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# Effects of suction on CPT results and soil classification

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**ABSTRACT:** Laboratory-controlled CPT results for an unsaturated sand and silty sand show suction has a pronounced influence. The penetration resistances increase significantly due to the presence of suction when compared to those for saturated or dry states, for a given relative density and net confining stress. Also, for unsaturated sands, when suction is included in the effective stress, the same semi-empirical expressions used for saturated conditions are found to link penetration resistance to the relative density and effective confining stress. However, the same is not true for unsaturated silty sands, as saturated and unsaturated silty sands behave very differently around a penetrating cone. Suction also has implications when using classification charts. CPT results from a range of soils for both saturated and unsaturated conditions are plotted in one type of chart. Suction induced increases in penetration resistances, and changes of the soil behaviours from partially drained/undrained to drained, cause incorrect soil classifications and assessments that soils are dilative.

## 1 INTRODUCTION

Unsaturated soils are widely encountered and need to be dealt with in many engineering problems (e.g. foundations, fills, embankment dams, pavements and slopes). Their behaviour is complex and influenced by many factors including externally applied stresses, soil type, structure, density and suction. Suction, in particular, increases the shear strength and may contribute to the stiffening of the soil.

The characterisation of unsaturated soils may involve expensive and time-consuming investigations including soil borings and undisturbed sampling for laboratory testing. Performing the cone penetration test (CPT), a widely used in situ test, in an unsaturated soil may enable less costly and more rapid characterisation of the unsaturated soil properties.

The evaluation of soil properties from CPT results has been an area of intense practical interest. For both coarse- and fine-grained saturated soils, many correlations have been developed that link cone penetration resistance ( $q_c$ ) to in situ state and shear strength (e.g. Baldi et al. 1982, Robertson & Campanella 1983a, b, Jamiolkowski et al. 2003).

However, it is only in the past few years that correlations have emerged for the CPT that take into account the effects of unsaturation (Pournaghiazar et al, 2013, Yang & Russell, 2016). So far these have been limited to a clean quartz sand and a silty sand (decomposed granite). These and other studies (Le-

hane et al. 2004, Collins & Miller 2014) show that suction significantly increases  $q_c$ .

Most notably, Pournaghiazar et al. (2013) showed for the unsaturated sand, as long as suction is correctly incorporated in the effective stress, correlations developed for the sand when saturated/dry to establish the peak friction angle and relative density from the measured  $q_c$  are equally applicable to the sand when unsaturated. This is because the sand when unsaturated did not exhibit suction hardening (where suction hardening only exists if the yield surface size and isotropic normal consolidation line location depend on suction (Russell & Khalili 2006a)), and the sand behaved like a drained material during cone penetration irrespective of whether it was dry, saturated and unsaturated.

The same was not true for the unsaturated silty sand. Yang & Russell (2015, 2016) showed that the unsaturated silty sand behaved like a drained material. This is because the air in the pore space permitted volumetric compression to take place around a penetrating cone. Furthermore, since the silty sand exhibited suction hardening, the relationships which link penetration resistance to effective stress and relative density for drained saturated conditions would not apply to drained unsaturated conditions. These are a highly significant features to note and are the main reasons why CPT charts and relationships developed for saturated silty sands (or any soil with say more than 5% fines (i.e. sub 75 micron in size)) have no applicability to unsaturated silty sands. Any correla-

tions for CPTs in a saturated silty sands, where undrained or partially drained conditions prevail around the penetrometer, are inapplicable to the interpretation of the CPT in the same soil when unsaturated.

This paper summaries the laboratory-controlled CPT results for the two unsaturated soils obtained. The effects of suction are highlighted.

The paper goes on to demonstrate how not accounting for suction can lead to incorrect soil classifications and incorrect estimates of a soil's tendency to dilate or contract. This may lead to unsafe consequences, for example when using the CPT to assess a soil's potential for liquefaction.

