Equal Strain Vertical Drain Consolidation with Creep

R.T.R. Walker
Centre for Geomechanics and Railway Engineering, University of Wollongong, NSW 2522, Australia.

ABSTRACT: Analytical solutions to vertical drain consolidation problems are only available for constitutive models where strain is a function of effective stress. Strain rate dependent soil models are not amenable to analytical solution due to their non-linear nature. By relaxing the requirement for an analytical solution, the equal strain radial consolidation equations provide a simple numerical technique of investigating such non-linear models. The equal strain assumption produces a second order non-linear ordinary differential equation in terms of strain, strain rate, and strain acceleration which is straightforward to solve using freely available ODE solvers. Strain dependent permeability and time dependent loading/vacuum are easily incorporated into the numerical model. This paper presents a parametric study of the new analysis method using the creep model of Yin and Graham (1996).

1 INTRODUCTION

There are many analytical available for consolidation with vertical drains: Conte and Troncone (2009); Hansbo (1981); Leo (2004); Tang and Onitsuka (2000); Walker and Indraratna (2009); Walker et al. (2012); Zhu and Yin (2001). However, none of the above solutions include creep/visco-plastic behavior. Rather than seek an analytical solution to radial consolidation with creep, a simple numerical approach is sought.

2 DERIVATION

Fig. 1 shows the geometry and material properties of a unit cell and a piecewise linear loading regime:

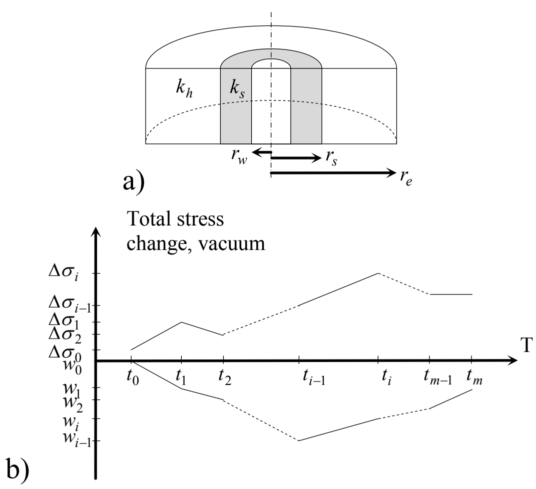

Fig. 1 a) Axisymmetric unit cell b) Piecewise linear surcharge and vacuum loading.

2.1 *Equal strain*

The equal strain radial consolidation equation of Hansbo (1981) can be re-derived with a non-zero pore pressure in the drain, w, to give the following relationship between strain-rate, $\dot{\varepsilon}$, and average excess pore pressure, $\bar{u}$:

$$\dot{\varepsilon} = k_h \eta (\bar{u} - w) / \gamma_w \tag{1}$$

Where k_h is the strain dependent horizontal permeability; γ_w is the unit weight of water, and η is the lumped drainage parameter given by Walker (2011):

$$\eta = 2 / r_e^2 \mu \tag{2}$$

For a smear zone with constant permeability μ is given by:

$$\begin{aligned}\mu = {} & \frac{n^2}{n^2-1}\left(\ln\frac{n}{s} + \kappa \ln s - \frac{3}{4}\right) \\ & + \frac{s^2}{n^2-1}\left(1 - \frac{s^2}{4n^2}\right) + \frac{\kappa}{n^2-1}\left(\frac{s^4-1}{4n^2} - s^2 + 1\right)\end{aligned} \tag{3}$$

$$n = r_e / r_w, \quad s = r_s / r_w, \quad \kappa = k_h / k_s \tag{4}$$

Other expressions for μ can be found for non-constant smear zone permeability distributions, Basu et al. (2006); Walker and Indraratna (2006), (2007); Walker (2011).

Anticipating the viscoplastic constitutive model Eq. (1) is differentiated with respect to time:

$$\ddot{\varepsilon} = \eta \left(\dot{k} (\bar{u} - w) + k (\dot{\bar{u}} - \dot{w}) \right) / \gamma_w \tag{5}$$

2.2 *Strain dependent permeability*

If void ratio changes with the natural logarithm of permeability then the following holds:

$$k_h = k_r \exp\left[-\left(\varepsilon - \varepsilon_0^{rk}\right)\frac{V}{\tilde{c}_k}\right] \tag{6}$$

Where k_r is a reference permeability defined at reference strain ε_0^{rk}, V is the volume of the soil element. Differentiating with respect to time yields:

$$\dot{k}_h = -k_h \dot{\varepsilon} V / \tilde{c}_k \tag{7}$$

2.3 *Constitutive model*

Effective stress, σ' and the time rate of change in effecitve stress, $\dot{\sigma}'$ are given by the following:

$$\sigma' = \sigma_0' + \Delta\sigma - \bar{u} \tag{8}$$

$$\dot{\sigma}' = \Delta\dot{\sigma} - \dot{\bar{u}} \tag{9}$$

Where σ_0' is the initial effective stress and $\Delta\sigma$ is the change in total stress.

