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Cylindrical cavity expansion analysis applied to the interpretation of variable rate cone penetration in tailings

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ABSTRACT: The assessment of rate effects on piezocone test in tailings is a necessary step in the design of tailings storage facilities, since these geomaterials often exhibit coefficients of hydraulic conductivity in the range of transitional soils. The present paper describes the application of a non-linear poroelastic model conceived to capture the transient flow effects of the soil around an expanding cylinder (Dienstmann et al, 2015), to the interpretation of piezocone tests in gold tailings. Tests carried out at constant penetration rates ranging from 0.3mm/s to 57 mm/s are used in the analysis. Results are interpreted in a space that correlates dimensionless velocity V_h to cone resistance Q and degree of drainage U showing that penetration is essentially drained at normalized velocities less than about 0.1 and essentially undrained at normalized velocities greater than 10.

1 INTRODUCTION

The design of tailings storage facilities (TSF) is still a challenge to geotechnical engineers because of the high complexity involving the tailings geotechnical behavior coming from the heterogeneous nature of waste products, the hydraulic depositional processes and the change in the constitutive parameters during the lifetime of deposits. The experience gathered in the last 20 years (e.g. Schnaid et al. 2013 Jamiolkowsky & Masella, 2014) shows that the depositional process often produces tailings in the so-called intermediate permeability range of 10^{-5} to 10^{-8} m/s. In intermediate soils including natural clayey and sandy–silts, tailing and others geomaterials, a partially drainage behavior is likely to occur when in-situ tests as cone penetration are performed at the standardized velocity (20mm/s), introducing errors in interpretation.

In this context, theoretical solutions can be an essential tool helping understanding what controls the drainage behavior taking place during cone penetration tests executed in silty materials. Among the existing methods, cavity expansion approach is extensively used and accepted for modeling the cone penetration (e.g. Gibson and Anderson, 1961; Baligh, 1985; Teh and Houlsby, 1991; Salgado et al., 1997; Yu and Mitchell, 1998; Burns and Mayne, 2000; Chang et al. 2001; Cao et al., 2001; Chen and Abousleiman, 2012, 2013; Zhang et al., 2016), with a variety of soil stress–strain models. Although, there is an extensively literature on cavity expansion solutions applied to the interpretation of piezocone tests, there is not a complete understanding of what controls the transient behavior during penetration. The present paper describes the use of a cylindrical cavity solution proposed by Dienstmann et al. (2015), to assist the drainage behavior interpretation of experimental data from a gold TSF.

2 BRIEF DESCRIPTION OF THE MODEL

The analytical cylindrical cavity expansion solution proposed in Dienstmann et al. (2015) is structured as a consolidation analysis of a rigid cylinder deeply embedded within an isotropic fully saturated poroelastic medium of infinite extent. The analysis is based on the assumption of plane strain conditions in the cross-section of a system defined by the cylinder and surrounding porous medium, restricting displacements and flow to two dimensions. The soil surrounding the cylinder is model as a fully saturated poroelastic material undergoing infinitesimal strains.

The problem of consolidation around infinite expanding cylinder is idealized in Figure 1. Starting from an initial stress and pore pressure states (σ_0, p_0), a prescribed radial displacement of magnitude $\alpha(t)R$ is applied at the cylinder wall $r = R$. The concept of influence zone $R \geq r \geq a$ is also introduced in this analysis to characterize the extent of the region whose poromechanical state is no longer affected by cylinder installation and subsequent expansion (e.g. Blight, 1968; Randolph and Wroth, 1979; Osman and Randolph, 2012). This means in particular that displacement and pore pressure vanish at r = a.

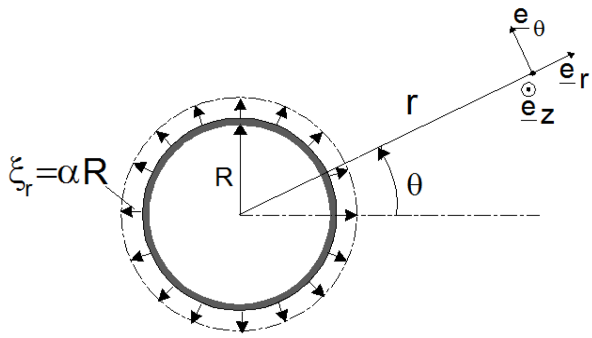

Figure 1. Idealized geometry and loading conditions for consolidation around infinite expanding cylinder.

