Expected precision of the pressuremeter results

J. Monnet
Gaiatech , 22 rue Antoine Chollier, Seyssinet F-38170, France ; monnet. jacques@wanadoo.fr

ABSTRACT: The development of numerical methods and the need to define precision for calculations means that the admissible margins of error for calculations must be defined. specialy on the pressuremeter parameters (E_M, p_{LM}), which are used for foundation design. The pressuremeter parameters which are used in the design of the foundations are assumed here to follow a Gaussian distribution curve. The error theory is used to define the precision of the pressuremeter module E_M, and the limit pressure p_{LM} . It is applied to a series of pressuremeter tests used as a benchmark for the ARSCOP French National Project. The precision of the pressuremeter modulus is shown highy related to the acuracy on the pressure and the volume, whereas the limit pressure is mostly linked to the determination of the creep pressure.

Keywords: pressuremeter, precision, modulus, limit pressure

1. Introduction

In the field of civil engineering, the pressuremeter is widely used for the design of structures such as shallow foundations [1] or the settlement of foundations (Appendix H [1]. These methods use the geotechnical characteristics of the soil related to the pressuremeter test, namely the Ménard p_{LM} limit pressure and the Ménard E_M pressuremeter module. These quantities are used for the design of foundation but the new requirements of dimensioning needs to define for these values a precision, even a tolerance so as to specify the dimensions of the geotechnical works

The aim of this study is to define the precision related to the pressuremeter module and the limit pressure, as a function of the accuracy of the unit pressure and volume measurements, but also according to the methods used to measure them in a standard way [2] .

2. Hypothesis

2.1. Essential notions

Before studying Error, it is important to identify some essential notions, and to distinguish Errors from Mistakes:

• ***True measurement does not exist***; there is always a measurement uncertainty related to the imperfection of our senses and / or our instruments

• ***Errors always add up***, and never remove one from the other

• ***The error is inevitable***; every measurement is vitiated by an error related to the imperfection of our senses and / or our instruments. The error always exists when one makes a measurement.

• ***The fault is a gross error*** related to the non respect of the measurement protocol

• ***Tolerance is the maximum limit of the error***; beyond which we consider a fault

2.2. What is a result of a measure?

When we measure n times the same size:

• for an ideal world, the average value of a measurement series and the true value are close (Fig. 1)

• In the real world we obtain a dispersion of the measurements (Fig. 2)

• Measures may have different biases (Fig. 3)

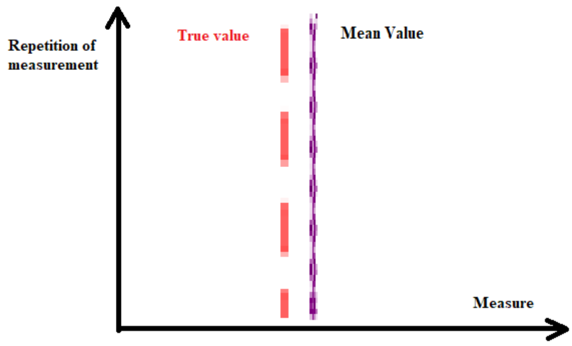

Figure 1. Ideal match between measurements and true value

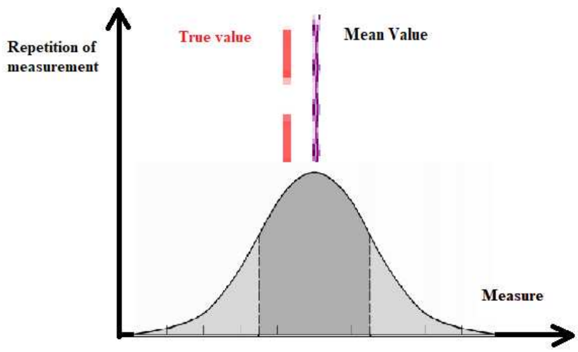

Figure 2. Real match between measurements and true value

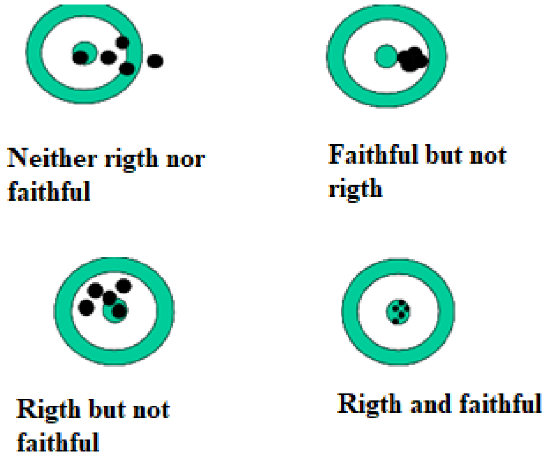

Figure 3. Correspondence between the measurements and the true value, for the dart game

2.3. Distribution of measurements - Gauss curve

We assume that the number of repeated measurements for the same measurand is distributed (Fig.4) according to a Gaussian curve (or normal law).

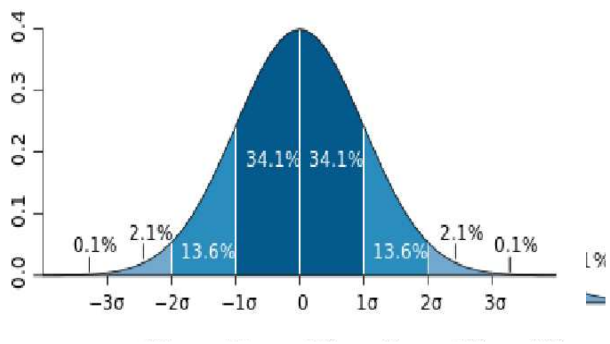

Figure 4. Distribution of measurements according to the Gaussian curve: σ is the standard deviation

2.4. Characteristics of Gauss curve

The characteristic quantities of the Gauss curve are:

