_n' + 0.19$
Disturbance parameters	D_u^t	0.8
	ξ_D^{*t}	0
	A^t	$A^t = -0.22 \ln \sigma_n' + 0.88$
	Z^t	$Z^t = 0.35e^{-0.085\sigma_n'}$

These parameters were used to predict the stress-strain and volume change behaviour of 0.2%, 0.6% and 1.2% lignosulfonate treated silty sand at 10kPa and 22kPa effective normal stresses (Fig. 4). They were considered to be independent predictions because the shear test results at 10kPa and 22kPa effective normal stresses were not used to calculate the parameters.

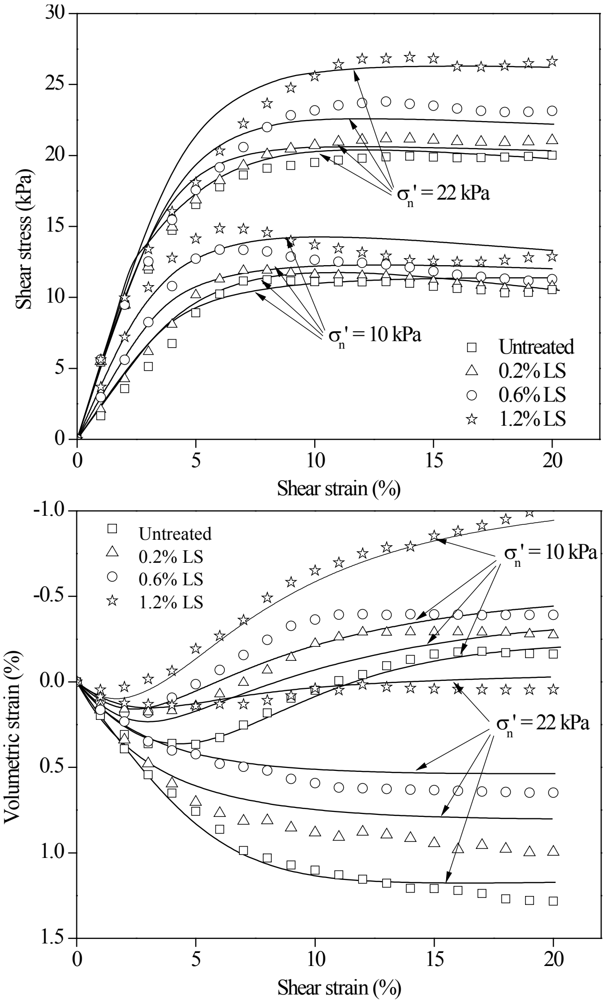

Fig. 4 Comparison of stress-strain and volume change behaviour of lignosulfonate treated silty sand with independent test data (after Athukorala et al; 2014).

The direct shear test results revealed that with lignosulfonate treatment, the shear strength of silty sand increased at a given effective normal stress. However, the stiffness only increased slightly due to treatment, and the change in ductility was negligible. Also, the volume changes showed more dilation after treatment. Comparisons between the predicted stress-strain curves and the experimental data shown in Fig. 4 verified that the experimentally observed stress-strain behaviour was captured very well by the proposed DSC model. Fig. 4 also confirms that the predicted volume changes agreed with the experimental observations.

5 CONCLUSION

In this paper a constitutive model for lignosulfonate treated silty sand was proposed, incorporating the original Disturbed State Concept. The δ_0-version of the HiSS plasticity models was used to simulate the RI behaviour and the lignosulfonate bond strength was considered as the FA state for lignosulfonate stabilized soil. The bond strength was modelled separately using a linear elastic RI response and zero strength state as the reference states. A series of drained direct shear tests on silty sand treated with lignosulfonate were carried out, and then the test results were used to calculate the relevant model parameters. The model predicted stress-strain and volume change behaviour was compared with an independent set of experimental test results. The comparisons verified that the proposed DSC model can accurately capture the changes in shear strength, stiffness, and volume changes due to lignosulfonate treatment.

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