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Dynamic Response of a Pile within an Anisotropic Homogeneous Soil

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ABSTRACT: This paper presents a mathematical model for the study of time-harmonic vibrations of a pile embedded in a transversely isotropic soil. The pile-soil interaction problem requires the derivation of Green's functions corresponding to buried distributed vertical loading in a half-space. The motion of the pile is represented by a time-harmonic polynomial function containing a set of generalized coordinates. The body force field in the layered medium corresponding to each polynomial shape function of the pile is obtained numerically by solving a flexibility equation system based on the Green's functions corresponding to buried distributed vertical body forces in the layered soil. The generalized coordinates of the pile displacement function are thereafter computed by formulating the Lagrange's equations of motion for the pile. The present formulation can be extended to study pile groups as well as piles under lateral and torsional excitations.

1 INTRODUCTION

A review of the literature reveals that an exact analytical formulation of the problem of embedded piles is not available even for the case of static loadings. A limited number of reports are available in the literature on the solution of the dynamic case using numerical schemes. Novak (1977) has investigated the harmonic response of piles embedded in layers of soil. Apsel (1979) solved the problem of a rigid cylinder embedded in an elastic half-space. Sen et al. (1985) presented a scheme to deal with the problem of a finite elastic bar under lateral dynamic loads. Rajapakse and Shah (1987) presented solutions for the longitudinal, rocking, transverse and coupled rocking-transverse behaviors of an elastic bar embedded in an elastic isotropic half-space. Models of finite beam elements have been used by Barros (2003) and Barros (2004) to study respectively the axial and transverse response of piles embedded in a half-space. In these two works, the surrounding medium of the piles were transversely isotropic continua. More recently, Lu et al. (2009) have investigated the axial response of piles embedded in multilayered poroelastic media.

This paper presents a mathematical model for the study of time-harmonic vibrations of an elastic pile embedded in a transversely isotropic half-space (Fig. 1). The formulation is presented within the framework of linear theory of elastic wave propagation that is applicable for most cases of foundation vibration problems under external excitations.

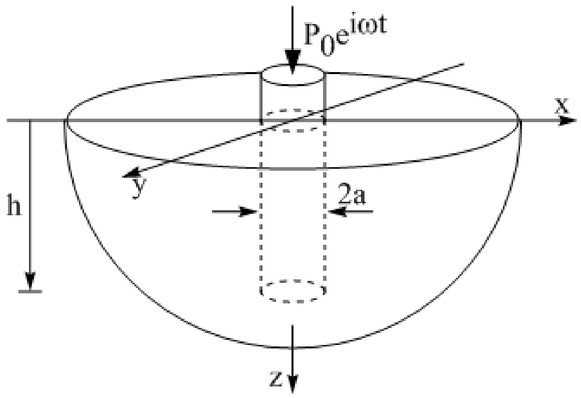

Fig. 1 Geometry of the bar and its surrounding medium.

The pile-soil interaction problem requires the derivation of Green's functions corresponding to buried distributed vertical loading in the half-space. The motion of pile is represented by a time-harmonic exponential function containing a set of generalized coordinates. The body force field in the half-space corresponding to each exponential shape function of the pile is obtained numerically by solving a flexibility equation system based on the Green's functions corresponding to buried distributed vertical body forces in the half-space. The generalized coordinates of the pile displacement function are thereafter computed by formulating the Lagrange's equations of motion for the pile. Numerical solution of the equation motion results in the solutions for generalized coordinates. The pile displacements and axial load profiles are determined from the solutions for generalized coordinates. The present formulation can be extended to study pile groups (Taherzadeh, Clouteau and Cottereau, 2009) as well as piles under lateral and torsional excitations.

2 GREEN'S FUNCTION AND BOUNDARY VALUE PROBLEM

Consider an elastic, transversely isotropic, three-dimensional full-space. The problem is governed by the Cauchy-Navier differential equations, which couples the displacement components $u_i=u_i(r,\theta,z,\omega)$ $(i=r,\theta,z)$. Rajapakse and Wang (1993) proposed a solution for this coupled problem in terms of Hankel integral transforms and series expansion. The solution is written in terms of arbitrary functions, the values of which are determined from the boundary and continuity conditions of a given problem.