- The average $\bar{X}$ of measurements Eq. (1)
- The standard deviation σ or variance or squared mean error E_{mq} Eq. (2)
- The probable error E_p Eq. (3)
- The Tolerance T Eq. (4); beyong it the measurement is faulty

$$\bar{X} = \left(\sum_{i=1}^{n} X_i\right)/n \quad (1)$$

$$\sigma = E_{mq} = \sqrt{\left\{\sum_{i=1}^{n} (X_i - \bar{X})^2 / (n-1)\right\}} \quad (2)$$

$$E_p = 2/3.E_{mq} \quad (3)$$

$$T = 4.E_p \quad (4)$$

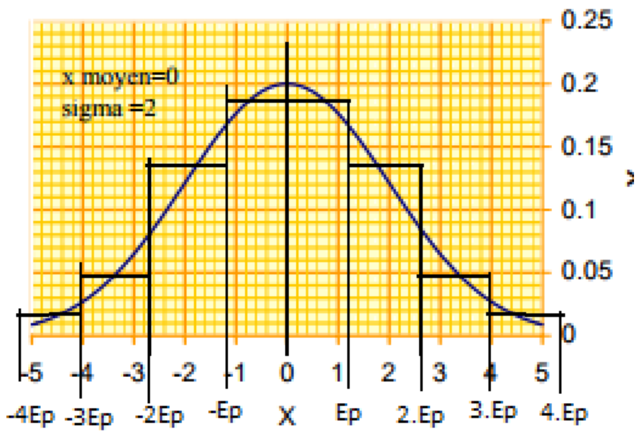

Figure 5. Gauss curve with an average of 0 and a standard deviation σ of 2

2.5. Composition of errors

When one wants to calculate a particular physical quantity F, one uses the measurement of different variables x, y, z which each have their own variance σ_x, σ_y, σ_z. The errors related to each measurement have an influence on the error of the magnitude that one seeks to calculate. It is calculated differently in the case of a simple function F (5) as addition or subtraction by adding the squares of the variances of each variable Eq. (6) or in the case of a composed function Eq. (7), as multiplication, or division, then its variance is calculated according to Eq. (8). A more detailed description of these calculations can be found in [3]. For a simpler application it is recommended to consult [4,[5].

$$F = x + y - z \quad (5)$$

$$\sigma_F^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \quad (6)$$

$$F = (a.x.b.y)/(c.z) \quad (7)$$

$$\sigma_F^2 = \sum_{i=1}^{n} (\partial F/\partial x_i)^2 . \sigma_i^2 \quad (8)$$

Especially in the case where we use a large measure into n smaller measures of precision σ_x (like to chaine a great distance of 90m by carrying out 5 times a decameter of 20m) the precision σ_{Total} on the sum of the n measures is then Eq. (9).

If we repeat the same precision measurement σ_x n times to reach a mean value with a better precision, the error σ_{Moy} on the average is Eq. (10)

$$\sigma_{Total} = \sigma_x . \sqrt{n} \quad (9)$$

$$\sigma_{Moy} = \frac{\sigma_x}{\sqrt{n}} \quad (10)$$

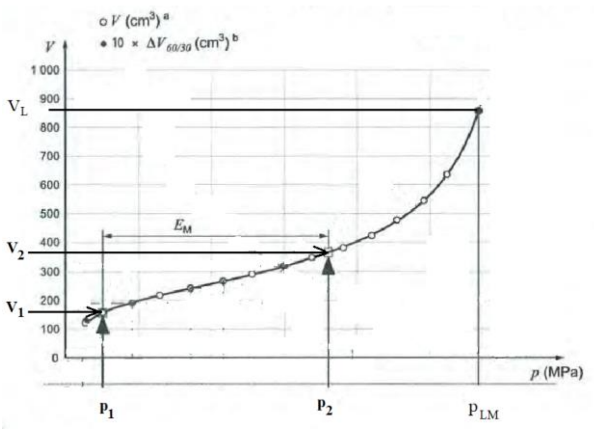

Figure 6. Determination of the pairs of points (p_1, V_1) and (p_2, V_2), (AFNOR, 2015)

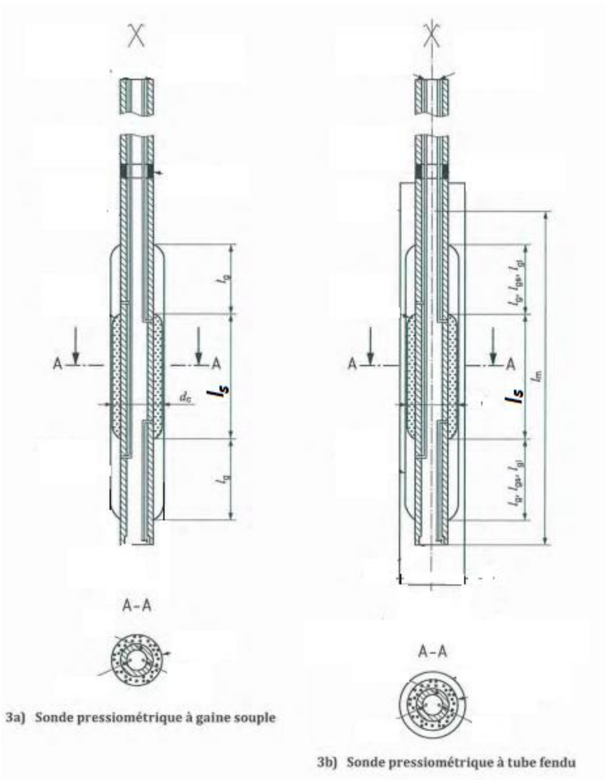

Figure 7. Determination of the probe length l_s (AFNOR, 2015)

1 : Calibration tube
2 : Pressuremeter probe

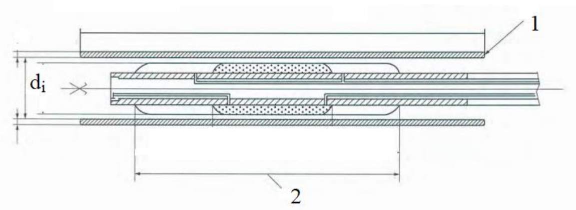

Figure 8. Determination of the diameter of the calibration tube (AFNOR, 2015)

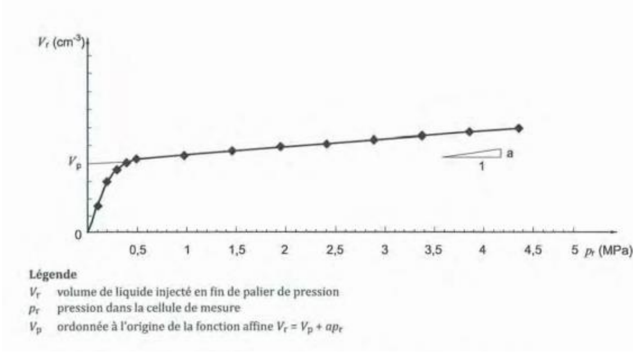

Figure 9. Calibration in dilatation of apparatus (AFNOR, 2015)

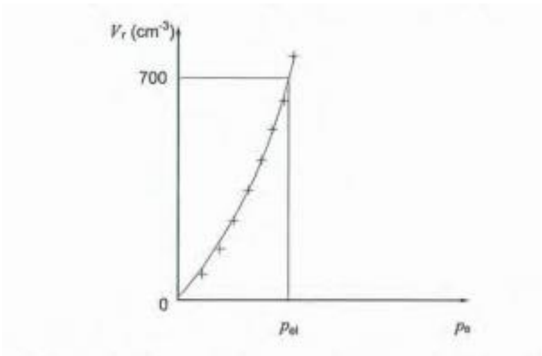

Figure 10. Calibration in pressure loss (AFNOR, 2015)

3. Ménard Pressuremeter modulus

3.1. Formula of the E_M Pressuremeter Modulus

The pressuremeter modulus is calculated by the expression Eq. (11) on the pseudo elastic range (Fig. 6) between the points p1 and p2.

