Detailed site effect estimation using multi-level modeling of earthquake strong-motion amplitudes and uncertainties

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ABSTRACT: Ground motion records from over 1700 earthquakes of $M_w 6.3$ at distances of 0.9-2.3 km on a small aperture (1.9 km) urban strong-motion array in Iceland (ICEARRAY I) exhibit a considerable variation in the spatial distribution of peak ground motion acceleration (PGA) and spectral response (PSA) on a uniform site condition characterized as lava-rock. We develop a Bayesian hierarchical model (BHM) that partitions the residual ground motions into earthquake event effects, station effects and event-station effects. The BHM thus quantifies to what extent the earthquakes and the recording sites contribute respectively to the overall variation in ground motions. The results show that on lava-rock the site effects dominate the spatial distribution of PGA and PSA at certain frequencies, while at others the event effects dominate, in agreement with results of physical modeling of the lava-rock geological structure as a dynamic system.

Keywords: Site effects; Bayesian Hierarchical Model; Markov Chain Monte Carlo; Uncertainty

1. Introduction

Estimating the spatial variability of strong ground motions plays a critical role in mitigating future earthquake disasters. It has been well investigated that the variation of geological conditions largely accounts for the spatial variability of earthquake ground motions. The importance of estimating the spatial variability of site response has been highlighted by some of the recent catastrophic events such as 1985 Mexico City and 1989 Loma Prieta, California earthquakes. Where the lack of information and inaccurate modeling of site effects was one of the most important reasons for a large loss of lives and economic crises. To date, estimation of ground motion variations is of great engineering interest, important implications for earthquake-resistant design, seismic hazard assessment, and urban planning, where the main purpose is to reduce human and economic losses that ground motions may cause [1].

In earthquake resistant design of structures in Iceland, site effects are generally considered being simple and associated with a binary site classification of stiff soil or rock, partly because in many cases the relatively thin and topsoil overlying the competent rock (e.g., lava-rock, hyaloclastite, dolerite, etc.) is easily removable. This assumption needs revision at sites underlain by the complex geological feature of alternating sedimentary and lava-rock layers with depth, introducing velocity reversals. Namely, repeated sedimentary deposits from glaciation and deglaciation together with sea-level fluctuations on top of which basaltic lavas have flowed over during volcanic eruptions are predominant in and near the relatively young volcanic zone. Atakan et al. [2] carried out a pioneering site effect investigation in an area of $400\ \mathrm{km}^2$ in the westernmost part of the south of Iceland. They argued that most of the site amplification is associated with the buried unconsolidated sedimentary layer. Later, Bessason and Kaynia [3] illustrated the considerable variation in site amplification due to different subsoil structure beneath the east and west abutments of the 80-meter long Thjorsa Bridge in South Iceland. More recently, extensive near-field ground motion recordings from over 1700 aftershocks of the $M_w 6.3$ Ölfus earthquake of 29 May 2008, recorded on a small aperture ($D = 1.9$ km) urban strong-motion array in the town of Hveragerði in south Iceland (ICEARRAY I [4]) showed a considerable variation in the spatial distribution of peak paremeters of earthquake ground motion and the associated structural response (peak ground acceleration, PGA, peak ground velocity, PGV, and pseudo-spectral acceleration response, PSA) despite being all recorded on the "rock" site condition, which holocene lava-rock was and still is considered to be [5, 7]. More recently, Rahpeyma et al. [5, 6] implemented a detailed analysis of the frequency dependence of site effects across ICEARRAY I in order to determine site-specific characteristics over ICEARRAY I. The findings of theses studies emphasized the importance of quantitatively characterizing the specific site amplification of the repeated lava/sedimentary layer soil structure, and in particular over relatively short distances.