In order to invoke the appropriate boundary conditions for the modeling of the pile problem, consider a half-space of Young's modulus E_h and mass density ρ_h, containing a buried vertical load of unit intensity. In the r direction, the load is concentrated on the circumference of a circle of radius *a*. In the z direction, the load is distributed from $z_1 \le z \le z_2$. The load is, therefore, uniformly distributed on a finite cylindrical shell within the half-space. In this paper, the vertical displacement of a ring of points of coordinates (r_i, z_i) due to this cylindrical vertical load is called $u_{ZZ}(r_i, z_i, a, z_1, z_2)$, in reference to the position of the load and the measuring point. A detailed description of these calculations is presented by Labaki et al. (2014).

3 MODEL OF AN ELASTIC BAR

Consider an elastic cylindrical bar of radius *a* and length h embedded within an elastic half-space (Fig. 1). The bar is assumed to behave like a 1-dimensional continuum with Young's modulus E_b and mass density ρ_b. The surface of the bar, described by $(r=a, 0\le\theta\le 2\pi, 0\le z\le h)$ and $(z=h, 0\le r\le a)$ is perfectly bonded with its surrounding half-space. The top surface of the bar is under axisymmetric, time-harmonic vertical load of intensity P_0.

The linear partial differential equation that relates the deflection profile w(z) of the 1D pile and the loading p(z) applied on it is:

$$A_b E^* \frac{d^2}{dz^2} w(z) - \rho^* A_b \ddot{w}(z) = -p_z(z) \quad (1)$$

in which A_b is the cross-sectional area of the bar, $A_b=\pi a^2$, and $E^*=E_b-E_h$ and $\rho^*=\rho_b-\rho_h$. A trial deflection profile is established for the bar in the following form:

$$w(z,t) = \sum_{n=1}^{N} \alpha_n(t) e^{-(n-1)z/h} \quad (2)$$

The summation in Eq. (2) comprises an approximation for the deflection of the pile w(z) by a series of exponential functions. Each exponential profile is weighed by a generalized coordinate α_n (n=1,N). The corresponding derivative of the deflection profile with respect to time is given by:

$$\dot{w}(z,t) = \sum_{n=1}^{N} \dot{\alpha}_n(t) e^{-(n-1)z/h} \quad (3)$$

Based on Eqs. (2) and (3), the strain and kinetic energy of the bar can be expressed as:

$$U_a = \sum_{m=1}^{N}\sum_{n=1}^{N} D_{mn}\alpha_m\alpha_n \quad (4)$$

$$T_a = \sum_{m=1}^{N}\sum_{n=1}^{N} C_{mn}\dot{\alpha}_m\dot{\alpha}_n \quad (5)$$

in which

$$D_{mn} = \frac{\pi E^* a^2 (n-1)(m-1)}{2(m+n-2)h}\left(1-e^{-(m+n-2)}\right) \quad (6)$$

for $m+n\neq 2$

$$D_{mn} = 0 \quad \text{for } m+n=2 \quad (7)$$

$$C_{mn} = \frac{\pi\rho^* h a^2}{2(m+n-2)}\left(1-e^{-(m+n-2)}\right), \ m+n\neq 2 \quad (8)$$

$$C_{mn} = \frac{\pi\rho^* h a^2}{2}, \ m+n=2 \quad (9)$$

Let $B_z(z)$ represent the body force along the body of the bar in the vertical direction. It is assumed that the influence of radial body forces is negligible (Rajapakse, 1988). Let $B^n_z(z)$ be the vertical body force corresponding to each term within the summation in Eq. (2). The resulting total body force will be given by all these individual terms by:

$$B_z = \sum_{n=1}^{N} \alpha_n B_z^n \quad (10)$$

A solution for $B^n_z(z)$ by analytical methods is not feasible due to the complexity of the problem. A numerical solution is obtained by considering a discretization of the pile.

Rajapakse and Shah (1987) presented an extensive, detailed model of this problem and discussed the implications of the discretization models proposed by Fowler and Sinclar (1978) and Sen et al. (1985) to deal with this problem. Rajapakse and Shah (1987) considered a discretization of the body of the pile into a number of toroidal elements and a discretization of its end surfaces (z=0) and (z=h) into a number of concentric annular discs. For the present case in which long bars are considered and frequencies within practical ranges are assumed, a simplified discretized model suffices. The present model considers that the body of the bar is discretized into M shaft elements, within which the body forces are assumed to be uniformly distributed.

