# A Constitutive Model for Soft Bangkok Clay

V. Wijeyakulasuriya
*SPL Consultants, Toronto, Canada*

A.S. Balasubramaniam
*Griffith University, Queensland, Australia*

ABSTRACT: The isotropic volumetric work hardening Cambridge critical state constitutive models (CCM), i.e. (original Cam clay (OCC) and the modified Cam clay (MCC)), are based on associative plasticity, and invoke different empirical plastic work equations to define the shape of the respective yield curves. These models depend only on three material constants M, λ and κ. This paper reports a model applicable for monotonic loading paths based on the simulation of an incremental stress path as being made up of an incremental undrained path and an incremental anisotropic compression path, using the same three material constants as the CCM. As part of the validation, the proposed model is initially applied on a constant p′ stress path for kaolin. It is then applied on a previously published stress path in triaxial compression for soft Bangkok clay. Improved predictions compared to MCC are observed for the reported test results.

## 1 INTRODUCTION

For deformation analyses in soft clays involving nearly monotonic loading paths, the Cambridge critical state constitutive models (CCM), i.e. the original Cam clay model (OCC) (Schofield and Wroth, 1968) and the modified Cam-Clay model (MCC) (Burland, 1965), have been particularly successful. Historically these models can be described as the first plastic hardening models that have received wider acceptance. However, they are proven to be more successful for normally consolidated (NC) and lightly over-consolidated (LOC) regions of clay behaviour under monotonic loading conditions.

Any hardening plastic model is characterised by four ingredients, namely,
(a) Description of elastic behaviour
(b) A yield criterion which defines the current boundary in stress space to the region of elastic behaviour; the consistency condition which requires that the stress state must remain on the yield surface when plastic strains are being generated
(c) A flow rule which describes the mechanism of plastic deformation
(d) A hardening rule which describes the dependence of the size of the yield locus on the plastic strains.

In each Cam clay model, the yield locus is derived using an empirical, experimentally based basic plastic work equation coupled with the use of the normality condition (associative plasticity). The MCC model is widely regarded as having addressed some of the shortcomings of the OCC model. Balasubramaniam (1969) has shown that in general, whilst the OCC largely over-predicts shear strains, the MCC under-predicts.

This paper reports a two-staged investigation carried out with the following objectives:
(a) To gain an insight into the relative merits of the basic plastic work equations in the two CCM models
(b) Based on the outcome of stage (a), to explore the potential for a proposal for an incremental stress-strain model and to validate it against previously published data in the triaxial space.

## 2 REVIEW OF THE PLASTIC WORK EQUATIONS

The following empirical power dissipation functions for basic plastic work, i.e. $p'\delta\varepsilon_v^p + q\delta\varepsilon_s^p = \delta W_{dissipated}$ are used in OCC and MCC respectively. The superscript, p stands for plastic strains.

$$\delta W_{dissip.} = Mp'|\delta\varepsilon_s^p| \quad \text{(Thurairajah, 1961)} \quad (1)$$

$$\delta W_{dissip.} = p'[(\delta\varepsilon_v^p)^2 + (M\delta\varepsilon_s^p)^2]^{1/2} \quad \text{(Burland,1965)} \quad (2)$$

For axial symmetry, the stress invariants are the mean effective stress, $p'$ (= $(\sigma'_a + 2\sigma'_r)/3$) and the deviator stress, $q$ (= $\sigma'_a - \sigma'_r$), and the corresponding strain increments are, $\delta\varepsilon_v$ (= $\delta\varepsilon_a + 2\delta\varepsilon_r$ : volumetric) and the $\delta\varepsilon_s$ (= $2/3(\delta\varepsilon_a - \delta\varepsilon_r)$: shear). M is the critical state stress ratio.

In this study, a scheme is proposed wherein the elastic-plastic principles of the CCM are retained but the work equation is replaced by the data of undrained axial compression tests. The resulting scheme is used for strain prediction and these are compared with those of OCC and MCC models.