## 2 EFFECTIVE STRESS

Following the work of Bishop (1959) the effective stress  $\sigma'$  is defined as:

$$\sigma' = \sigma + \chi s \quad (1)$$

where a prime symbol denotes the stress invariant to be effective,  $\sigma$  is the total stress in excess of pore-air pressure ( $u_a$ ) also referred to as the net stress,  $s$  is the suction, being the difference between pore-air and pore-water pressure ( $u_a - u_w$ ), and  $\chi$  is the effective stress parameter, having a value of 1 for saturated soils and 0 for dry soils.

## 3 CPT RESULTS IN UNSATURATED SYDNEY SAND AND LYELL SILTY SAND

The calibration chamber used was detailed by Pourhangiazar et al. (2011). The testing was conducted using a constant stress boundary condition.

The chamber accommodated cylindrical specimens with a height of 840 mm and diameter of 460 mm. The chamber has high-air entry ceramics embedded in the base plate for imposing suction in a specimen using the axis translation technique (i.e. imposing suction by elevating the pore air pressure above a positive pore water pressure). The axis translation technique was used for some specimens, while others were tested with suction being measured using vibrating wire piezometers.

CPTs were conducted using a miniature electrical cone with a diameter of 16 mm and cone tip area of 2 cm<sup>2</sup>. The cone was pushed into the soil at constant rate of 2 cm/s. The miniature cone was used to increase the chamber to cone diameter ratio ( $R_D$ ), which has a value of 29 in this study.

Sydney sand is a predominantly quartz sand sourced from the dunes around Kurnell, Sydney, Australia. An extensive experimental program was conducted by Russell & Khalili (2006a) to characterise saturated and unsaturated Sydney sand including

the soil-water characteristic curves and mechanical behaviour. Index properties of the soil include a particle density of  $\rho_s = 2.65 \text{ g/cm}^3$ , a minimum dry density of  $\rho_{\min} = 1.38 \text{ g/cm}^3$  corresponding to the maximum voids ratio  $e_{\max} = 0.92$ , and a maximum dry density  $\rho_{\max} = 1.66 \text{ g/cm}^3$  corresponding to the minimum voids ratio  $e_{\min} = 0.60$ . Peak friction angles  $\phi'$  observed in drained triaxial compression, for both constant cell pressure and constant  $p'$  load paths, may be estimated using:

$$\phi' - \phi'_{cs} = 3 \left[ D_r \left( 3.7 - \ln \left( \frac{p'_i}{p_a} \right) \right) - 0.9 \right] \quad (2)$$

in which  $D_r = (e_{\max} - e)/(e_{\max} - e_{\min})$  is the relative density,  $\phi'_{cs} = 36.3^\circ$  is the critical state friction angle and  $p'_i$  the mean effective stress at the beginning of shear. Russell & Khalili (2006a) found for Sydney sand that:

$$\chi = \begin{cases} 1 & \text{for } \frac{s}{s_e} \leq 1 \\ \left( \frac{s}{s_e} \right)^{-0.55} & \text{for } 1 < \frac{s}{s_e} \leq 25 \\ 25^{0.45} \left( \frac{s}{s_e} \right)^{-1} & \text{for } 25 < \frac{s}{s_e} \end{cases} \quad (3)$$

where  $s_e$  is the suction value separating saturated from unsaturated states.

Lyell silty sand is a decomposed granite from the catchment area of Lyell dam, NSW, Australia. It contains 27% fines, including 4% clay (sub 2 micron). Index properties of the soil include  $\rho_s = 2.55 \text{ g/cm}^3$ ,  $\rho_{\min} = 1.51 \text{ g/cm}^3$  corresponding to  $e_{\max} = 0.69$ , and  $\rho_{\max} = 2.02 \text{ g/cm}^3$  corresponding to  $e_{\min} = 0.29$ .  $\phi'$  observed in the triaxial compression tests on unsaturated samples with constant suctions may be estimated using:

$$\phi' - \phi'_{cs} = 3 \left[ D_r \left( 4.9 - \ln \left( \frac{p'_i}{p_a} \right) \right) - 1.0 \right] \quad (4)$$

where  $\phi'_{cs} = 35.7^\circ$  and  $p'_i$  is the mean effective stress at the start of shearing.  $\chi$  may be expressed as:

$$\chi = \begin{cases} 1 & \text{for } \frac{s}{s_e} \leq 1 \\ \left( \frac{s}{s_e} \right)^{-0.55} & \text{for } 1 < \frac{s}{s_e} \end{cases} \quad (5)$$

Numerous CPTs were conducted on saturated and unsaturated Sydney sand within the calibration chamber (Pournaghiazar et al. 2013). Dry sand specimens were prepared by the pluvial deposition technique. Unsaturated specimens were formed by first saturating the specimens and then letting the moisture content reduce and then the applying axis translation technique to achieve a target suction.

Two initial relative densities ( $D_r = 0.61$  and  $0.33$ ) and five net confining stresses (25 kPa, 30 kPa, 50 kPa, 100 kPa and 150 kPa, each applied isotropically) were used to test specimens. For a given relative density and confining stress combination, a test on a saturated specimen was performed, along with a number of tests on unsaturated specimens with initial suction measurements ranging from slightly larger than  $s_e$  to 200 kPa. It is noted that unsaturated Sydney sand specimens prepared in this way do not exhibit suction hardening (Russell & Khalili 2006a).

Samples of Lyell silty sand were prepared using static compaction of soil cured at certain moisture contents. After compaction of some samples a CPT was conducted immediately so that the sample suction was equal to the as-compacted value. For other samples the axis translation technique was used to reduce moisture content and increase suction. The axis translation technique enables more direct control of the suction applied. But the test soil had a low hydraulic conductivity (around  $3 \times 10^{-7}$  m/s for saturated conditions, and much lower for unsaturated conditions), which prevented uniform moisture and suction profiles from being achieved in reasonable time frames (Yang et al. 2014). Therefore, the moisture contents of the samples measured after a CPT was conducted were used to infer suctions. In samples in which CPTs were conducted without use of axis translation, the suction values were measured by three vibrating wire piezometers. Before inserting the piezometers their tips comprising high air-entry-value disks were carefully saturated.

For Sydney sand Pournaghiazar et al. (2013) presents the plots of  $q_c$  versus depth for saturated and unsaturated specimens. Suction has a significant influence on  $q_c$ . In all cases, suction causes the  $q_c$  to increase above the value measured in a saturated specimen for a given combination of relative density and net confining stress. For loose specimens, suctions of 25 kPa and 200 kPa increased  $q_c$  by 24% and 50% when the net confining stress was 50 kPa, and by 14% and 31% when the net confining stress was 100 kPa. For medium-dense specimens, the suction-induced increase of  $q_c$  was less than that for loose specimens for all the net confining stresses considered.

To interpret the Sydney sand results further it may be assumed that cone penetration in Sydney sand occurs under drained conditions; that is, when suction is constant, even though a constant moisture content condition actually exists. The differences in Sydney

sand's stress-strain behaviour for constant suction and constant moisture content conditions are negligible, as long as the degree of saturation is less than about 10% (which is almost always the case in unsaturated sands) (Russell & Khalili 2006b). The constant suction assumption in Sydney sand means that the suction around the cone tip can be assumed equal to the initial or far-field value. Also, as  $s_e$  in the  $\chi$  relationship does not vary significantly with sand deformation it can also be assumed that  $\chi_s$  around the cone tip is also equal to the initial or far-field value. This greatly simplifies interpretation of the results. A plot of  $q_c$  versus imposed mean effective stress  $p'$  is presented in Figure 1, where  $p' = p - u_w$  for saturated sands ( $u_w$  is the pore-water pressure),  $p' = p + \chi_s$  for unsaturated sands, and  $p$  is the mean net stress. An air entry value of  $s_e = 7$  kPa was used in the calculations of  $\chi$ . The data can be reasonably well fitted by the power law expression in the figure caption.