With the proposed discretization, the following flexibility equation can be established for the bar:

$$\left[A_{ij}\right]\left\{B_z^n\right\} = \left[w^1 \quad w^2 \quad \cdots \quad w^N\right]_{M\times N} \tag{11}$$

$$A_{ij}\Big|_{M\times M} = u_{zz}\left(0, z_i, a, z_{j1}, z_{j2}\right) \tag{12}$$

$$\left\{w^n\right\} = \left\langle e^{-(n-1)z_1/h} \quad \cdots \quad e^{-(n-1)z_N/h}\right\rangle^T \tag{13}$$

In Eqs. (11) to (13), M is the number of shaft elements used to discretize the bar. The function u_{ZZ} is described in Section 2. w^n is the vertical displacement corresponding to the n^{th} term inside Eq. (2) when α_n=1.

The Lagrangian of the embedded bar problem is written as:

$$L_h = \frac{1}{2}\frac{\partial}{\partial t}\int_V \rho w_i \dot{w}_i \, dV - \frac{1}{2}\int_{V^*} w_i B_i \, dV \tag{14}$$

The application of the Lagrange's equation of motion to the bar–half-space system results in the following algebraic equations:

$$\sum_{n=1}^{N}\alpha_n\left[-2\omega^2 C_{ni} + 2D_{ni} + \pi\sum_{j=1}^{M} r_j\left(B_z^i e_{nj} + B_{zj}^n e_{ij}\right)\Delta z_j \Delta r_j\right] = P_0 \tag{15}$$

In which $e_{nj}=e^{-(n-1)zj/h}$ and $e_{ij}=e^{-(i-1)zj/h}$.
The solution of Eq. (15) results in the generalized coordinates from which the deformation profile of the pile can be obtained (Eq. 2). For example, the displacement of the top end of the bar can be obtained from Eq. (2) by making z=0:

$$\Delta_0 = \sum_{n=1}^{N}\alpha_n \tag{16}$$

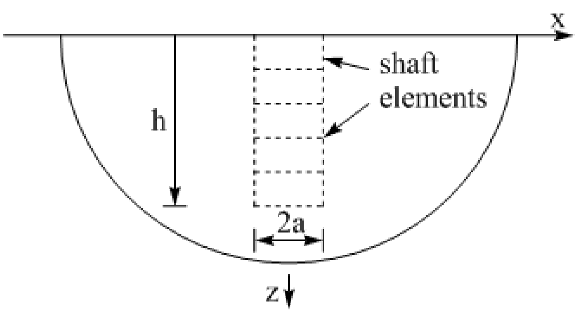

Fig. 2 Discretized model of the bar.

4 NUMERICAL RESULTS

This paper presents two selected numerical results for the vibration of the embedded bar. Figs. 3 and 4 show respectively the vibration of a bar of length h/a=5 and h/a=15. These cases employ a discretization of M=20 and M=60 cylindrical shell elements, respectively. In all cases, the Poisson ratio of the half-space is ν=0.25, and the relative mass density of the bar and half-space is $\rho'=\rho_b/\rho_h=1$. Figs. 3 and 4 show that the axial impedance Kv ($Kv=P_0/\mu_h a\Delta_0$) of the bar is significantly affected by the relative stiffness of the bar, $E'=E_b/E_h$. All cases consider a profile described by N=6 generalized coordinates (see Eq. 2).

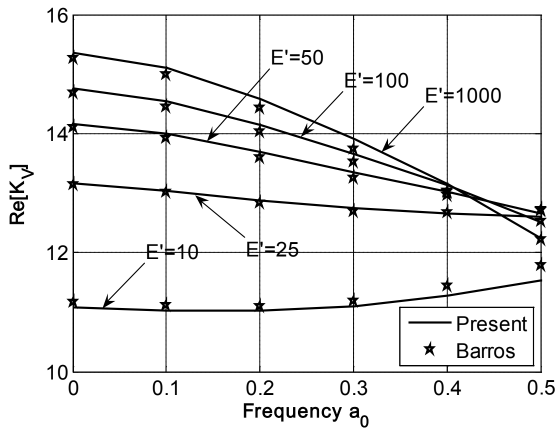

Fig. 3a Real part of the vertical impedance $Kv(a_0)$ of the elastic bar of length h/a=5 for varying E'.