## 2.1 *The Proposed Decoupling Scheme*

### 2.1.1 *Invoking the associative flow rule*

Two stress probes, QR and QS are considered, as shown in Fig. 1(a). From the associative flow rule, the incremental plastic strain ratios for the two stress probes are independent of the stress increment directions and hence should be equal, Fig. 1(b). It follows then,

$$(\delta\varepsilon_v^p / \delta\varepsilon_s^p)_{QR} = (\delta\varepsilon_v^p / \delta\varepsilon_s^p)_{QS} \quad (3)$$

The assumption of normality for NC and LOC clays has been shown to be reasonable by Atkinson and Richardson (1985). From the volumetric hardening rule of Calladine (1963), since the two stress probe increments have the same κ shift (κ is the slope of the unload – reload lines), the incremental plastic volumetric strains, $\delta\varepsilon_v^p$ should be the same, Fig. 1(c):

$$(\delta\varepsilon_v^p)_{QR} = (\delta\varepsilon_v^p)_{QS} \quad (4)$$

From (3) and (4), we get, $\delta\varepsilon_s)_{QR} = \delta\varepsilon_s)_{QS}$ (5)

Eq. 5 implies that the incremental shear strains are the same when the increments in κ are made very small and when recoverable shear strains are neglected as in the OCC. It follows from Eq. 5 that the normalized shear stress-strain response of a conventional undrained stress path can be used, to predict the incremental shear strains along any other monotonic loading path, through an incremental marching process. The scheme outlined above is used to decouple the plastic work equation from the elastic-plastic assumptions.

### 2.1.2 *Marching scheme*

Consider for example, a triaxial compression, conventional drain path $A_1X_1$ of a NC clay in the (q, p') space where $A_1$ is on the isotropic consolidation line and $X_1$ is on the critical state line (q = Mp′) at failure (in the (q, p′) space), Fig. 2(a). The segment $A_1B_1$ is along the drained path $A_1X_1$ and the segment $A_1C_1$ is a segment of the undrained stress path through $A_1$, such that $B_1$ and $C_1$ reside on the same yield locus on the state boundary surface in the (p′, e, q) space. The projections of these stress paths on the (e, $\log_e$(p′) plane are denoted by the subscript "2", in Fig. 2(b). Therefore $A_2$ and $C_2$ on the (e, $\log_e$(p′) plane correspond to $A_1$ and $C_1$ respectively. The points on the state boundary surface (not shown on Fig. 2) are referred to without subscripts.

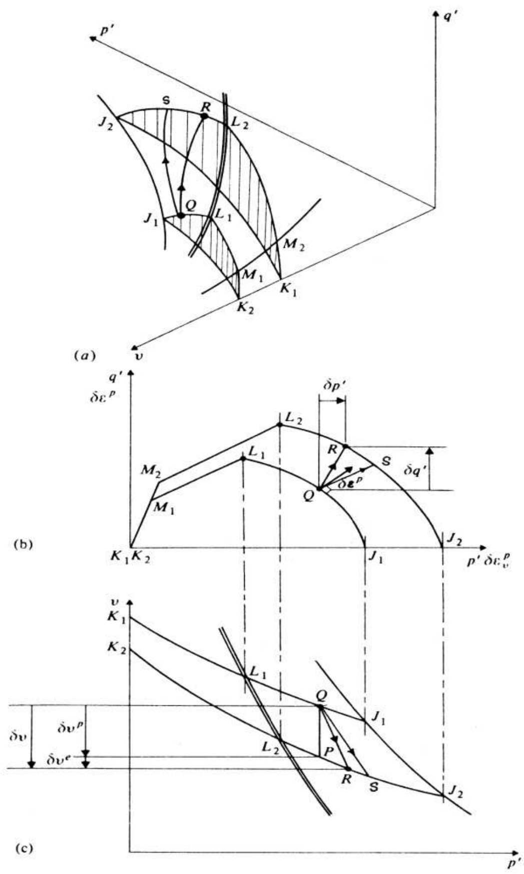


Fig. 1 Plastic straining of normally consolidated clay (after Atkinson, 1981)