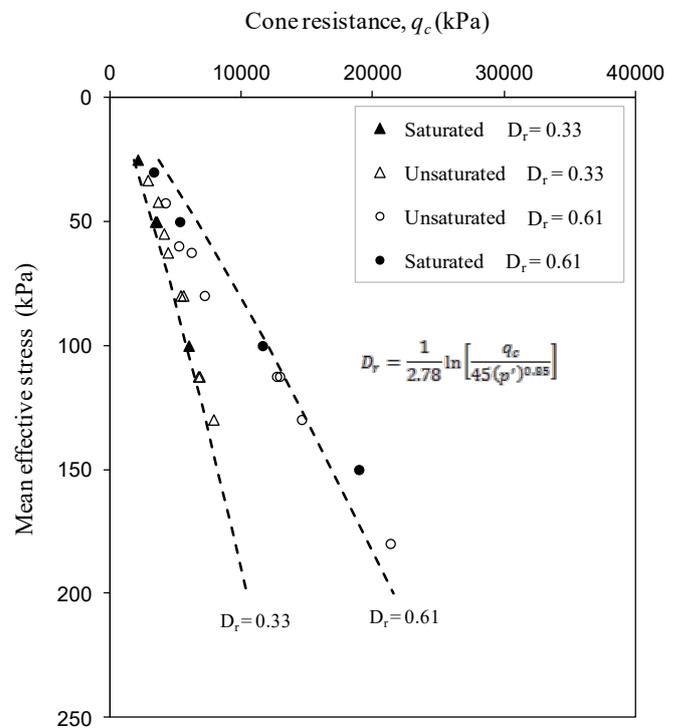


Figure 1. Comparison of CPT results for Sydney sand and estimations using the power law relationship  $q_c = 45(p')^{0.85} \exp(2.78D_r)$ .

Lyell silty samples were subjected to a range of isotropic net confining stresses. Yang & Russell (2016) presents plots of  $q_c$  versus depth for CPTs performed in unsaturated samples. Larger suctions caused larger cone penetration resistances. A suction of 37 kPa is associated with a  $q_c$  that is 50 % larger than the value for a suction of 24 kPa. A suction of 72 kPa is associated with a  $q_c$  that is 55 % larger than the value for a suction of 24 kPa.

Saturated CPTs in samples at void ratios comparable to those used in unsaturated CPTs were not conducted. However, insight in to the differences

that would exist in CPT results from saturated and unsaturated Lyell silty sand can be obtained from cavity expansion analysis - an analog to the CPT. The pressure at the wall of an expanding spherical cavity ( $\sigma_L$ ) is related to  $q_c$ . The most relevant findings of Yang & Russell (2015) who studied cavity expansions in unsaturated Lyell silty sand are summarised here. They studied the effects of three different drainage conditions (constant suction, constant moisture content and constant  $\chi_s$ ) on  $\sigma_L$ . For each drainage condition they observed that a significant change of void ratio occurs around an expanding cavity in the unsaturated soil, and analogously occurs around the tip of a penetrating cone. They found that, for a constant moisture content condition and full hydro-mechanical coupling, the changes to  $\chi$  and  $s$  mostly counteract each other and constant  $\chi_s$  may be assumed to give a reasonable approximation. The assumption of  $\chi_s$  being a constant may be used to simplify interpretation of the CPT results.