Looking closer into the character of the variation of peak parameters, Rahpeyma et al. [9] proposed a Bayesian inference scheme in order to estimate the contribution of source, path, and site effect, and applied it to the PGA recorded across ICEARRAY I. The proposed model, the so-called Bayesian Hierarchical Model (BHM), offers a flexible probabilistic framework for multilevel modeling of earthquake ground motion parameters, in which a collection of random variables can be decomposed into a series of conditional models. Since the BHM represents the model parameters in terms of the posterior probability densities, the BHM model successfully quantified the relative contribution of source, path, and site effects and more importantly represented their associated uncertainties. Later, Rahpeyma et al. [10] implemented the BHM in order to estimate and compare site effects on two Icelandic strong-motion arrays in the south (ICEARRAY I) and north (ICEARRAY II) of Iceland with different geological settings. The BHM results indicated that although the "rock" site conditions believed to apply to the lava-rock surface layer found across ICEARRAY I and thought to represent a low and uniform site amplification the station terms, which indicate to what extent the ground motion amplitude can be expected to be either higher or lower than the mean over the array, only contribute around 13% to the total variability found in the amplitudes of predicted ground motions across the array. While, the contribution of site effect variability across ICEARRAY II (characterized by more variable soil structure and topography) was found to be much larger, or ~57%. Thus, with respect to PGA which captures the high-frequency character of seismic motions, the localized site condition of ICEARRAY I on lava-rock indeed appears to be associated with more uniform site effects that influence only to limited extend the overall variability of PGA variations. The opposite is found for ICEARRAY II, predominantly due to the different site conditions and topography.

In the current study, we extend our approach to cover PSA at multiple periods of engineering interest (i.e., $T =$ 0.05-3.0 s) on the basis of the ICEARRAY I strong-motion data recorded on lava-rock. The BHM is setup hierarchically in three levels: (1) data level, (2) latent level, and (3) hyperparameter level. The data level describes the distributional model for the observation conditioned on the model parameters. Here, the earthquake PGA and PSA response of simple structures are considered comprising the data level. The latent level describes the distribution of the latent parameters, some of which are found in the distributional model of the observations. Finally, the hyperparameter level generally consists of the prior information for the hyperparameters at the data and latent levels and typically are defined as standard deviations of the distribution describing the corresponding latent parameters.

The BHM is found to be very practical as it allows the partitioning of a ground motion model into event, station, and event-station terms, which in turn allows the relative contributions of source, path, and site effects to be quantified, respectively. It will improve our understanding of earthquake ground motions. The results confirm the results of Rahpeyma et al. [5] and highlight the importance of regionalization of the ground motion in the effort of understanding the underlying sources of the aleatory variability, which consequently has implications for seismic hazard analysis.

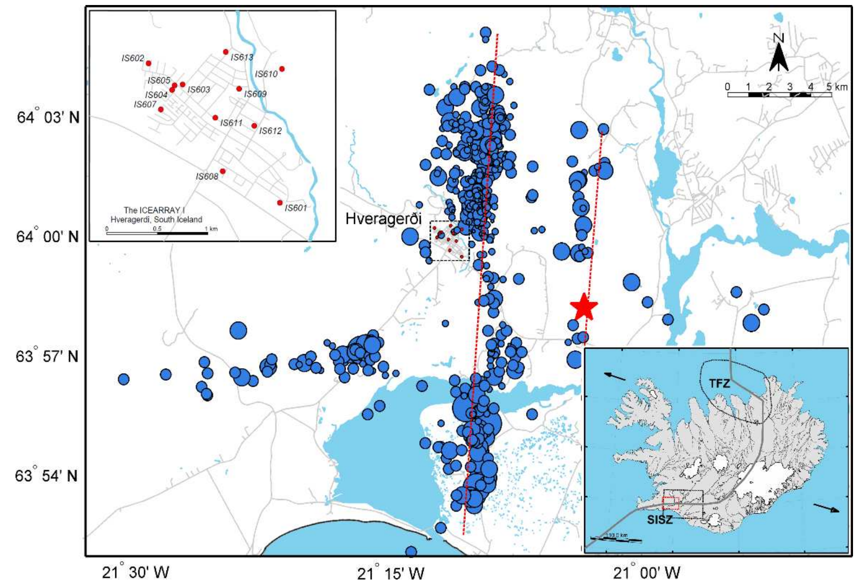

Figure 1. Mainshock epicenter (red star) and aftershock distribution (blue circles) of the Ölfus earthquake on May 29, 2008 in SISZ recorded by ICEARRAY I located in the town of Hveragerði (black dashed rectangle). The top left figure shows the ICEARRAY I strong-motion stations (red symbols) [9]