Roscoe and Poorooshasb (1963) have shown that the undrained stress-strain behavior can be normalized, hence:

$$(\delta\varepsilon_s)_{AC}=\int_{\eta_A}^{\eta_C} f(\eta)d\eta=(\delta\varepsilon_s)_{AB} \quad (6)$$

where (η = q/p′, is the stress ratio), $\eta_A = 0$ (on the isotropic consolidation line), $\eta_C$ remains to be determined, and f is a unique function for a clay.

From the drained loading segment, $A_1B_1$, since stress point B is known (assumed δq and δp′ increments), $\eta_B$ is known. Using the data of the undrained stress path, the two-dimensional representation of the state boundary surface due to Roscoe and Poorooshasb (1963) can be established in the (ξ, η) space. The variable $\xi = p'_e / p'$, where $p'_e$ is the Hvorslev equivalent pressure (Schofield and Wroth, 1968). Accordingly, $(p'_e)_B$ is the mean effective stress on the isotropic compression line which has the same void ratio as B. Knowing $\eta_B$, $\xi_B$ can be determined from the (ξ, η) plot.

Now, since $\xi_B = (p'_e)_B / p'_B$, $(p'_e)_B$ is known, hence the corresponding void ratio can be found. This requires the slope, λ, of the isotropic compression line on the e vs $\log_e$(p′) plane to be known (or the slopes of anisotropic compression lines in general as they are assumed to be parallel in the Cam clay models). As this is the same void ratio as $e_B$, i.e. at point B, hence $e_B$ is known. The knowledge of $e_B$ enables us to define the κ line through $B_2$, on which $C_2$ is also located.

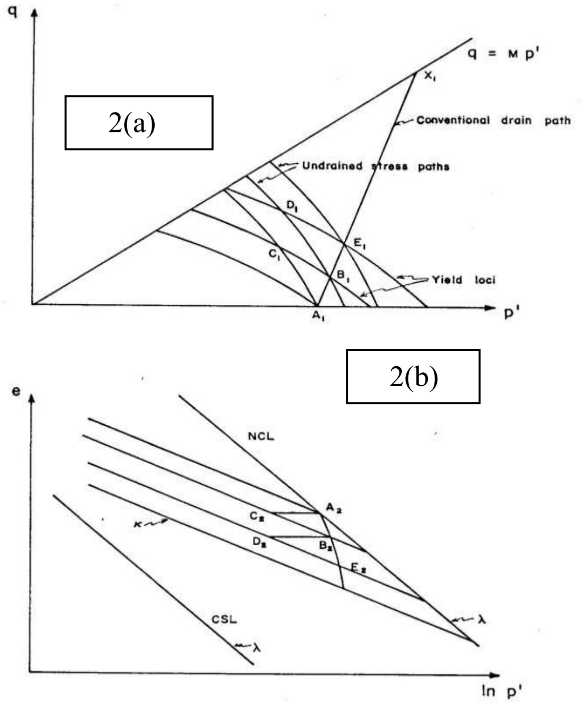


Fig. 2 Projections of conventional undrained stress paths, fully drained path and yield loci on (q, p′) and (e, $\log_e(p')$ planes for NC clay

The projections of $B_1$ and $C_1$ are on the same κ line (shown as $B_2$ and $C_2$ on the e vs $\log_e(p')$ plane) and $A_1$ and $C_1$ are on the same undrained stress path (this gives $e_C = e_A$). This enables us to compute the mean effective stress at $C_1$, $p'_C$. Once $p'_C$ is known, $\xi_c$ ($(p'_e)_C / p'_C$); $(p'_e)_C = p'_A$) can be computed, which in turn, via the (ξ, η) plot, enables us to determine $\eta_C$. Hence the integral in Eq. 6 can be evaluated.