The calculation variables are the volume of the V_s probe Eq. (12) which depends on the length of the probe l_s (Fig. 7) of the diameter of the calibration tube d_i (Fig. 8) of the volume correction V_p (Fig. 9). The corrected pressures p_1 and p_2 are calculated by Eq. (13) which depend on the measured pressure p_{r1}, the self-resistance relationship of the pel probe (V_e) (Fig. 10) and the depth z of the probe in the borehole. The corrected volumes V_1 and V_2 are calculated by Eq. (14) and depend on the measured volume V_{r1}, the device's own expansion coefficient a (Fig. 9), the ordinate at the origin of the equipment expansion line (Fig. 9). For more precision on these different variables, see [2]

$$E_M = 2.66.\left[V_s + \frac{V_1 + V_2}{2}\right].\frac{(p_2 - p_1)}{(V_2 - V_1)} \quad (11)$$

$$V_s = 0,25.\pi.l_s.d_i^2 - V_p \quad (12)$$

$$p_1 = p_{r1} - p_{el}(V_e) + \gamma_w.z \quad (13)$$

$$V_1 = V_{r1} - a.p_{r1} \quad (14)$$

3.2. Precision of the main variables

In this paper the term error indicates the standard deviation (in the Gaussian sense) of the measurement error. The precision on the main variables is also calculated by the law of propagation of the errors for the composed functions Eq. (8). We then obtain for the calculation of the volume of the probe V_s Eq. (12):

- The derivatives with respect to the length of the probe l_s Eq. (15), with respect to the diameter of the tube of qualibration d_i Eq. (16)
- The derivative with respect to V_p is equal Eq. (17)
- the accuracy on the volume of the V_s probe (21) whose definition is Eq. (12). Note that V_p is obtained by the ordinate at the origin of the indicated regression line (Fig. 9). Its accuracy follows the same rule as that of the limit pressure (see 4.2.2). The average error of estimation is Eq. (18)
- the standard deviation of the error on V_p is (with y = V and x = p and $\tilde{y}$ the volume estimated by the linear regression) given by Eq. (20). Finally the error on the volume of the probe V_s is Eq. (21)

$$\partial V_s/\partial p_{el} = 0{,}25.\pi.d_i^2 \tag{15}$$

$$\partial V_s/\partial d_i = 0{,}5.\pi.l_s.d_i \tag{16}$$

$$\partial V_s/\partial V_p = -1 \tag{17}$$

$$\sigma_r = \sqrt{\sum_{i=1}^{n} (y_i - \tilde{y})^2/(n-2)} \tag{18}$$

$$R = (x-\bar{x})^2 \sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \tag{19}$$

$$\sigma_{Vp} = t_{1-\alpha/2}^{n-2}.\sigma_{Vp}.\sqrt{1 + 1/n + (x-\bar{x})^2/R} = \Delta x \tag{20}$$

$$\sigma_{Vs} = \sqrt{\{0{,}0625.\pi^2.d_i^4.\sigma_{ls}^2 + 0{,}25.\pi^2.l_s^2.d_i^2.\sigma_{di}^2\}} \tag{21}$$

- The precision on the corrected pressures p1 and p2 Eq. (27) whose definition is Eq. (13), with Ve being written Eq. (22)

$$V_e = a_0 + a_1.p_{el} + a_2.p_{el}^2 + a_3.p_{el}^3 \tag{22}$$

$$\partial V_e/\partial p_{el} = a_1 + 2.a_2.p_{el} + 3.a_3.p_{el}^2 \tag{23}$$

$$\partial p_{el}/\partial V_e = 1/(a_1 + 2.a_2.p_{el} + 3.a_3.p_{el}^2) \tag{24}$$

$$\partial p_1/\partial p_{r1} = 1 \tag{25}$$

$$\partial p_1/\partial z = \gamma_w \tag{26}$$

$$\sigma_p = \sqrt{\{\sigma_{pr}^2 + \sigma_{Ve}^2/(a_1 + 2.a_2.p_e + 3.a_3.p_e^2) + \gamma} \tag{27}$$

- the derivative of the corrected volume V_1 or V_2 with respect to the measured volume Vr1 or Vr2 Eq. (28), with respect to the coefficient a of the linear regression Eq. (29), with respect to the measured pressure p_{r1} Eq. (30)
- The accuracy of the corrected volumes V_1 and V_2 Eq. (32) depends on the accuracy on the slope of the regression line σ_a Eq. (31) which is a function of σ_r the mean error of estimation Eq. (18)

$$\partial V_1/\partial V_{r1} = 1 \tag{28}$$

$$\partial V_1/\partial p_{r1} = -a \tag{29}$$

$$\partial V_1/\partial a = -p_{r1} \tag{30}$$

$$\sigma_a = \sigma_r \Big/ \left\{\sum_{i=1}^{n} (x_i - \bar{x})^2\right\}^{0.5} \tag{31}$$

$$\sigma_{V1} = \sqrt{\sigma_{Vr1}^2 + a^2.\sigma_{pr1}^2 + p_{r1}^2.\sigma_a^2} \tag{32}$$

3.3. Precision of the Pressuremeter Modulus E_M

The calculation of uncertainty requires the determination of the partial derivatives of the E_M module Eq. (11) with respect to the main variables:

• compared to the volume of the Vs probe Eq. (33)
• to corrected volumes V_1 and V_2 Eq. (34-(36)
• compared to corrected pressures p_1 and p_2 Eq. (37)