Yang & Russell (2015) explained that the unsaturated silty sand around a cavity behaves more like a saturated drained soil than a saturated undrained soil as the air in the pore space permits volumetric compression. However, as Lyell silty sand exhibits suction hardening, there is not a 1:1 correlation between saturated drained behaviour and unsaturated drained behaviour. This is in contrast to unsaturated Sydney sand, which did not exhibit suction hardening, meaning unsaturated drained and saturated drained behaviours were virtually identical. This is a significant feature to note and the main reason why CPT charts and relationships developed for saturated silty sands (or any soil with, say, more than 5% fines) have no applicability to unsaturated silty sands. The saturated soil is partially drained or undrained, whereas the unsaturated soil behaves like it is drained causing very different stress-strain behaviours.

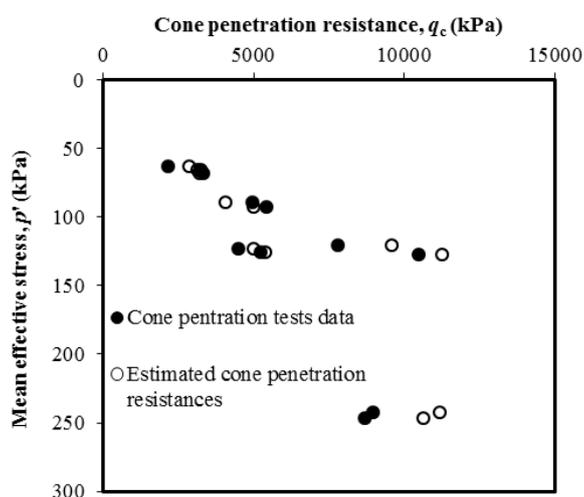


Figure 2. Comparison of CPT results for Lyell silty sand and estimations using the power law relationship  $q_c = 162(p')^{0.65} \exp(2.6D_r)$ .

The CPT results are presented in Figure 2 in the  $q_c$  versus  $p'$  plane using solid symbols. The initial  $\chi_s$  was used in the computation of  $p'$ . The data can be reasonably well fitted by the power law expression in the caption of Figure 2 in which  $q_c$  and  $p'$  have units of kPa. The predicted  $q_c$  values from this expression are shown using hollow symbols. The error associated with the expression to obtain  $q_c$  is less than 30%.

#### 4 SUCTION INFLUENCES ON SOIL CLASSIFICATION

A chart for classifying soil type and assessing whether a soil being tested is contractive or dilative has been proposed by Robertson (2010). Only that chart will be focussed on here, although many others exist in the literature. The findings made here would also be evident in other interpretation frameworks, if they were put under the same scrutiny.

In using the chart the normalised cone resistance,  $Q_m$ , is plotted against the normalised friction ratio,  $F_r$ , where each is defined as:

$$Q_m = \frac{q_c - \sigma_v}{p_a} \left( \frac{p_a}{\sigma'_v} \right)^n \quad (6)$$

$$F_r (\%) = \frac{f_s}{q_c - \sigma_v} \times 100 \quad (7)$$

The exponent  $n$  is the exponent that relates  $q_c$  to  $\sigma'_v$  or  $p'$  in power law relationships, thus  $n = 0.85$  for Sydney sand while saturated and unsaturated and  $n = 0.65$  for Lyell silty sand while unsaturated.  $f_s$  denotes the sleeve friction. The chart is plotted as Figure 3, and it can be seen that the location of the  $Q_m$ ,  $F_r$  coordinate enables estimation of soil type and whether it is contractive or dilative.

Figure 3 also shows the Sydney sand data with the effects of suction correctly incorporated in to  $\sigma'_v$ , with solid circular symbols representing saturated or dry tests and hollow circular symbols the unsaturated tests. Also shown is the Sydney sand data when the effects of suction are ignored, that is when  $\sigma'_v$  is taken to be equal to  $\sigma_v$  for the unsaturated tests, with cross symbols representing the unsaturated tests. It can be seen that the circular symbols, hollow and solid, are located in region 6 belonging to clean sands. Correct interpretation of the CPT results using the effective stress concept results in the correct soil classification. The crosses, representing data when suction is ignored, plot a little higher than the hollow circular symbols, indicating the soil is more dilative than it actually is, although not to the extent that an incorrect soil classification would result. Care should be taken and account given to  $\chi_s$  in the effective stress before charts like this are used, even for clean sands.