Following a similar approach over successive stress increments along the drained stress path, $A_1X_1$, the solution can be marched on the κ lines to estimate the shear strains. The parameters λ and κ are determined from oedometer tests and M from conventional CIU tests.

### 2.2 *Shear Strain Prediction using the Proposed Decoupling Scheme*

Shear strain predictions were made for a drained stress path using the proposed scheme, using previously established stress-controlled undrained axial compression test data on Nong Ngoo Hao clay in Bangkok. These predictions are shown in Fig. 3 along with predictions from OCC and MCC.

The values of λ, κ, and M as reported in the literature for this clay are 0.51, 0.091 and 1.05 respectively which were used in the predictions. All test results are given in terms of natural strains $\varepsilon_n$.

It is seen that the proposed scheme yielded overprediction of shear strains with respect to the experimental values. The OCC gave large overprediction whilst the MCC gave some underprediction. What is striking is that the prediction by the proposed scheme almost coincided with that of the OCC.

In the application of elastic-plastic principles, the proposed scheme is identical with Cambridge critical state models (use of normality principle and definition of volumetric hardening). The only difference is whilst CCM use plastic work dissipation rules to characterize the yield loci, the proposed scheme uses undrained test results throughout.

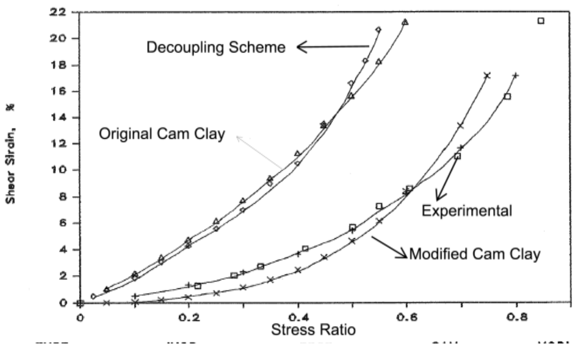


Fig. 3 Shear strain prediction using the decoupling scheme

In view of the close agreement in predictions between the proposed decoupling scheme and the OCC, it would not be unreasonable to hypothesize that the OCC energy dissipation rule and the energy dissipation rule inherent in undrained tests possess a close resemblance, at least, in so far as the prediction of shear strains are concerned. The implications of this hypothesis are developed next.

## 3 THE PROPOSED CONSTITUTIVE FORMULATION

Balasubramaniam (1969) has shown that the basic computational schemes in the CCM are similar and model an incremental stress path as made up of an incremental undrained path and an incremental anisotropic compression path, i.e. a η constant path.

Roscoe and Poorooshasb (1963) used experimental results of anisotropic compression paths in conjunction with undrained test results to predict shear strains.

Following the above, an incremental shear strain model is proposed on similar lines but based only on the parameters M, $\lambda$ and $\kappa$ as follows:

$$\delta\varepsilon_s=(\delta\varepsilon_s)_{undrained(\delta v=0)}+(\delta\varepsilon_s)_{compression(\eta=const.)} \quad (7)$$

$$(\delta\varepsilon_s)_{undrained} = (\delta\varepsilon_s /\delta\eta)_v\, \delta\eta \quad \text{(from OCC)} \quad (7.1)$$

$$(\delta\varepsilon_s)_{compression}=(\delta\varepsilon_s/\delta\varepsilon_v)_\eta\,\delta\varepsilon_v \quad \text{(from MCC)} \quad (7.2)$$

The following expression is obtained in terms of the parameters M, $\lambda$ and $\kappa$ for incremental shear strains based on the incremental marching scheme discussed, where $\Lambda = (1 - \kappa/\lambda)$ in Eq. 8.

$$\delta\varepsilon_s = [\kappa\Lambda\delta\eta] / [M(1 + e)(M - \eta)] + [2\eta\Lambda\lambda\delta p']/[(M^2-\eta^2)(1+e)p'] \quad (8)$$