Which allows to have the precision on E_M Eq. (38) thanks to the relation of propagation of the errors for the composed functions

$$\partial E_M/\partial V_s = 2.66.(p_2 - p_1)/(V_2 - V_1) \tag{33}$$

$$\partial E_M/\partial V_1 = 2.66.\partial[Vs + (V1 + V2)/2]/(\partial V1).((p2 - p1))/((V2 - V1)) + 2.66.[Vs + (V1 + V2)/2].(p2 - p1)\ \partial/(\partial V1).1/((V2 - V1)) \tag{34}$$

$$\partial E_M/\partial V_1 = 2.66.1/2.((p2 - p1))/((V2 - V1)) + 2.66.[Vs + (V1 + V2)/2].(p2 - p1).1/(V2 - V1)\text{^}2 \tag{35}$$

$$\partial E_M/\partial V_1 = 1.33.(p2 - p1)/(V2 - V1)).\{1 + 2/((V2 - V1))[Vs + (V1 + V2)/2]\} \tag{36}$$

$$\partial E_M/\partial p_1 = 2.66.[Vs + (V1 + V2)/2].1/((V2 - V1)) \tag{37}$$

$$\sigma_{EM} = \{\partial E_M/\partial V_s . \sigma_{Vs} + 2.\partial E_M/\partial V_1^2 + 2.\partial E_M/\partial p_1^2 + \sigma_p{}^2\}\text{^}0.5 \quad (38)$$

4. Limit pressure

4.1. Calculation of the Limit Pressure p_{LM}

4.1.1. Direct calculation of the Limit Pressure p_{LM}

In this case, the measurement of the limit pressure is done when the doubling of the volume of the hole is reached, that is to say when the injected volume V_L corresponds to Eq. (39) with V_1 the volume injected for the contact. of the probe to the borehole.

$$V_L = V_s + 2.V_1 \quad (39)$$

4.1.2. Calculation by linear regression

In this case, the pressuremeter curve is transformed into a straight line by linear regression Eq. (40) of the corrected values {p; 1 / V}. This equation is transformed into a hyperbolic equation Eq. (41), which allows to determine the Ménard limit pressure by Eq. (42), with A slope of the regression line at V^{-1} and B at its ordinate at the origin:

$$V^{-1} = A.p + B \quad (40)$$

$$p = -B/A + 1/(A.V) \quad (41)$$

$$p_{LM} = -B/A + 1/(A.(V_s + 2.V_1)) \quad (42)$$

4.1.3. Calculation by the double hyperbola method

In this case, the pressuremeter curve is approximated by a double hyperbola whose coefficients are (A1, A2, A3, A4, A5, A6). The Ménard limit pressure is then the solution of Eq. (43) positive and lower than A6

$$0 = -A_2.p_{LM}^3 + [V_L - A_1 + A_2.(A_5 + A_6)].p_{LM}^2 + [(A_1 - V_L).(A_5 + A_6) - A_5.A_6.A_2 + A_3 + A_4].p_{LM} + [(V_L - A_1).A_5.A_6 - A_3.A_6 - A_4.A_5] \quad (43)$$

4.2. Precision of the Limit Pressure p_{LM}

4.2.1. Precision on the calculation of p_{LM} by method in V^{-1} and by the composition of the errors

In this case we use the inverse linear relation of V and we find the relation Eq. (42). By applying the law of composition of the errors to Eq. (42), one obtains successively the partial derivatives with respect to the variables Vs, V1, A and B Eqs. (44), (45), (46), (47) then the precision on the pressure limit p_{LM} Eq. (48)

$$\partial p_{LM}/\partial V_s = 1/[A.(V_s + 2.V_1)^2] \quad (44)$$

$$\partial p_{LM}/\partial V_1 = 2/[A.(V_s + 2.V_1)^2] \quad (45)$$

$$\partial p_{LM}/\partial A = (B + 1/(V_s + 2.V_1))/A^2 \quad (46)$$

$$\partial p_{LM}/\partial B = 1/A \quad (47)$$

$$\sigma_{Plm} = \{1/(A^2.(V_s + 2.V_1)^4).\sigma_{Vs}^2 + 4/(A^2.(V_s + 2.V_1)^4).\sigma_{V1}^2 + 1/A^4\ [B + 1/(V_s + 2.V_1)\ ]^2.\sigma_{A-}^2 + 1/A^2.\sigma_B^2\}^{0.5} \quad (48)$$

4.2.2. Precision on the calculation of p_{LM} by linear regression

In this case, the limit pressure p_{LM} and the volume V_L lie on the regression line beyong the points P_2 with a line of slope A and ordinate at the origin B. The equation Eq. (40) is represented in the theory of the regression line by Eq. (49), with y for V^{-1} and x for p, β_0 for B and β_1 for A. As we search for precision on p_{LM}, we must determine the precision from x and back to the theoretical equation Eq. (50), whose coefficients β'_0 and β'_1 are given by Eq. (51) and Eq. (52) [6], [7].

To determine the confidence interval of x, we find its value for a given risk α by means of the relation Eq. (54) in which t represents the Student's law with n-2 degree of freedom. To reduce it to the case of the pressuremeter and the limit pressure, we can notice that the variable y corresponds to V_L^{-1}, $\bar{y}$ corresponds to à $\overline{V^{-1}}$, y_i corresponds to V_i^{-1}, Δx corresponds to the permissible variation of the pressure p_{LM}, n corresponds to the number of measurements. Assuming a Gaussian distribution of the measures, we assume that the variance corresponds to a measurement expectation of 34.1% (Fig. 4).

Under these conditions, the theoretical relationship that allows to determine the variation of the limit pressure with a probability of α = 34.1%, which corresponds to the standard deviation of the limit pressure is obtained by Eq. (58). If we also take into account the variability on V_i^{-1} and V_L^{-1}, assuming that the mean values are not affected by the measurement errors, we get the partial derivatives of the variance with respect to $V_i^{-1}$and V_L^{-1} Eqs. (59) (60), then the value of the variance on the limit pressure is Eq. (61):

$$\hat{y} = \beta_1 . x + \beta_0 \quad (49)$$

$$\hat{x} = \beta'_1 . y + \beta'_0) \quad (50)$$

$$\beta'_1 = (\sum_{i=1}^{n}(x_i - \bar{x}).(y_i - \bar{y})/(\sum_{i=1}^{n}(y_i - \bar{y})^2) \quad (51)$$

$$\beta'_0 = \bar{x} - \beta'_1.\bar{y} \quad (52)$$

$$\sigma_r = \{\sum_{i=1}^{n}(x_i - \hat{\bar{x}})^2 / (n-2)\}^{0.5} \quad (53)$$

