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# Generalization and Standardization of Multi-station Surface Wave Method for Site Investigation

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**ABSTRACT:** The application of surface wave method for site investigation becomes more and more popular in practical uses due to its non-intrusive tests and convenient operations. However, various methods have been developed with different approaches in the field testing and dispersion analysis, with the SASW and MASW methods being the two major groups. The data reduction method for dispersion relation in a surface wave testing is conventionally associated with certain method of data acquisition. In this paper, it is pointed out that, while the channel number of a seismograph may restrict the field testing procedure, it does not necessarily prescribe the method of dispersion analysis. Limited to the two channel data, conventional dispersion analysis of SASW suffers from possible phase un-wrapping errors, inefficient data filtering and synthesis, and inability to distinguish multiple modes. The MASW method is generalized to accommodate SASW data. Numerical simulations were performed to demonstrate its feasibility and advantages for analyzing SASW data. The MASW method is further examined to show tradeoffs involved in the testing configuration when spatial resolution, effect of lateral heterogeneity, spectral resolution, investigation depth, and near and far field effects are considered. Some efforts are made in this paper to resolve the dilemma and put forth a more definitive guideline for MASW testing and analysis.

## 1 INTRODUCTION

Field testing methods conventionally used in geotechnical site investigations are mainly penetration methods (e.g. standard penetration test, SPT, cone penetration test, CPT, and dilatometer test, DMT). Those methods are primarily large-strain methods that provide better prediction for ground strength than ground stiffness. Shear wave velocity ( $V_s$ ) from seismic tests, on the other hand, measures small-strain modulus. Soils exhibit nonlinear variation in shear modulus with shearing strain ( $G - \log \gamma$  curve) and in shear stress with shearing strain ( $\tau - \gamma$  curve). However, in addition to dynamic response, the importance of small-strain modulus from  $V_s$  measurement on static deformation analysis has also been pointed out, especially for analyses of settlement and soil structure interaction (see, for example, Shibuya et al. 1994; Jardine et al. 1998; Jamiolkowski et al. 2001; Di Benedetto, et al. 2003). Furthermore, Stokoe et al. (2004) showed that  $V_s$  measurement in the field is a critical component in evaluating sample disturbance and in predicting nonlinear  $G - \log \gamma$  and  $\tau - \gamma$  curves.

The main advantage of surface wave method for  $V_s$  measurements is essentially related to its non-destructive and non-invasive nature that allows the

characterization of hard-to-sample soils without the need for boreholes that makes the subsurface seismic methods (such as down-hole and cross-hole methods) expensive and time consuming. Surface wave testing is not affected by sample disturbance or insertion effects and is capable of sampling a representative volume of the ground even in difficult materials such as fractured rock or gravelly deposit. The method has been successively applied to various problems, such as profiling sub-ground stiffness (Foti 2003), delineating potential liquefaction area (Lin et al. 2004), evaluating thickness and condition of pavement (Ryden et al. 2004), evaluating efficiency of soil improvement (Lin et al. 2012), and characterizing waste disposal site (Haegeman and Van Impe 1999).

Three steps are involved in a surface wave test: (1) field testing for recording surface waves, (2) determination of the experimental dispersion curve from the field data, and (3) inversion of shear wave velocity profile from the experimental dispersion curve. The data reduction method for dispersion relation in a surface wave testing is conventionally associated with a certain method of data acquisition. At present, the two-station spectral analysis of surface wave and multi-station analysis of surface wave are the most popular methods used worldwide. The

two-station SASW method is based on the phase difference between two receivers as a function of frequency (Nazarian and Stokoe 1984), while Multi-station MASW method are based on the relation between phase angles and source-to-receiver offset (Lin and Chang 2004), or 2D wavefield transformation of surface wave (see, for example, McMechan and Yedlin 1981; Gabriels et al. 1987; Park et al. 1998; Xia et al. 2007). It can be confusing to geotechnical engineers as to their differences and which method performs better.