2.1 *Constitutive formulation*

A non-linear poroelastic model is adopted for the problem of the cylindrical cavity solution. The model assumes the local equivalence between the response of a perfectly plastic behavior to monotonic loading and an appropriate fictitious non-linear poroelastic behavior.

The non-linear poroleastic behavior is conceived as an asymptotic representation of the plastic yielding of the material. Adopting a Drucker-Prager yield condition for the material strength, Equation 1 describes the non-linear shear modulus as a function of both volumetric and equivalent deviatoric strains (ε_v and ε_d, respectively) and pore pressure p.

$$G(\varepsilon_d, \varepsilon_v, p) = \frac{1}{2}\left[T(h - K\varepsilon_v - (1-b)\,p)\right]\;\frac{1/\varepsilon_{ref}}{1+\varepsilon_d/\varepsilon_{ref}} \tag{1}$$

where $\varepsilon_{ref} \ll 1$ is a reference strain that physically represents the order of magnitude of the shear strain mobilized at yielding, parameters h and T respectively characterize the tensile strength and the friction coefficient of the medium, and K is a constant value for the bulk modulus.

The pore flow problem is solved observing first that the combination of the fluid mass balance equation, the second poroelastic state equation and Darcy's law yields:

$$b\frac{\partial \varepsilon_v}{\partial t} + \frac{1}{M}\frac{\partial p}{\partial t} = k\nabla^2 p \tag{2}$$

where M is the material Biot modulus, and k is the permeability coefficient. Considering the non-linear poroelastic medium the following generalized Navier equation can be derived (assuming irrotational displacement field).

$$\left[K + \tfrac{4}{3}G\right]\nabla\varepsilon_v + 2\,\nabla G\cdot(\boldsymbol{\varepsilon} - \tfrac{1}{3}\varepsilon_v\mathbf{1}) = b\nabla p \tag{3}$$

The above equation emphasizes the coupling between skeleton strains and pore-fluid pressure. The formulation differs from the classical formulations in which only the volumetric part ε_v of skeleton strains affects pore-fluid pressure. In the present case, pore pressure is also related to the deviatoric part ε_d of skeleton strains through the shear modulus $G(\varepsilon_d,\varepsilon_v,p)$. In classical formulations considering a constant shear modulus, the volume strain ε_v is eliminated between Equations (2) and (3) to get an uncoupled pressure diffusion equation governing the evolution of pore pressure, what is not possible on the derived formulation. A time incremental procedure has been elaborated and implemented in Dienstmann et al. (2015) to handle semi-analytically this problem, making use of a specific iterative algorithm within each time step. The incremental procedure determines the pore pressure distribution p(r,t) and displacement function f(r,t) for each time increment by solving the coupled system defined by the set of non-linear partial differential equations (2) and (3), together with associated boundary and initial conditions.

2.2 *Initial excess pore-fluid pressure distribution*

The solution of equations (2) and (3) requires the initial field of pore pressure to be determined to represent the initial excess of pore pressure u_0 (or equivalently p_0) generated by insertion of a rigid cylinder within the medium. Observing that none of the expressions proposed in literature (e.g. Randolph and Wroth, 1979, Morris and Williams, 2000), complies with the condition of null fluid flow through the cylinder wall a new expression shall be proposed. It extends classical expressions to account for the flow restriction at cylinder wall:

$$u_0(r) = u_{0,\max}\frac{\mathfrak{F}(r)}{\mathfrak{F}(R)} \tag{4}$$

$$\textit{with}\quad \mathfrak{F}(r) = 1 - \frac{a}{r} + \frac{a}{R}\ln\frac{a}{r} \qquad \text{for } R \le r \le a$$

where $u_{0,\max}$ refers to the maximum value of pore pressure generated by cylinder insertion. In the present paper $u_{0,\max}$ is estimated by the pore pressure generated during the consolidation phase in an undrained triaxial test

$$u_{0,\max} = \frac{p_{c0}}{2}\left(1 + M_{cs}\right) \tag{5}$$

where p_{c0} denotes a reference initial consolidation pressure, and M_{cs} is the critical state line inclination.