$$R1 = \left(V_L^{-1} - \overline{V^{-1}}\right)^2 / \sum_{i=1}^{n}\left(V_i^{-1} - \overline{V^{-1}}\right)^2) \quad (54)$$

$$R2 = t_{(1-\alpha/2)}^{n-2} . . \sqrt{\{1 + 1/n + R1\}} \quad (55)$$

$$R3 = (V_L^{-1} - \overline{V^{-1}}) / \sum_{i=1}^{n} (V_L^{-1} - \overline{V^{-1}})^2 \quad (56)$$

$$\Delta x = t_{(1-\alpha/2)}^{n-2} . \sigma_r . \sqrt{\{1 + 1/n + R1\}} \quad (57)$$

$$\sigma_{pLM} = t_{(1-\alpha/2)}^{n-2} . \sigma_r . \sqrt{\{1 + 1/n + R1\}} \quad (58)$$

$$\partial p_{LM}/\partial V_L = 2. R2. \underline{V_L^{-2}} . (V_L^{-1} - \overline{V^{-1}}) / \sum_{i=1}^{n} (V_L^{-1} - \overline{V^{-1}})^2 . \quad (59)$$

$$\underline{\partial p_{LM}}/\partial V_1 = R2. R3^2 . \sum_{i=1}^{n} 2. \left(V_i^{-1} - \overline{V^{-1}}\right) . V_i^{-2} \quad (60)$$

$$\sigma_{pLM} = t_{(1-\alpha/2)}^{n-2} . \sigma_r . \sqrt{\{1 + 1/n + R1\}} + [(\partial\sigma_{pLM}/\partial V_L)^2 . \sigma_{VL}^2 + (\partial\sigma_{pLM} / \partial V_i)^2 . \sigma_i^2]\text{^}0.5 \quad (61)$$

4.2.3. Precision on the calculation of p_{LM} by by the double hyperbole

In this case we use the polynomial equation of the double hyperbola for the volume V_L Eq. (43), which can be written in the form of Eq. (62) which allows to have the partial derivatives with respect to each unit variable Eqs. (63) to (70)

$$E(V_L, p_{LM}) = -A_2 . p_{LM}^3 + [V_L - A_1 + A_2 . (A_5 + A_6)] . p_{LM}^2 + [(A_1 - V_L) . (A_5 + A_6) - A_5 . A_6 . A_2 + A_3 + A_4] . p_{LM} + [(V_L - A_1) . A_5 . A_6 - A_3 . A_6 - A_4 . A_5] = 0 \quad (62)$$

$$\widehat{\partial E/\partial p_{LM}} = -3. A_2 . p_{LM}^2 + 2. [V_L - A_1 + A_2 . (A_5 + A_6)] . p_{LM} + [(A_1 - V_L) . (A_5 + A_6) - A_5 . A_6 . A_2 + A_3 + A_4] \quad (63)$$

$$\widehat{\partial E/\partial V_L} = p_{LM}^2 + (A_5 + A_6) . p_{LM} + (A_5 . A_6) \quad (64)$$

$$\widehat{\partial E/\partial A_1} = p_{LM}^2 + (A_5 + A_6) . p_{LM} \quad (65)$$

$$\widehat{\partial E/\partial A_2} = p_{LM}^3 + (A_5 + A_6) . p_{LM}^2 + A_5 . A_6 . p_{LM} \quad (66)$$

$$\widehat{\partial E/\partial A_3} = p_{LM} + 1 \quad (67)$$

$$\widehat{\partial E/\partial A_4} = p_{LM} + 1 \quad (68)$$

$$\partial E/\partial A_5 = A_2 . p_{LM}^2 + [(A_1 - V_L) - A_6 . A_2] . p_{LM} + [(V_L - A_1) . A_6 - A_4] \quad (69)$$

$$\partial E/\partial A_6 = A_2 . p_{LM}^2 + [(A_1 - V_L) - A_5 . A_2] . p_{LM} + [(V_L - A_1) . A_5 - A_3] \quad (70)$$

The law of composition of the errors applied to the relation Eq. (62), gives Eq. (71); we notice that equation (62) is null by definition so as its variation Eq. (71). We pose by definition a null variance, since the nullity of E is a hypothesis of resolution. This equation, allows to have the precision on the limit pressure by Eq. (72)

$$\sigma_E{}^2 = (\partial E/\partial p_{pLM})^2 . \sigma_{pLM}{}^2 + (\partial E/\partial V_L)^2 . \sigma_{VL}{}^2 + (\partial E/\partial A_1)^2 . \sigma_{A1}{}^2 + (\partial E/\partial A_2)^2 . \sigma_{A2}{}^2 + (\partial E/\partial A_3)^2 . \sigma_{A3}{}^2 + (\partial E/\partial A_4)^2 . \sigma_{A4}{}^2 + (\partial E/\partial A_5)^2 . \sigma_{A5}{}^2 + (\partial E/\partial A_6)^2 . \sigma_{A6}{}^2 \quad (71)$$

$$\sigma_{pLM} = 1/(\partial E/\partial p_{pLM}) . \{(\partial E / \partial V_L)^2 . \sigma_{VL}{}^2 + (\partial E/\partial A_1)^2 . \sigma_{A1}{}^2 + (\partial E/\partial A_2)^2 . \sigma_{A2}{}^2 + (\partial E/\partial A_3)^2 . \sigma_{A3}{}^2 + (\partial E/\partial A_4)^2 . \sigma_{A4}{}^2 + (\partial E/\partial A_5)^2 . \sigma_{A5}{}^2 + (\partial E/\partial A_6)^2 . \sigma_{A6}{}^2\}^{0.5} \quad (72)$$

5. Application - Precision of the Ménard Modulus on typical examples

Typical examples were provided by Fondasol [8]. In the following calculations, we use the following accuracies (or standard deviation) (Tab.4). deduced from the standard (Tab.3).