2. Array strong-motion data

In 2007, the first small-aperture Icelandic strong-motion array (ICEARRAY I) was deployed in the town of Hveragerði in South Iceland Seismic Zone (SISZ) with the aim of (1) monitoring and recording strong events in the region, (2) quantifying spatial variability of strong-motion over short distances, and (3) shedding lights on earthquake source processes [4]. As shown in **Fig. 1**, the array covers around 1.23 km^2 and consists of 12 accelerometric stations with inter-station distances ranging from 50-1900 m [4]. According to the geological information (see **Fig. 2**) and borehole data in Hveragerði, the majority of ICEARRAY I stations are located directly upon a lava-rock layer which in turn lies on top of a softer sedimentary layer, introducing a significant shear wave velocity reversal [5, 7]. At deeper depths, this structure is essentially repeated with the underlying lava-rock layer sitting on softer sediments on top of older bedrock. Hence, the geologic structure beneath the ICEARRAY I is characterized by at least two significant shear wave velocity reversals with depth.

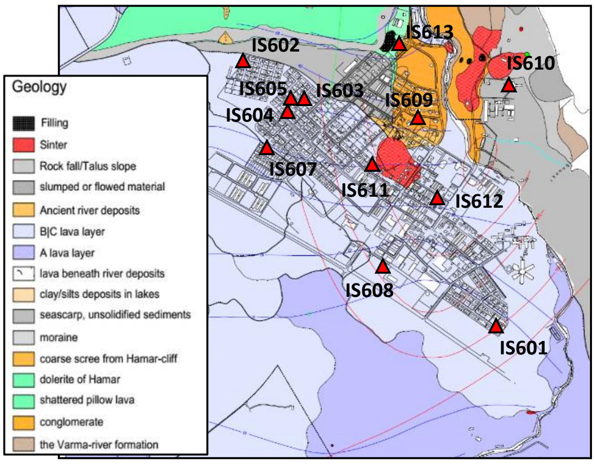

Figure 2. Geological map of ICEARRAY I area adopted from Sæmundsson and Kristinsson [8]. Red triangles represent ICEARRAY I strong-motion stations.

On 29 May 2008 at 15:45 local time, the $M_w 6.3$ Ölfus earthquake occurred in the western part of the SISZ. The ICEARRAY I recorded the main-shock and more than 1700 of its aftershocks over the following year (cf. **Fig. 1**). Preliminary analysis revealed consistent and significant variation in the spatial distribution of the ground motion amplitudes across ICEARRAY I, even for motions recorded on seemingly similar layered geologic profiles. Although the spatial distribution of events may introduce an azimuthal effect on the amplitudes of ground motions, the recorded aftershocks covered almost the entire azimuthal range around ICEARRAY I and no azimuthal dependency on amplitudes was observed.

3. BHM setup

The most common and classical model for the prediction of a ground-motion parameter dependent on source, path and site parameters can be defined as Eq. (1) [9-12]:

$$\log Y_{es} = \log \mu_{es}(M_e, R_{es}, D_e) + \delta B_e + \delta W_{es} \quad (1)$$

the subscripts $e = 1, \ldots, N$ and $s = 1, \ldots, Q$ represent earthquake and station, respectively. Y_{es} is the strong-motion parameter of interest follows a Gaussian distribution. The predictive model, $\log \mu_{es}$, known as ground motion model (GMM) provides median ground motion in terms of independent seismic variables. Although GMMs come in a variety of different functional forms, here, we used a commonly used and parsimonious linear model that links ground motion amplitude to the local magnitude of the eth earthquake, M_e; the hypocentral distance from the eth event to the sth station, R_{es}; and the depth of the origin of the eth earthquake, D_e, as follows:

$$\log \mu_{es} = \beta_1 + \beta_2 M_e + \beta_3 \log_{10}(R_{es}) + \beta_4 D_e \quad (2)$$

the coefficients, $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4)$ of the GMM, consists of the parameters that capture the characteristics of the seismic region and the geological structure.

δB_e, the inter-event residuals (so-called event term in the BHM modeling) represents the average shift, corresponding to an individual earthquake, of the observed ground motions from corresponding median estimates of the ground motion model. The event terms are assumed to be independent of each other, normally distributed with zero mean and variance of τ^2.