2.3 *Cone resistance estimation from the cavity approach*

The conversion of the cavity expansion results into cone resistance values was defined according the approach formulated by Rohani & Baladi (1981) and adopted by Silva (2005) and LeBlanc & Randolph (2008). The approach estimates the cone tip resistance from the vertical projections of all forces acting on the cone. Equation 6 presents the general estimation of q_c, which is derived considering that constant values of σ and τ act over the whole length of the cone.

$$q_c = \sigma + \tau \cot\ (\alpha) \tag{6}$$

For cylindrical cavity expansion is more convenient to write Expression 6 in terms of the radial stresses:

$$q_c = \sigma_r'(1+\tan(\delta)/\tan(\alpha))/(1-\tan(\delta)\tan(\alpha))+u \tag{7}$$

Equation 7 is obtained by the application of the equilibrium condition in a point along the cone, in which α is half of the angle of the cone tip (60°/2), δ is the interface friction taken as ϕ' and σ_r' is the radial stress at the expanding cylinder radius.

3 GOLD TAILINGS DEPOSIT

The *Fazenda Brasileiro* disposal plant from Bahia State, located in northeast Brazil is being a subject of research for the past 10 years, as reported by a series of papers (Bedin et al., 2012, Schnaid et al., 2013) and MSc and PhD thesis (Bedin, 2010; Klahold 2013, Nierwinski 2013, Dienstmann, 2015). This set of studies indicates that the material disposed in ponds is predominantly silty sand (see Figure 2) with an average in situ solids content (in weight) of about 30%, in situ water content of 35%, low to non plastic and high specific gravity (2.89<Gs<3.2 g/cm3).

To evaluate drainage effects during piezocone penetration test a total of 14 variable penetration rate cone tests in the 0.3mm/s to 57mm/s range was performed at the *Fazenda Brasileiro* testing site (see Table 1).

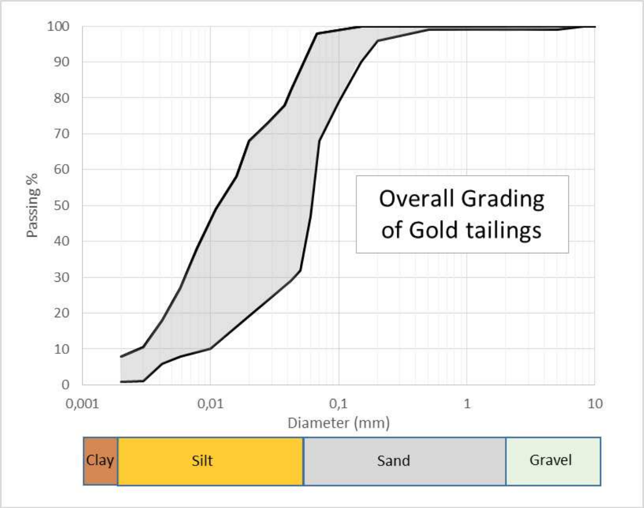

Figure 2. Grain size distribution

Table 1. Piezocone penetration rate

Study	Location	Penetration rate (mm/s)	Distance between tests (m)
Bedin (2006)	PZC 03	1; 20; 35	2
	PZC 07	20; 45	
	PZC 08	1,5; 10; 20	
Klahold (2012)	Island 01	0.3 to 0.7 $^{(*)}$; 20; 57	1.5
	Island 02	0.3 to 0.7 $^{(*)}$; 20; 57	

(*) test only executed between 3 to 5m

Figure 3 presents the gold tailings variable rate piezocone database interpreted in terms of a normalized resistance $Q=q_{cnet}/\sigma'_{v0}$ with $q_{cnet}=q_c - \sigma_{v0}$, and normalized pore pressure $U=\Delta u_2/\sigma'_{v0}$ plotted against the dimensionless velocities V_h, where q_c is the cone tip resistance and u_2 is the pore pressure. The normalized velocity V_h is defined according Equation 8, where d is the cone diameter, v the velocity of penetration and c_h the horizontal coefficient of consolidation.