The determination of the volume of the probe (Procedure A) through the expansion test (resistance) is indicated for borehole test A at 29m (Fig. 11) for both borefole B at 21 and 22m (Fig. 12) for borehole test E at 1.5m (Fig. 13). In these Fig.s, the Minimum and Maximum lines corresponding to the mean line +/- a standard deviation (34.1% precision) were also carried

Table 1. Measuring Range and Accuracy for Measuring Devices of a Ménard Pressuremeter (AFNOR, 2015)

Measurement	unit	Area of measurement	Process A	Process B
Depth	m		0.2	0.2
Time	s		1	0.5
Pressure	kPa	0 - 5000	25	15
Voluùe	Cm^3	0 - 700	2	1

Table 2. Precision used in the calculations of the typical examples

Measu	unit	Process A	Process B	Process C
Length	mm	1	1	1
Diameter calib. tube	mm	0.1	0.1	0.1
Depth	m	0.2	0.2	0.2
Time	s	1	0.5	0.5
Pressure	kPa	25	15	10
Voluùe	cm3	2	1	0.1
Unit weigth	kN/m3	0.1	0.1	0.1

Table 3. Precision calculations for the main variables - procedure A

	Value	Precision
d_i (mm)	66	0.1
l_s (mm)	210	1
γ_w (kN/m^3) à 20°	97,89	0.1
V_c (cm3)	139,2	1
Borehole A , 29m		
p_{rl} (MPa)	3,000	0,025
p_l (MPa)	3,076	0,0253
V_{rl} (cm^3)	262	2
V_l (cm^3)	248.8	2
Borehole B, . 21m		
p_{rl} (MPa)	1,300	0,025
p_l (MPa)	1,398	0,0253
V_{rl} (cm^3)	165	2
V_l (cm^3)	159.9	2
Borehole B, 22m		
p_{rl} (MPa)	0,800	0,025
p_l (MPa)	0,860	0,0253
V_{rl} (cm^3)	202	2
V_l (cm^3)	198,9	2
Borehole E, 1,5m		
p_{rl} (MPa)	0,300	0,025
p_l (MPa)	0,210	0,0253
V_{rl} (cm^3)	258	2
V_l (cm^3)	257,5	2

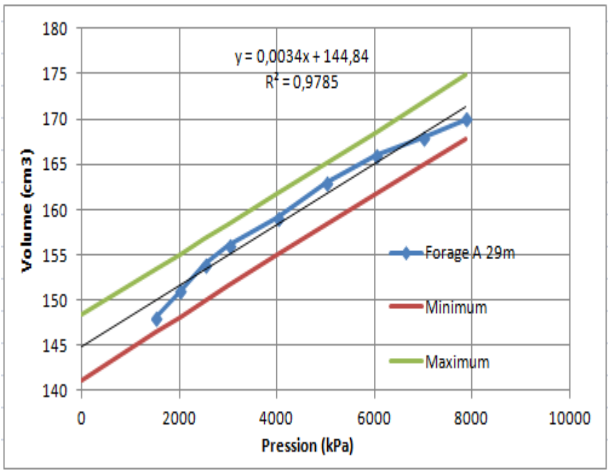

Figure 11. Precision of V_p by linear regression related to the own dilation (resistance) - Test at 29m, hole A.

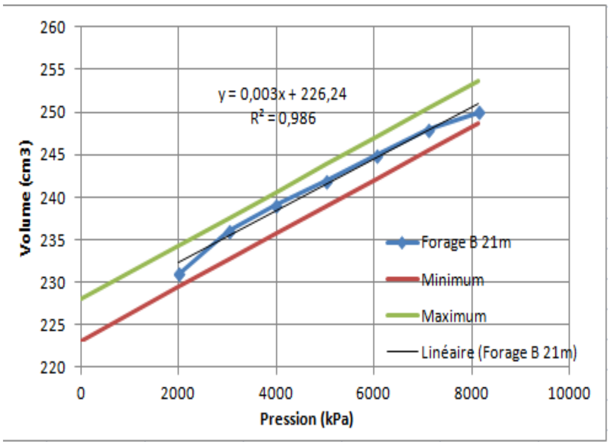

Figure 12. Precision of V_p by linear regression related to the own dilation (resistance) - Test at 21m and 22m, hole B..

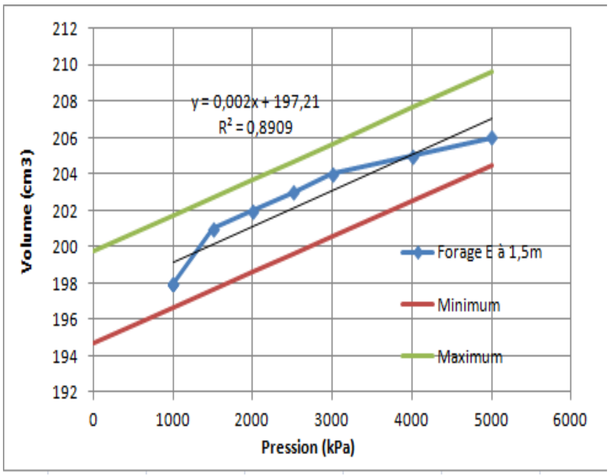

Figure 13. Precision of V_p by linear regression related to the own dilation (resistance) - Test at 1.5m hole E.

5.1. Precision on E_M - Procedure A for σ_p=25kPa and σ_v=2cm³

A first series of calculations is based on the assumption that the operator carries out visual measurements with a precision of reading of the pressure p_r with an accuracy of 25kPa and a precision on the volumes V_r of 2cm3. The accuracy on the z-side of the probe (Tab.2) is assumed to be 0.2m. Tab.3 shows the precision calculation for the main variables and in Tab.4 the precision of the calculation for the pressuremeter module E_M which varies between 4% and 17% (col.9, Table4)

The precision on the Pressuremeter module E_M allows to notice:

- an error related to the linear regression σ_p greater than that on the unit volumes of procedure A (col.3, Tab.4).
- The linear regression error is not improved if the accuracy is better. The values of σ_s do not change when proceeding from procedure A to B or C
- There is a significant error in the linear regression procedure that cannot be reduced by improving the quality of the measurement (see Tab.4 to Tab.6). The improvement of the accuracy passes by a better approximation of V_c which cannot be linear.

5.2. Precision on E_M - Procedure B for σ_p=15kPa and σ_v=1cm³

A second series of calculations is based on the assumption that the measurements are carried out automatically with a precision of pressure reading p_r with an accuracy of 15kPa and a precision on volumes V_r of 1cm³. The accuracy on the z-side of the probe is assumed to be 0.2m. The precision of the calculation for the pressuremeter module E_M is shown and varies between 2% and 10%. (col.9, Tab.5)

.