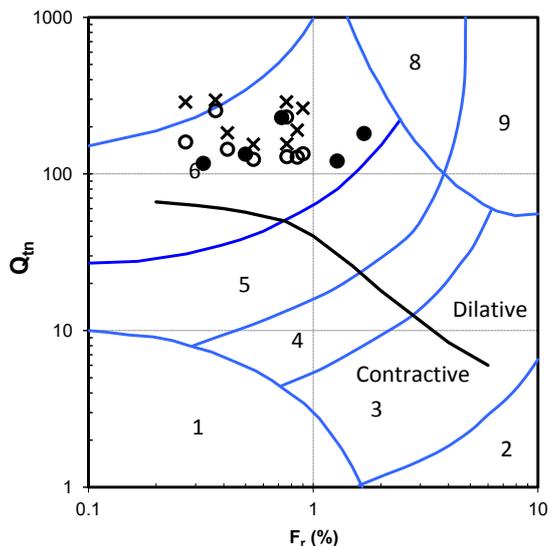


Figure 3. CPT data for Sydney sand with the effects of suction correctly incorporated in the circular symbols (solid symbols representing saturated or dry tests and hollow symbols representing the unsaturated tests). Also shown is data when the effects of suction are ignored in the unsaturated tests using the cross symbols. (Key: 1. Sensitive, fine grained, 2. Organic soils – peats, 3. Clays – clay to silty clay, 4. Silt mixtures – clayey silt to silty clay, 5. Sand mixtures – silty sand to sandy silt, 6. Sands – clean sand to silty sand, 7. Gravelly sand to sand, 8. Very stiff sand to clayey sand\*, 9. Very stiff, fine grained\*; where (\*) indicates heavily overconsolidated or cemented.)

Figure 4 shows data for a quartz marine sand, including up to 8% fines, taken from a reclaimed land site in Hong Kong (Lee et al. 1999). It was not possible to incorporate the effects of suction correctly into  $\sigma'_v$ . Calibration chamber tests on that soil enabled the exponent  $n = 0.8$  to be determined. Solid circular symbols represent saturated test data and cross symbols the unsaturated test data (where  $\sigma'_v$  is taken to be equal to  $\sigma_v$ ). It can be seen that the solid circular symbols are located across regions 4 and 5, belonging to silt and sand mixtures, and all but one data point indicate that the soil is slightly dilative. The cross symbols are located in region 6, incorrectly implying that the soil is a clean sand and very dilative. The drastic shift of the unsaturated data for this soil most probably comes from the pronounced affect suction has on the effective stress, arising from the significant fines content, and also from the sand behaving more like a partially drained/ undrained material when saturated as penetration occurs compared to it behaving more like a drained material when unsaturated as penetration occurs.

Figure 5 shows data for a gold tailings. For this material, CPT probing was undertaken in the same location on a tailings facility at regular intervals, across which the phreatic surface (inferred through dissipation tests) was seen to vary. Again it was not possible to incorporate the effects of suction correctly in to  $\sigma'_v$ . Solid circular symbols represent saturated test data and cross symbols the unsaturated test data (where  $\sigma'_v$  is taken to be equal to  $\sigma_v$ ). It can be

seen that the solid circular symbols are located across regions 3 and 4, belonging to clays and silt mixtures, and all data points indicate that the tailings is highly contractive and susceptible to liquefaction. The cross symbols locate in region 4, incorrectly implying that the tailings is a silt mixture that is slightly contractive or slightly dilative. Again suction causes a drastic shift of the unsaturated data for this tailings, and can give an unsafe and false indication of the tailings' susceptibility to contractive behaviour and liquefaction if not accounted for.