The viscoplastic soil model of Yin and Graham (1994), (1996) can be rearranged in terms excess pore pressure rate:

$$\dot{\bar{u}} = \Delta\dot{\sigma} - \left(\sigma_0' + \Delta\sigma - \bar{u}\right)\frac{V}{\kappa} \times \left[\dot{\varepsilon} - \frac{\psi}{V}\frac{1}{t_0}\exp\left[-\left(\varepsilon - \varepsilon_0^{rc}\right)\frac{V}{\psi}\right] \times \left(\frac{\sigma_{z0}' + \Delta\sigma - \bar{u}}{\sigma_0'^{rc}}\right)^{\lambda/\psi}\right] \tag{10}$$

Where κ and λ are the slope of the instant and reference time lines in $\varepsilon - \ln(\sigma')$ space. A point positioning the reference time line is ($\sigma_0'^{rc}$, ε_0^{rc}). ψ represents the creep deformation for a natural log cycle of time. t_0 is the reference time for the reference time line. t_0 is often 24 hrs based on standard oedometer tests.

2.4 *Solution*

By substituting Eqs. (1), (6), (7), and (10) into Eq. (5) a second order ordinary differential equation in strain ε is revealed. The author uses the *odeint* function of the *python* based *scipy.integrate* package (Jones et al. (2001)) which uses the Fortran ODEPACK (Hindmarsh (1983)) to numerically solve the second order ODE. Solution requires recasting the ODE as a system of first order ODE as well as initial values of ε and $\dot{\varepsilon}$, the latter of which can be obtained from Eq. (1).

3 VALIDATION AGAINST NON-CREEP ANALYTICAL MODELS

To test the validity of the ODE model, results are compared against the following analytical radial consolidation models that do not incorporate creep: Hansbo (1981) with constant soil properties; Tang and Onitsuka (2000) with constant soil properties under multiple ramp loads; Walker et al. (2012) with void ratio dependent compressibility and permeability. The properties for each case are listed in Table 1. The loading regimes and calculated excess pore pressures are displayed in Figure 2. Results are in excellent agreement.

Setting ψ close to zero removes any creep and the reference time line becomes the conventional compression line (Using ψ =0 leads to division by zero errors). In order to model properties that do not vary with strain the initial effective stress is artificially made very high. The logarithm stress strain relationship will then mean increases in effective stress will only cause miniscule strain values to material properties will stay at their initial values. Properties defined using a log10 stress axis can be converted to natural log properties simply by dividing by ln(10).

Table 1. Parameters for validation cases

Property	Hansbo (1981)	Tang and Onisuka (2000)	Walker et al. (2012)
Hori. coeff. of consol, c_{h0} (m^2/yr)	7	20	20
r_e (m)	0.525	0.846	0.525
μ	8.539	8.273	8.539
C_r	-	-	0.1
C_c	-	-	1
C_k	-	-	0.8
σ'_0	100000	100000	30
σ'_p	-	-	60
ψ/V	0.0001	0.0001	0.0001
κ/V	1	1	0.0434
λ/V	1	1	0.4343
ε_{0rc}	0	0	0
σ'_{rc} (kPa)	120000	120000	60
k_r *(m/yr)*	0.0007	0.002	0.2895
ε_{0rk}	1	1	0
$\tilde{c}_k$	1e100	1e100	0.3474
t_0	1	1	1

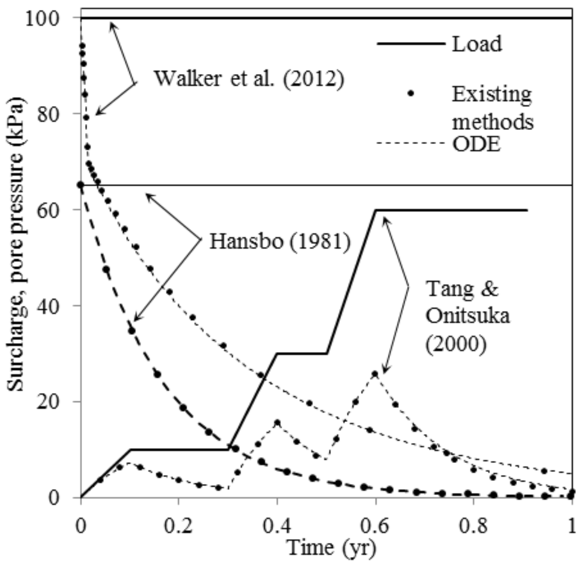

Fig. 2 Validation of ODE model.

4 PARAMETRIC STUDY

The effect of varying the creep parameter ψ is briefly explored in relation to constant loading conditions as well as an additional sinusoidal load. Properties used in the analysis are listed in Table 2. The material properties for analysis are taken from a Norwegian clay investigated by Yin and Graham (1996). Parameters related to vertical drains as well as the reduction in permeability with strain are assumed.