δW_{es}, the intra-event residuals, on the other hand, represents the difference between an individual observation at station s from the earthquake specific median prediction [12, 13]. The BHM allows the intra-event residuals be further divided into three terms:

$$\delta W_{es} = \delta S2S_s + \delta WS_{es} + \delta R_{es} \quad (3)$$

$\delta S2S_s$, the inter-station residuals (so-called station term in BHM modeling) represents the average intra-event residual at each station. The station terms are modeled a priori with a mean zero Gaussian distribution with an exponential covariance function from the Mátern family with inter-station variability of ϕ_{S2S}^2 and fixed range parameters of Δ_{S2S}. δWS_{es}, the even-station residuals captures record-to-record variability and can be investigated for other repeatable effects. We define the event-station residuals as spatially correlated variables from a zero mean Gaussian field governed by a covariance function from Mátern family with a marginal variance of ϕ_{SS}^2 and a range parameter of Δ_{SS}. Finally, δR_{es}, referred to as unexplained terms, accounts for effects that are not modeled by the other terms. The unexplained terms are modeled independently follow a mean zero Gaussian distribution with a variance of ϕ_R^2.

The total variability of the BHM can be separated into two main parts: the inter-event variability and the intra-event variability in which the intra-event variability can further be divided into inter-station variability (i.e., station-to-station variability), event-station variability (i.e., variability between stations within an event), and other unexplained variability (e.g., measurement and model error, etc.). This ability is of great importance for many of the engineering applications in particular for removing the ergodicity assumptions. Effectively, therefore, the total (aleatory) variability of the model in Eq. (1) can be quantified with the variance of the sum of the four independent terms presented in Eq. (4):

$$\sigma_T^2 = \tau^2 + \phi_{S2S}^2 + \phi_{SS}^2 + \phi_R^2 \quad (4)$$

The inter-event variance (τ^2) quantifies the variation between events relative to the average ground motion level predicted by the GMM for each event. The inter-station variance (ϕ_{S2S}^2) quantifies the variability between stations, which then is primarily a manifestation of the localized variations such as the geological profiles beneath the stations. The event-station variance (ϕ_{SS}^2) can be defined as a measure of the spatial variability in the ground-motion amplitudes between stations within an event after taking into account the event and station terms. The purpose of this term is to quantify the remaining variations not already captured by the GMM or the event-station terms. Finally, the variance of the (ϕ_R^2) quantifies the variability in the measurement errors and other deviations that are not accounted for by other terms of the model.

The records of 10 stations that are directly located on the lava-rock layer are used in the present study. The Markov Chain Monte Carlo (MCMC) simulations using the Metropolis steps [14] provided reliable estimates of the posterior distribution of the latent parameters, $\boldsymbol{\eta} = (\boldsymbol{\beta}, \boldsymbol{\delta B}, \boldsymbol{\delta S2S})$, along with the hyperparameters, $\boldsymbol{\theta} = (\tau, \phi_{S2S}, \phi_{SS}, \phi_R, \Delta_{SS})$, of our BHM model using ICEARRAY I dataset. For more details about the BHM modeling read [6,9,10].

4. Results

4.1. Posterior inference of station terms

Contrary to the commonly used classical regression methods designed to decouple the observed ground motion parameters and provide point estimates of the unknown model parameters, the BHM represents the model parameters in terms of the posterior probability densities. This ability lets us determine the uncertainty of the model parameters given the observed data. **Fig. 3** represents the posterior distributions of the station terms, $\delta S2S_s$ (i.e., inter-station residuals).

The station terms serve as indicators to what extent the ground-motion amplitude can be expected to be either higher or lower than the mean over the array. As can be seen in **Fig. 3**, posterior probability densities of the station terms are all unimodal and have shapes that are close to that of a Gaussian probability distribution. However, there is a noticeable growth in the posterior variability (i.e., station term's posterior standard deviation) of the station terms for the period range of $T =$ 0.1-0.3 s in which the standard deviation is about 0.06-0.07 that is ~2.5 times larger than the posterior standard deviation for the rest of periods (~0.03-0.04). On close scrutiny, except stations, IS609, IS611, and IS612, the remaining stations show the similar outline of increasing the station term amplitudes at the period range of $T =$ 0.1-0.3 s then slightly decreasing at longer periods.