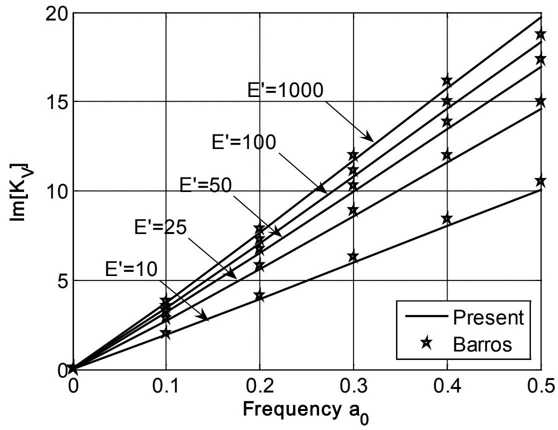

Fig. 3b Imaginary part of the vertical impedance $Kv(a_0)$ of the elastic bar of length h/a=5 for varying E'.

These results have been compared with a solution by Barros (2006). Barros solved the problem of vertical vibration of the elastic bar by an indirect formulation of the Boundary Element Method. In Figs. 3 and 4, the results by Barros are shown by black markers. These results also agree with the former model by Rajapakse and Shah (1987).

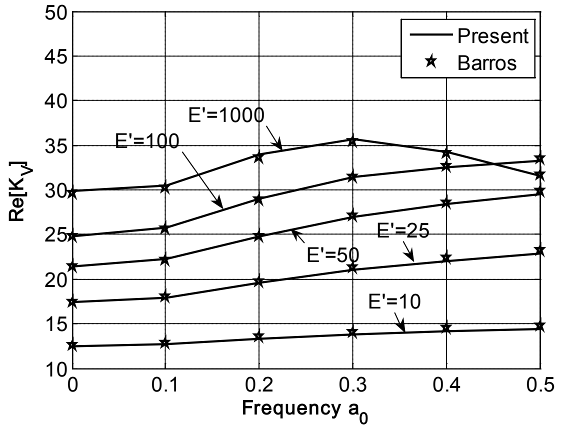

Fig. 4a Real part of the vertical impedance Kv(a_0) of the elastic bar of length h/a=15 for varying E'.

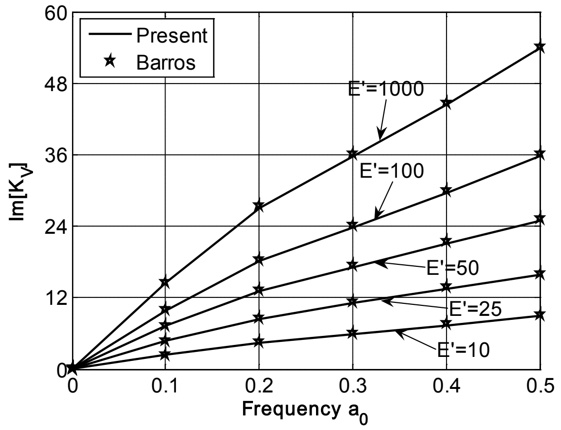

Fig. 4b Imaginary part of the vertical impedance Kv(a_0) of the elastic bar of length h/a=15 for varying E'.

5 CONCLUDING REMARKS

The present paper showed the mathematical formulation of the time-harmonic response of an elastic pile embedded in a transversely isotropic half-space. The model is based on a simplification of a previous model from the literature. The present model considers that the body of the bar is discretized into shaft elements, within which the body forces are assumed to be uniformly distributed. For the present case in which long bars are considered and frequencies within practical ranges are assumed, this simplified discretized model suffices. The model was used to investigate the variation of the axial impedance with frequency of excitation. Different cases of relative stiffness between the pile and its surrounding half-space have been considered. The results show good agreement with another model in which the indirect-BEM has been used to solve the embedded pile problem.

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