This study attempted to clarify their differences and suggest preference. It is pointed out that, while the channel number of a seismograph may restrict the field testing procedure, it does not necessarily prescribe the method of dispersion analysis. Limited to the two channel data, conventional dispersion analysis of SASW suffers from possible phase unwrapping errors, inefficient data filtering and synthesis, and inability to distinguish multiple modes. The MASW method is generalized to accommodate SASW data. Numerical simulations were performed to demonstrate its feasibility and advantages for analyzing SASW data. The MASW method is further examined to show tradeoffs involved in the testing configuration when spatial resolution, effect of lateral heterogeneity, spectral resolution, investigation depth, and near and far field effects are considered. Some efforts are made in this paper to resolve the dilemma and put forth a more definitive guideline for MASW testing and analysis.

## 2 GENERALIZATION OF DISPERSION ANALYSIS

In a conventional two-station SASW test, the two recording stations are in line with the source and typically have the same spacing as the near offset (distance between the source and nearest receiver). By Fourier transform, the phase shift between the two signals can be determined for each frequency. The phase velocity  $v_a(f)$ , or more precisely the apparent phase velocity since the wave may propagate in more than one mode, can then be calculated as

$$v_a(f) = \frac{2\pi f}{\frac{\Delta\phi(f)}{\Delta x}} \quad (1)$$

where  $f$  is the frequency in Hz,  $\Delta x$  is the receiver spacing, and  $\Delta\phi$  is the phase shift between the two receivers after unwrapping. To overcome the near field and far field effects, the tests are typically repeated with various receiver spacings, each of which analyzed for its associated appropriate frequency range. The testing sequence can be arranged in so-called common source array or common midpoint array.

The introduction of SASW method using only two-channel recording greatly contributed to the

wide-spread use of surface wave testing in geotechnical engineering in its time. However, both its strength and weakness are related to the minimum number of stations used for phase velocity determination. Using only a pair of receivers, different modes of propagation cannot be differentiated, and the data reduction for correcting possible unwrapping errors is tedious. As multi-channel recording system became readily available, methods based on multi-station receivers sprouted. Multi-station methods sample the wavefield at multiple locations. The sampling periods in the time and space domain are  $\Delta t$  and  $\Delta x$ ; and the numbers of samples in the time and space domain are  $M$  and  $N$ , respectively. The analysis of the multi-station signals may begin with Fourier transform as

$$U(f, x_n) = \sum_{m=0}^{M-1} u(t_m, x_n) \exp(-j2\pi f t_m) \quad (2)$$

where  $u$  is the ground motion (typically velocity) recorded by the receivers with space interval  $\Delta x$  and time interval  $\Delta t$ ,  $U$  is the DFT of  $u$ ,  $j = \sqrt{-1}$ ,  $t_m = m\Delta t$ , and  $x_n = n\Delta x$ . The subscripts  $n$  and  $m$  in Eq. (2) are integer indices to represent respectively discrete points in the space and time domain. The most straight-forward algorithm of dispersion algorithm is the 2-D Fourier transform, which is often referred to as the  $f$ - $k$  transform (Gabriels et al. 1987). For each frequency component of interest in Eq. (2), the wavefield  $U$  is a harmonic function of space. By taking another Fourier transform with respect to the space (i.e. spectral analysis in the space domain),

$$Y(f, k) = \sum_{n=0}^{N-1} U(f, x_n) \exp(-j2\pi k x_n) \quad (3)$$

where 2-D spectrum  $Y(f, k)$  represents the wavefield in the frequency-wavenumber domain, the wavenumbers (spatial frequency, inverse of wavelength) of propagating modes for each frequency can be identified at amplitude peaks of the spectrum  $Y(k)$ . The phase velocity is then determined by the definition  $v=2\pi f/k$ . Alternatively, the  $f$ - $v$  spectrum can be derived from Eq. (3) by simply changing the variable  $k = 2\pi f/v$  as

$$\hat{Y}(f, v) = \sum_{n=0}^{N-1} U(f, x_n) \exp\left(-j \frac{2\pi f}{v} x_n\right) \quad (4)$$

where 2-D spectrum  $\hat{Y}(f, v)$  represents the wavefield in the frequency-velocity domain, which will be used throughout this paper. The peaks of the amplitude of the frequency-velocity domain spectrum constitute the experimental dispersion curve. Spectra in other domains can be derived from (3) by simply changing the variable  $k=2\pi f/p$  for the frequency-slowness domain and  $k= 2\pi/\lambda$  for the frequency-

wavelength domain. Other 2-D transform algorithms are also available, such as,  $p$ - $f$  transform (McMechan and Yedlin 1981), the phase shift (Park et al. 1998), and the frequency decomposition and slant stacking (Xia et al. 2007). But they are essentially or physically equivalent to Eq. (3) and Eq. (4).