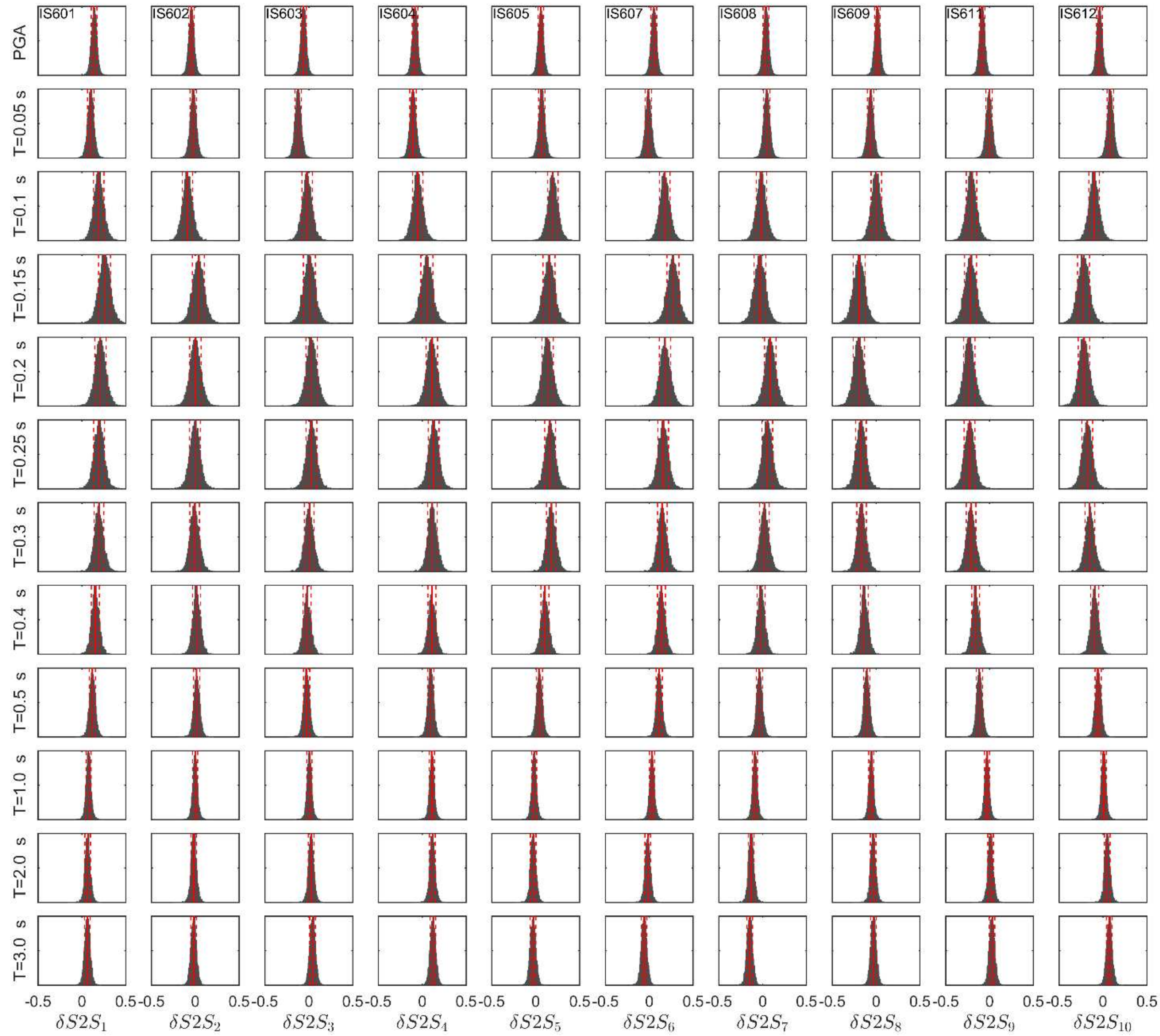

Figure 3. Posterior probability density functions of the station terms, $\boldsymbol{\delta S2S}$ for PGA and PSA (T = 0.05-3.0 s). The red solid and dashed lines show posterior mean and 16-84% posterior intervals, respectively.