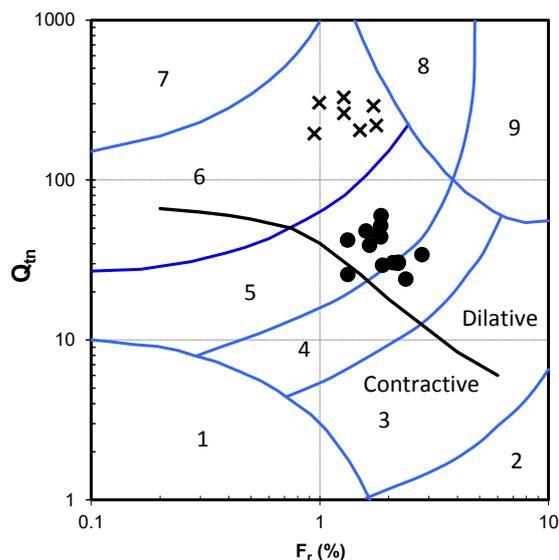


Figure 4. CPT data for marine sand with up to 8% fines. Solid circular symbols represent saturated test data and the cross symbols represent the unsaturated test data. The effects of suction were not incorporated.

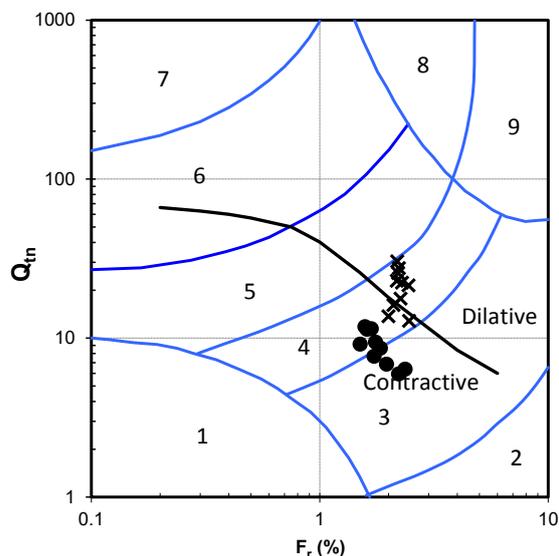


Figure 5. CPT data for a gold tailings. Solid circular symbols represent saturated test data and the cross symbols represent the unsaturated test data. The effects of suction were not incorporated.

## 5 CONCLUSION

CPTs conducted in saturated and unsaturated Sydney sand and unsaturated Lyell silty sand highlight the effects of suction on cone penetration resistance. In general the effects of suction were most pronounced for the lowest relative densities considered. Also, the effects of suction became more significant as the applied net confining stress decreased, and thus the effects of suction in field testing are most important for shallow penetrations (up to 5 m) where a soil is most likely to be unsaturated.

The unsaturated Sydney sand around a penetrating cone behaves just like the saturated Sydney sand. The same empirical power law relationship linking  $D_r$ ,  $q_c$ , and  $p'$  holds for saturated and unsaturated conditions, because unsaturated Sydney sand does not exhibit suction hardening.

Conversely, the unsaturated silty sand around a penetrating cone behaves more like a saturated drained soil than a saturated undrained soil as the air in the pore space permits volumetric compression. However, as Lyell silty sand exhibits suction hardening, there is not a 1:1 correlation between saturated drained behaviour and unsaturated drained behaviour. This is a highly significant feature to note and is the main reason why CPT charts and relationships developed for saturated silty sands (or any soil with say more than 5% fines) have no applicability to unsaturated silty sands.

This is also evident when CPT data from a range of soils for both saturated and unsaturated conditions are plotted in soil classification charts. For unsaturated soils having a significant amount of fines (say more than 5%) the suction induced increase in cone resistance, and the change of the soil behaviour from partially drained/undrained to more like drained, causes incorrect soil classification and an incorrect assessment that the soil is dilative.

## 6 ACKNOWLEDEMENTS

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