Table 2. Soil properties for parametric study

Property	Value
r_e (m)	0.525
μ	8.539
σ'_0	50
ψ/V	0.007
κ/V	0.004
λ/V	0.158
ε_{0rc}	0
σ'_{rc} (kPa)	79.2
k_r *(m/s)*	$1.67\text{x}10^{-9}$
ε_{0rk}	0
$\widetilde{c}_k$	0.158
t_0 (mins)	40

4.1 *Constant load*

A surcharge load of 100 kPa is applied instantly. The results of the consolidation analysis for various values of ψ / V are shown in Fig. 3.

The stress-strain plot shows that the apparent preconsolidation stress depends on the creep parameter. The cases with low ψ show a clear preconsolidation stress before which the soil is stiff and consolidation is rapid. The cases with higher ψ, i.e. more viscous soils, do not exhibit such recompression behavior rather they deform parallel to the compression line almost immediately after loading. The purely creep deformation is apparent from the vertical part of the stress-strain curve corresponding to when excess pore pressures have dissipated.

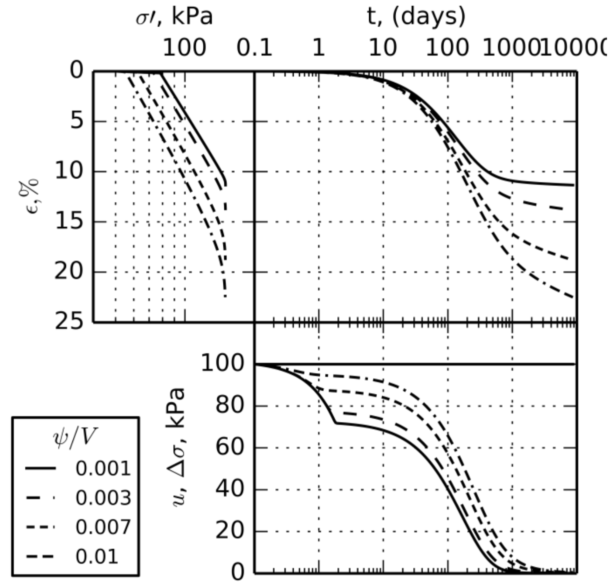

Fig. 3 Effect of ψ on consolidation, instant load

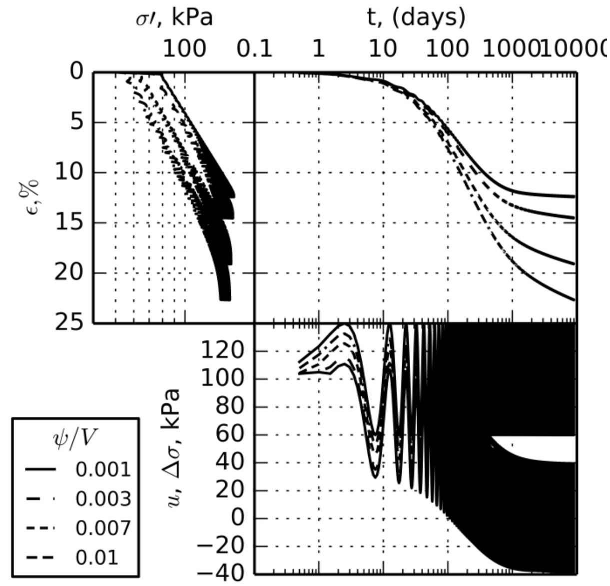

Fig.4 Effect of ψ on consolidation, cyclic load

4.2 *Cyclic load*

A sinusoidal load with amplitude 40 kPa and frequency of 0.1 days is added to the instant 100 kPa load. The results of the consolidation analysis for various values of ψ / V are shown in Fig. 4.

For high values of ψ the strain vs time plots in Fig. 3 and Fig. 4 are almost identical. The cases with lower ψ show more strain under the cyclic load because a portion of the positive excess pore pressure induced by the sinusoidal load has dissipated. It should be noted that this model does not account for a buildup of pore pressure due to undrained cyclic loading as discussed by Ni et al. (2013).

5 CONCLUSION

By abandoning the search for analytical solutions to radial consolidation equations the equal strain assumption leads to relatively straightforward, simple numerical solutions. Using a rate dependent soil model such as Yin and Graham (1994), (1996), yields a second order ordinary differential equation in terms of strain, strain rate, and strain acceleration. The ODE can be easily solved using freely available ODE solvers. A brief parametric study using Yin and Graham (1996) soil model shows that highly viscous soils may exhibit a lower apparent preconsolidation stress compared to rate independent soils. Also the viscous nature of such soils means their consolidation curves in terms of deformation do not respond to rapid oscillations in load.

The new method for analyzing radial consolidation problems can easily be adapted to other soil models allowing rapid analysis of complicated soil models.

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