For example, stations IS604 and IS611 show the lowest relative station terms of -0.08 ± 0.03 base-10 logarithmic units for PGA while at period 0.25 s station term for IS604 ($\delta S2S_4$) amounts to 0.13 ± 0.06 and station term for IS611 ($\delta S2S_9$) becomes -0.23 ± 0.06.

At longer periods ($T = 3.0$ s) station term for IS604 slightly decreases and equals to 0.12 ± 0.03 and station term for IS611 turns to 0.03 ± 0.03. It is worth mentioning that station IS609 is located on older bedrock while the other stations are located on a post-glacial lava-rock layer on top of a sedimentary layer according to the logs of shallow boreholes across the Hveragerði.

The BHM allows us to decompose the residuals and consequently determine the contribution of source, path, and site effects. In the BHM formulation, τ encodes differences of one particular event from the mean of all events, while ϕ_{S2S} and ϕ_{SS} are due to differences in the site and path related aspects, respectively [9,10,13,15,16]. The contribution of the standard deviations of inter-event (τ), inter-station (ϕ_{S2S}), event-station (ϕ_{SS}), and unexplained effects and other unaccounted factors (ϕ_R) relative to the total standard deviation are presented graphically in **Fig. 4**. A first look at **Fig. 4** reveals that the inter-event standard deviation has the main contribution to the total standard deviation over the range of periods of this study. On a close scrutiny, a generic high aleatory variability is observed at the period range of $T = 0.1$-0.3 s. While the event-station and unexplained standard deviations show rather constant contribution at this range of period, the inter-station variability shows a considerable boost in the moderate range of periods ($0.1 \text{ s} \leq T \leq 0.3 \text{ s}$) which accounts for the higher standard deviation (σ) in this range of periods. In particular, at high frequencies (e.g., PGA or spectral response at $T = 0.05$ s), the standard deviations of the inter-event, inter-station, event-station, and unexplained effects appear to have a constant contribution of ~60%, ~15%, ~20%, ~5%, respectively. However, for a period range of $T = 0.1$-0.3 s, the contribution of inter-station variability noticeably increases from ~20% to ~40% with a reduction in inter-event variability from ~60% to ~40% indicating a large station-to-station variability in the dataset. Then, at longer periods $T = 0.5$-3.0 s, the inter-station standard deviation evidently increases to while the inter-event standard deviation decreases.

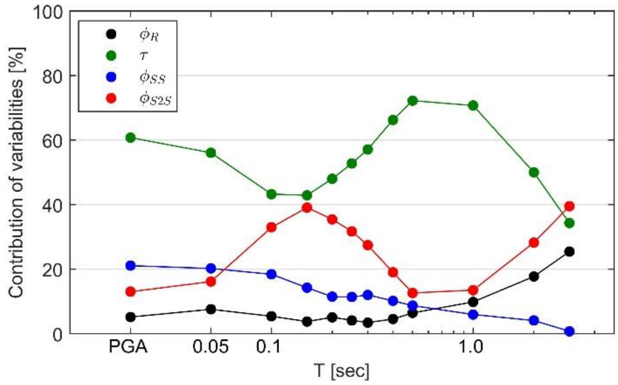

Figure 4. Contribution of inter-event (τ), inter-station (ϕ_{S2S}), event-station (ϕ_{SS}), unexplained (ϕ_R) standard deviations for PGA and PSA [17].

5. Discussion

As illustrated in **Fig. 4**, the inter-station standard deviation is conspicuously enlarged in the period range $T = 0.1$-0.3 s. Effectively, therefore, the increased variability can be expected to be the manifestation of different site amplifications at the array stations at this period range. Understanding this is essential due to the importance of characterizing site-to-site variability for the probabilistic hazard assessments and also defining practical site classification proxies. In order to scrutinize site-to-site variability, we need to explore site-specific characteristics beneath the ICEARRAY I stations.