$$V_h = \frac{v \times d}{c_h} \tag{8}$$

In the $Q \times V_h$ space it is possible to identify a region characterized by normalized velocities V_h in the range of about 0.01 to 10 where partial drainage appears to occur during CPTu penetration. The lowest (undrained) penetration resistance (Q_{UD}) is of the order of 2.0. The drained penetration resistance (Q_D) of 22.0 yields a drained to undrained ratio Q_D/Q_{UD} of about 11, which is consistent to previously reported data (Jager et al. 2010 and Lehane et al. 2010).

From the results in the $U \times V_h$ space a slightly larger scatter is observed, but some conclusions can be drawn from the CPTu response. Partial drainage takes place for V_h values in the 0.01 to 10 range, which is in agreement with the interpretation of cone penetration data. At fully undrained conditions the normalized pore pressure $\Delta u_2/\sigma'_{v0}$ ratio is of the order of 1.5 reducing to zero for drained penetration.

The adjusting curves presented in Figure 3 were constructed using the hyperbolic cosine function suggested by Schnaid (2005) in the form of:

$$Q = Q_{\min} + \left(a + (1-a)\frac{1}{\cosh(bV^c)} \right) \times (Q_{\max} - Q_{\min}) \quad (9)$$

$$\frac{\Delta u}{\sigma'_{v0}} = \frac{\Delta u_{\max}}{\sigma'_{v0}} - \left(a + (1-a)\frac{1}{\cosh(bV^c)} \right) \times \left(\frac{\Delta u_{\max}}{\sigma'_{v0}} - \frac{\Delta u_{\min}}{\sigma'_{v0}} \right) \quad (10)$$

Curve fitting coefficients for gold tailings are shown in Table 2.

Table 2. Curve fitting parameters

Curve fitting coefficients gold tailings			
	a	b	c
Lower Boundary	0.025	4	0.7
Upper Boundary	0.025	1	0.7

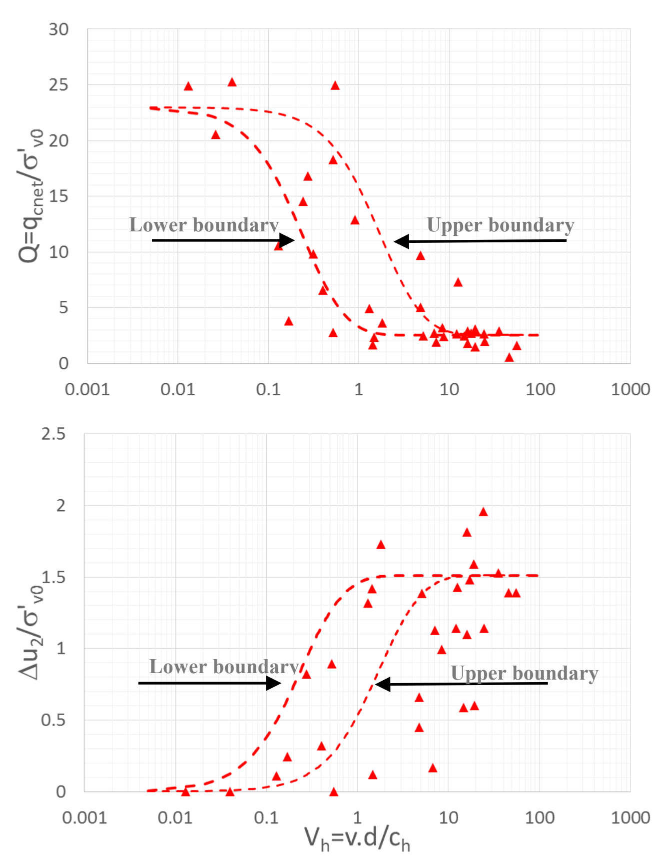

Figure 3. Rate effects in the (a) Q versus V_h space and (b) U versus V_h space

4 MODELING THE GOLD TAILINGS

The gold tailings average properties were used for modeling analytically and numerically the cylinder expansion problem with the aim of helping on assess rate effects on piezocone tests. The properties used in the analyses are summarized in Table 3. The parameters h and T from the Drucker Prager criteria are defined according to an inner Mohr Coulomb yield surface condition, with a friction angle $\phi' = 32°$.