5.3. Precision on E_M - Procedure C for σ_p=10kPa and σ_v=0.1cm³

A third series of calculations is based on the assumption that the measurements are carried out automatically with a precision of pressure reading p_r with an accuracy of 10kPa and a precision on volumes V_r of 0.1cm³. The accuracy on the z-side of the probe is assumed to be 0.2m. The precision of the calculation for the pressuremeter module E_M is shown and varies between 2% and 7%. (col.9, Tab.6)

Table 4: Precision calculations on the pressuremeter module; procedure A

Borehole	Depth m	σ_r cm³	σ_{Vp} cm³	σ_{Vs} cm³	a cm³/kPa	σ_a cm³/kPa	E_M MPa	σ_{EM} MPa	Error %	Tolérance MPa
A	29	1.2	3.6	7.7	3,380	0.1891	152,3	8.6	5.6	26,8
B	21	1.8	4.5	8.6	3,470	0.2793	21.6	0.6	3,9	2.3
B	22	1.8	4.5	8.6	3,470	0.2793	20,6	1.8	8.9	4.8
E	1,5	0.4	2.6	6.7	1.411	0.1282	5.3	0.9	16.6	2.4

Table 5 : Precision calculations on the pressuremeter module; procedure B

Borehole	Depth m	σ_r cm³	σ_{Vp} cm³	σ_{Vs} cm³	a cm³/kPa	σ_a cm³/kPa	E_M MPa	σ_{EM} MPa	Error %	Tolérance MPa
A	29	1.2	2.6	6.7	3,380	0.1891	152,3	4.4	3.0	11.9
B	21	1.8	3.5	7.6	3,470	0.2793	21.6	0.4	2,2	1.3
B	22	1.8	3.5	7.6	3,470	0.2793	20,6	1.0	4,9	2.7
E	1,5	0.4	1.6	5.7	1.411	0.1282	5.3	0.5	9,9	1,4

Table 6 : Precision calculations on the pressuremeter module; procedure C

Borehole	Depth m	σ_r cm³	σ_{Vp} cm³	σ_{Vs} cm³	a cm³/kPa	σ_a cm³/kPa	E_M MPa	σ_{EM} MPa	Error %	Tolérance MPa
A	29	1.2	1.7	5.8	3,380	0.1891	152,3	1.4	0.8	4,0
B	21	1.8	2.6	6.7	3,470	0.2793	21.6	0.2	2,2	1,3
B	22	1.8	2.6	6.7	3,470	0.2793	20,6	0.6	4,9	2,7
E	1,5	0.4	0.7	4.8	1.411	0.1282	5.3	0.4	9,9	1,4

6. Application - Precision of the Limit pressure on typical examples

6.1. Precision on p_{LM} using V^{-1} - Procedure A for σ_p=25kPa and σ_v=2cm³

Calculations are made using the relation Eq. (48), and estimating the variation of the regression coefficients A and B by varying the creep pressure p_f assumed to be p_2. Note that the test A at 29m is very sensitive to the variation of p_f which leads to a significant error on the limit pressure. This is not the case for all other tests.

Table 7 : Precision calculation on limit pressure - procedure A

Hole	Depth m	p_{LM} MPa	σ_{EM} MPa	Error %	Toler MPa
A	29	15,4	5.0	32.9	13.5
B	21	3,9	0,1	1,3	0,1
B	22	3,1	0,2	0,1	0,1
E	1,5	0.74	0,01	0,1	0,1

6.2. Precision on p_{LM} using V^{-1} - Procedure B for σ_p=15kPa and σ_v=1cm³

Table 8 : Precision calculation on limit pressure - procedure B

Hole	Depth m	p_{LM} MPa	σ_{EM} MPa	Error %	Toler MPa
A	29	15,4	5.2	32.4	13.8
B	21	3,9	0,1	1,3	0,1
B	22	3,1	0,2	0,1	0,1
E	1,5	0.74	0,01	0,1	0,1

6.3. Precision on p_{LM} using V^{-1} - Procedure C for σ_p=10kPa and σ_v=0.1cm³

Table 9 : Precision calculation on limit pressure - procedure C

Hole	Depth m	p_{LM} MPa	σ_{EM} MPa	Error %	Toler MPa
A	29	15,4	5.2	32.4	13.8
B	21	3,9	0,1	1,3	0,1
B	22	3,1	0,2	0,1	0,1
E	1,5	0.74	0,01	0,1	0,1

6.4. Precision on p_{LM} using linear regression - Procedure A for σ_p=25kPa and σ_v=2cm³

The calculations are based on equation (61), which takes into account the uncertainty associated with linear regression and pressure and volume uncertainty. We can see (Fig. 14 - Fig. 17) and (Tab.10 - Tab.12):

• a marked linear alignment of the experimental points and a small difference between the high and low limits linked to the standard deviation of the linear regression, calculated for a variation of 34.1% of the experimental values (ie a standard deviation of the Gaussian law).

• No influence of the accuracy of the pressure and volume measurement on the accuracy of the limit pressure

• The accuracy of the limit pressure is only affected by the quality of the alignment after the pressure p2, ie by the relation Eq. (55) and the poor accuracy on Vs

Table 10 : Precision limit pressure by linear regression - procedure A

Hol	Dept m	σ_r MPa	p_{LM} MPa	σ_r MPa	σ_{EM} MPa	Err. %	Toler MPa
A	29	0,08	15,4	0.32	0.32	2,0	0.86
B	21	0.02	3,9	0.17	0.17	4,3	0.46
B	22	0.01	3,0	0.06	0.06	2,0	0.16
E	1,5	0.02	0.79	0.06	0.06	7,5	0.16

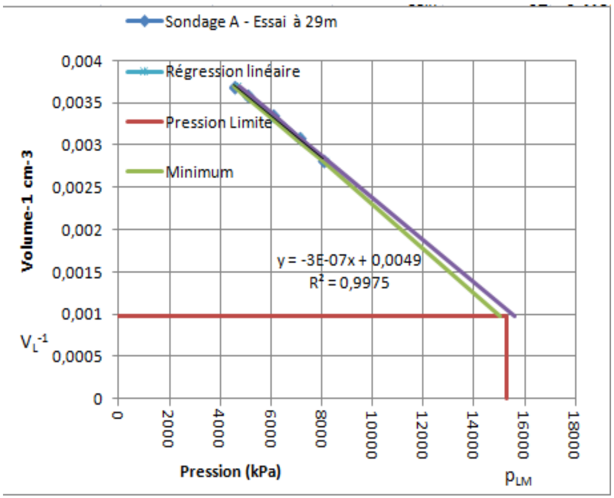

Figure 14. Accuracy of the linear regression related to the limit pressure p_{LM} - Test at 29m, Borehole A