The proposed model was applied to predict shear strains on a constant p′ stress path on a reconstituted specimen of kaolin (Balasubramaniam, 1969). As seen in Fig. 4, the model prediction compares favourably with the experimental results. The MCC prediction is also shown for comparison. The proposed model was then tested with undisturbed triaxial compression data of Balasubramaniam and Chaudry (1978). The results shown in Fig. 5 compare favourably with the test data especially at pre-failure stress levels. Modified Cam Clay predictions are also shown for comparison. Further shear strain predictions including triaxial extension are given in Wijeyakulasuriya (1986).

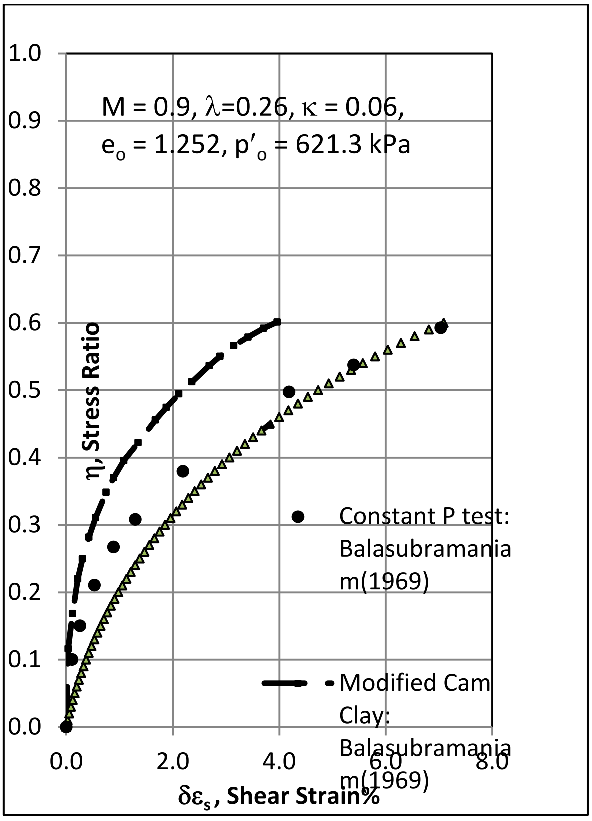


Fig. 4 Experimental shear strains with the predictions for reconstituted kaolin

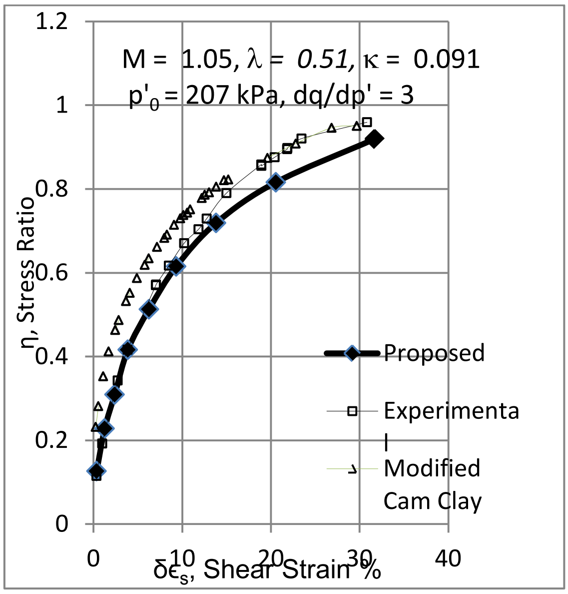


Fig. 5 Experimental shear strains with the predictions for soft Bangkok clay

## 4 CONCLUSIONS

The theoretical and experimental work reported in this paper has demonstrated that for the results of isotropically consolidated reconstituted kaolin and soft Bangkok clay, the proposed formulation has shown good strain predictability. Modified Cam Clay predictions are also shown for comparison.

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