Results of a cavity expansion modeled using a finite element software ABAQUS@ (Abaqus, 2009) are also introduced for analysis. The cavity expansion simulated in Abaqus was defined according an axisymmetric model, with unit weight and infinite extend (classical approach). The same initial and boundary conditions used in the analytical model were adopted in the finite element approach. For the constitutive model, the Drucker Prager combined with linear elasticity was considered. Elements used are 8-node axyssimetric quadrilateral, biquadratic displacement, bilinear pore pressure and reduced integration (CAX8RP).

Two radius of influence were considered for analysis:

- $a = (I_r)^{1/2} \times R$ with I_r=874 (defined from triaxial results, Bedin et al. 2012)), corresponding to a radius of influence of about 30 times the cylinder radius (a=30×R).
- and a=10×R to examine the possibility of better prediction of measured data and to comply with previous studies (Vesic, 1972; Randolph & Wroth, 1979; Osman & Randolph, 2010, 2012).

Maximum displacements were defined by limiting the local strains to 10% to try to comply with the model assumptions (infinitesimal strains). From a practical engineering perspective, strain levels as high as 10% are often admitted for geotechnical testing interpretation. It is therefore implicitly assumed that predictions obtained from both the simplified model and numerical approach are reasonable approximations for cylinder expansion, although a large strain formulation would be more appropriate.

The measured piezocone data are compared to analytical and numerical (finite element) predictions in Figures 4 and 5. The results presented in the present paper are focused on the interpretation of drainage effects in the normalized velocity space (for simplicity only V_h) combined with normalized cone tip resistance Q/Q_{ref} and normalized pore pressure $\Delta u/\Delta u_{ref}$, where Q_{ref} is the maximum value of Q (Q_{ref}=Q_{max}) corresponding to the drained penetration, and Δu_{ref} is the maximum value of mobilized pore pressure ($\Delta u_{ref} = \Delta u_{max}$), corresponding to undrained penetration.

From Figures 4 and 5 is possible to observe that predictions are generally able to capture the experimental trends shown in the Q×V_h and U×V_h spaces, but with some discrepancies. In terms of resistance (Q×V_h) both the analytical and numerical models underestimate the Q_D/Q_{UD} measured ratio of 11, but the models somehow capture the transitions from drained to partial drained to undrained. In the normalized pore pressure space U×V_h the models underestimate the drained normalized velocity. Transition from undrained to partial drainage is typically in the range of 1 to 10, but the onset of drainage is underestimated by at least one log-cycle.

The reduction proposed in the zone of influence (a=10×R instead of a=30×R) has a significant effect on the variation of pore fluid pressure with time. Adopting a zone of influence of 10 times the cavity radius produces a better approximation of the velocity V_h that characterized the transition for partially drained to drained behavior.

Table 3. Gold Tailings constitutive parameters

Constitutive parameters	
p_{c0}(kPa)	100
p_0' (kPa)	50
u_{max}	$p_{c0}(1+M_{cs})/2$
p_0 (kPa)	$p_0' + u_{max}$
γ_w (kPa)	10
k (m/s)	1.00E-08
K (KPa)	5814
ε_{ref}	0.01
e_0	1.34
K_s (GPa)	0.1
ϕ	32
ν	0.30
R (cm)	2.5
a (cm)	10R 30R