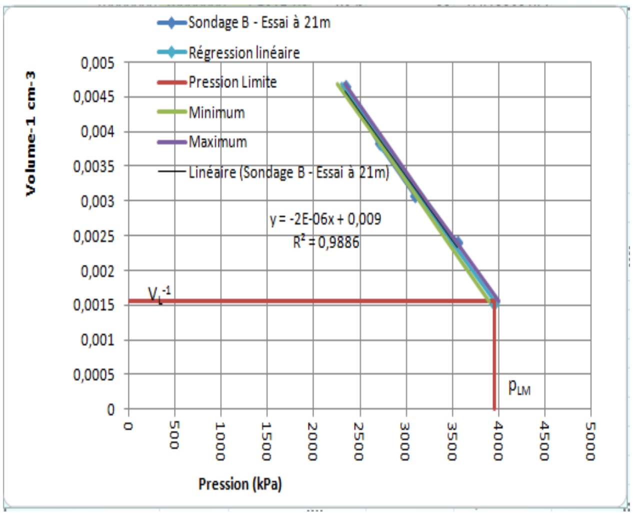

Figure 15. Accuracy of the linear regression related to the limit pressure p_{LM} - Test at 21m, Borehole B

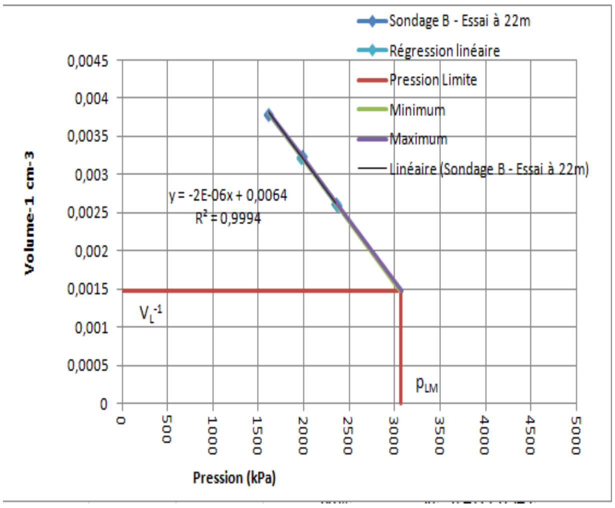

Figure 16. Accuracy of the linear regression related to the limit pressure p_{LM} - Test at 22m, Borehole B

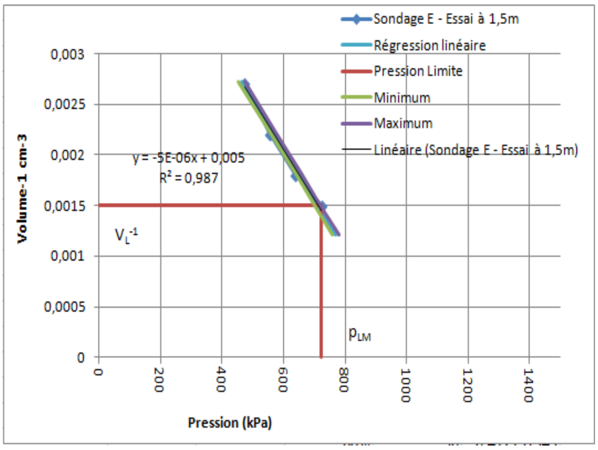

Figure 17. Accuracy of the linear regression related to the limit pressure p_{LM} - Test at 1,5m, Borehole E

6.5. Precision on p_{LM} using linear regression - Procedure B for σ_p=15kPa and σ_v=1cm^3

Table 11 : Precision limit pressure by linear regression - procedure B

Hole	Depth m	σ_r MPa	p_{LM} MPa	σ_r MPa	σ_{EM} MPa	Error %	Toler MPa
A	29	0.08	15,4	0.31	0.31	2,0	0.84
B	21	0.07	3,9	0.16	0.16	4,1	0.43
B	22	0.01	3,0	0.05	0.05	1,7	0.13
E	1,5	0.01	0.79	0.05	0.05	6,3	0,13

6.6. Precision on p_{LM} using linear regression - Procedure C for σ_p=10kPa and σ_v=0.1cm^3

Table 12 : Precision limit pressure by linear regression - procedure C

Hole	Depth m	σ_r MPa	p_{LM} MPa	σ_r MPa	σ_{EM} MPa	Error %	Toler MPa
A	29	0.08	15,4	0.30	0.30	1,9	0.81
B	21	0.07	3,9	0.15	0.15	3,8	0.40
B	22	0.01	3,0	0.04	0.04	1,3	0.11
E	1,5	0.01	0.79	0.04	0.04	5,0	0.11

7. Conclusions

This paper used three classes of precisions which allows to analyse the pressuremeter results. The main calculations were made with a precision on the pressures of 25kPa to 10kPa and 15 cm3 to 0.1 cm3 on volumes and could be carried out by the theoretical analysis of the standard deviations of the principal variables

Analysis of the accuracy of the pressuremeter module shows that the precision is highly related to the accuracy of the pressure and volume measurements. It is also linked to the precision on the volume of the Vs probe. There is a better accuracy for the high values of the pressuremeter modulus. Further more the accuracy of the cyclic module is done by the same formulas as that of the Ménard module by replacing p1 by p3 and p2 by p4

The analysis of the precision of the limit pressure shows that the accuracy of the pressure and volume measurements has no influence on the limit pressure and solely the variability of the creep pressure p_F can affect p_{LM}

Acknowledgement

The project presented in this article is supported by The French National Project ARSCOP

References

[1] AFNOR, 2013. NF P94-261 Justification des ouvrages géotechniques - Normes d'application nationale de l'Eurocode 7 - Fondations superficielles - Calcul géotechnique.
[2] AFNOR, 2015. NF EN ISO 22476-4 Essai pressiométrique Ménard. AFNOR, Reconnaissance et essais géotechniques.
[3] GUM, 2008. Evaluation des données de mesure – Guide pour l'expression de l'incertitude de mesure. JCGM 100, 132.
[4] Gachon, J.-C., 1969. Cours de topographie. INSA, Lyon.
[5] Lalère, B., 2011. Estimation de l'incertitude de mesure appliquée aux résultats de mesures des paramètres physico-chimiques de l'eau. LNE - ONEMA L010705, 40.
[6] Wikipedia, 2016. Régression linéaire [WWW Document]. Wikipedia.
[7] Rouaud, M., 2013. Calcul d'incertitudes. Creative Commons.
[8] Jacquard, C., 2017. Résultats de l'enquête sur l'interprétation de 4 essais pressiométriques. ARSCOP - Fondasol.