According to logs of shallow boreholes in the town, most of Hveragerði is located on a young and competent lava-rock layer, which in turn is underlain by a layer of softer sedimentary deposits [5]. Namely, stations IS609 is located on older bedrock while the remaining stations are located on a post-glacial lava rock layer on top of a sedimentary layer. The latter geologic profile is a typical layering encountered primarily in geologically younger parts of Iceland in the vicinity of the volcanic zones. In Hveragerði, the uppermost lava layer is relatively young basaltic lava of varying thickness. This lava layer lies on top of a sedimentary layer, which in turn lies on top of an older lava layer (see **Fig. 2**). No information about the thickness of the lower layer exists, but it flowed from the same volcanic fissure and based on the spatial extent of the lava it is likely of similar thickness as shallower Lava layer. From the typical layered structure of young geological formations in Iceland, it is likely that it is underlain by sediments, introducing the second velocity reversal with depth, above the bedrock. This geologic structure which is characterized by a significant shear wave velocity reversal is most probably repeated with depth down to the old bedrock. Thus, the site effects under this part of the town are expected to be affected by at least two velocity reversals.

Recently, a detailed site effect investigation across ICEARRAY I has been released by Rahpeyma et al. [5, 6] using Horizontal-to-Vertical Spectral Ratio (HVSR) as along with Standard Spectral Ratio (SSR) methods. The results showed that in spite of small aperture of ICEARRAY I there are noticeable differences in the site responses across the array. Some stations exhibit bimodal amplification curves with predominant frequencies one between 3-4 Hz and the other at 8-9 Hz. Moreover, one of the fundamental modes observed in the site response amplification curves at some stations was found to be more dominant and of relatively larger amplitude than the other (e.g., IS605, IS604, and IS608) while other stations have a single narrow-band peak of relatively low amplitudes (e.g., IS602) or amplification curves of relatively high amplification over a wide frequency range (e.g., IS601, IS603), or even very low and uniform amplification curves across the frequency range (IS609). Comparing **Fig. 4** and site effect investigation results [5,6] shows that the site-specific characteristics play a significant role around the resonant frequency bandwidth ($T = 0.1$-0.3 s or $f_0 = 3$-4 Hz and $f_0 = 7$-8 Hz in the frequency domain) of the subsoil structure of the region. The site amplification is therefore both more prominent at these frequency ranges due to the natural frequencies of the subsoil structure, and suggests that the subsoil structure may be highly variable even over short distances. This last result is consistent with the small-scale irregularity of lava-rock surfaces.

6. Conclusion

The strong-motion data recorded by the first Icelandic strong-motion array (ICEARRAY) during and after the 29 May 2008, M_w6.3 Ölfus earthquake in the south of Iceland exhibited notable variations in ground-motion amplitudes (e.g., PGA, PGV, and PSA) over a very short distances. This unexpected variation is of paramount importance due to the unique geological site condition underlying ICEARRAY I which is characterizing mainly as "rock". Therefore, this study investigated the variability of Icelandic earthquake ground motions and brought to light the role of site-specific characteristics.

For the purpose of comprehensive analyzing the strong-motions to explain their observed variations, we modeled them using a multi-level Bayesian Hierarchical Model (BHM) that partitions the motions into event, station, and event-station terms in order to quantify the relative contributions of source, site, and path effects. In this study, we analyzed the peak-ground acceleration along with the 5% damped spectral acceleration at oscillator periods of interest $T =$ 0.05-3.0 s.

BHM allows us to split up the total variability into an inter-event and intra-event variabilities. Furthermore, the intra-event variability partitions into an inter-station, event-station, and unexplained term variabilities. Decomposing the total standard deviation revealed that the inter-event standard deviation has the main contribution to the total standard deviation over the range of periods of this study; however, the results precisely showed that a generic high aleatory variability at the period range of $T =$ 0.1-0.3 s is largely associated to the bump in the inter-station standard deviation due to a large station-to-station variability in the dataset. This larger variability in inter-station standard deviation is due to the resonance frequencies (~3-4 Hz and ~8-9 Hz) that we observed in some stations such as IS604, IS605, and IS608.

The results of this study have improved our understanding of the key factors that affect the variation of seismic ground motions across a relatively small area. We believe our results find application in earthquake hazard assessment on a local scale, with practical implications for seismic risk and engineering decision making such as urban planning.

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