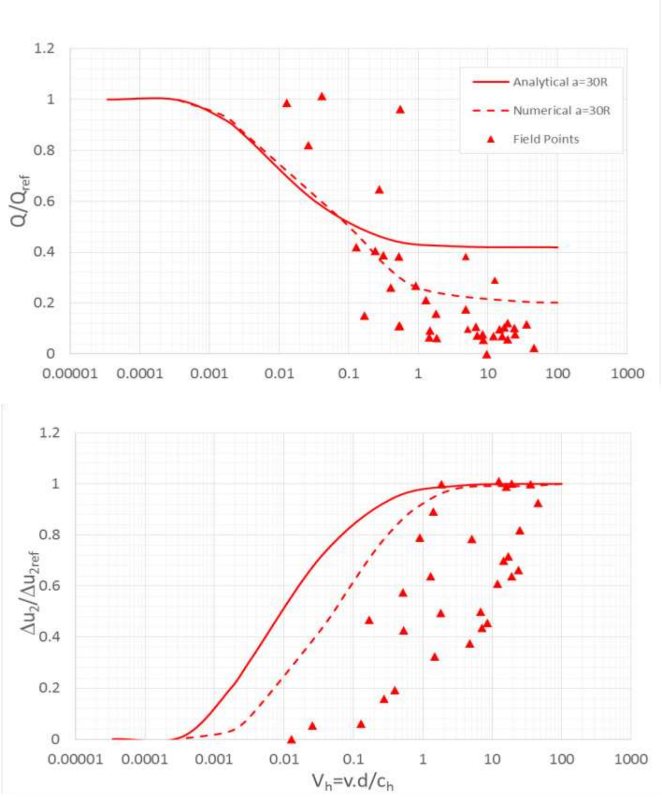

Figure 4. Comparisons between analytical and numerical predictions to field data in tailings for a/R=30

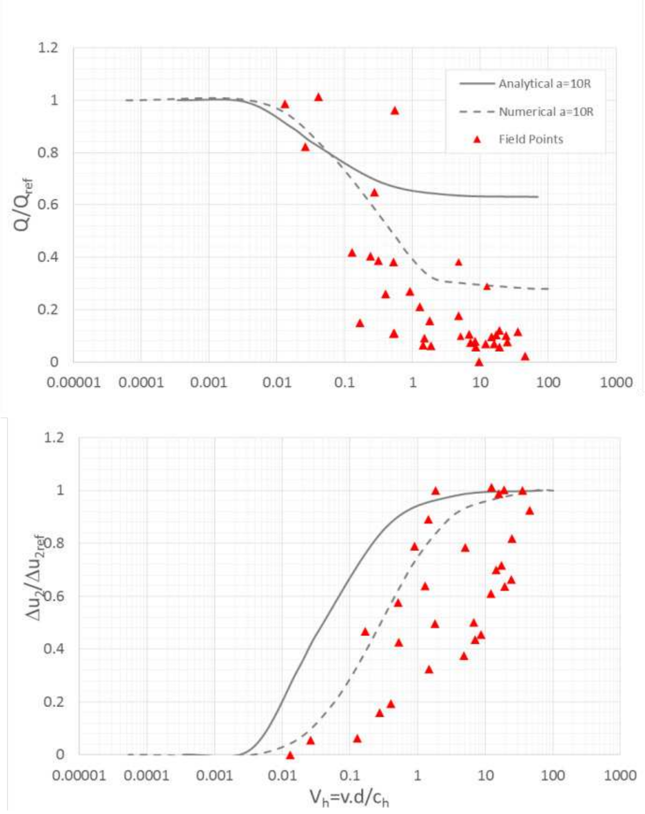

Figure 5. Comparisons between analytical and numerical predictions to field data in tailings for a/R=10

5 CONCLUSION

A non-linear poroelastic analytical solution for a cylindrical cavity expansion analysis developed in Dienstmann et al. (2015) is briefly described and applied to the investigation of rate effects into piezocone tests executed in a gold tailings deposit. Numerical simulations using finite elements are also used to enforce the validity of the analysis. The derived limit pressure (associated to q_c by the method of Rohani & Baladi, 1981) together with the pore pressure u_2 obtained from the analytical and numerical formulations are directly compared to field data. Results are shown to capture the experimental trends in the Q×V_h and U×V_h spaces: the ratio of drained to undrained resistance Q_D/Q_{UD} is underestimated in both models, and the velocity that defines the transition from drained to partially drained response is also under-predicted. Partially drained behavior occurs at a normalized velocities V_h in the range of 0.01 to 10. The practical recommendation is to perform tests at different penetration rates in the same location to establish the characteristic drainage curve of the soil to guide interpretation.

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