The dispersion analysis in a surface wave testing is conventionally associated with a certain method of data acquisition, for example, the phase angle analysis in the two-station SASW method and the 2-D multi-station wavefield transformation in the MASW method. While the channel number of a seismograph may restrict the field testing procedure, it does not necessarily prescribe the method of dispersion analysis. In fact, the 2-D wavefield transformation can be extended to accommodate the two-station SASW data. Two-D wavefield transformation formulated in the form of Eq. (3) or Eq. (4) can be used to process each pair of SASW data. To demonstrate this approach, numerical simulations were performed using two distinct earth models. One is a normal profile with a dominant fundamental mode and the other is a reversal profile with multiple dominant modes. The parameters of the earth models used are listed in Table 1. Both SASW and MASW testing are numerically simulated. The common-midpoint receiver spacings for the SASW testing include 2 m, 4 m, 8 m, 16 m, 32 m, and 64 m. The receiver spacing for the ordinary MASW testing is 2 m. Synthetic waveforms for the surface wave testing were simulated by the modal summation method programmed by Herrmann and Ammon (2002).

The results of 2-station MASW analysis for the normal case is shown in Fig. 1. The spectral resolution of the dispersion image increase with increasing receiver spacing. However, as receiver spacing gets longer, aliasing patterns are observed. Fortunately, the dispersion curve typically has a shape different and can be differentiated from the aliasings. Dispersion curve can be picked starting from the shortest receiver spacing at high frequencies.

Table 1. Parameters of earth models used in numerical simulations.

Model	Layer	$V_s$ (m/s)	$V_p$ (m/s)	Layer Thickness(m)	Density(g/cm <sup>3</sup> )
1	1	300	600	10	1.8
	2	400	800	$\infty$	1.8
2	1	300	600	4	1.8
	2	250	500	8	1.8
	3	400	800	$\infty$	1.8

Figure 2 shows the picked dispersion curve, in which the usual filtering criterion ( $\lambda/3 < \text{geophone spacing} < 2\lambda$ ) was applied to mitigate effects of near field and far field. The dispersion curve obtained from 2-station wavefield transformation is basically identical to that from the conventional SASW analy-

sis using Eq. (1). However, the wavefield transformation avoids the ticklish and error-prone phase-unwrapping procedure. It is also more tolerant to measurement errors (e.g. noises and waveform clipping) than the two-station phase analysis using Eq. (1). Similar results are obtained for the case of reversal profile with multiple dominant modes, as shown in Fig. 3 and 4.

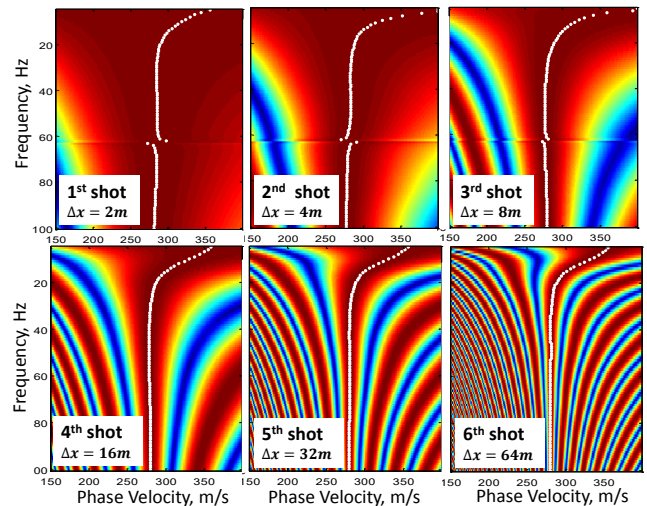


Figure 1. Results of 2-station wavefield transformation on each pair of SASW data for Model 1.

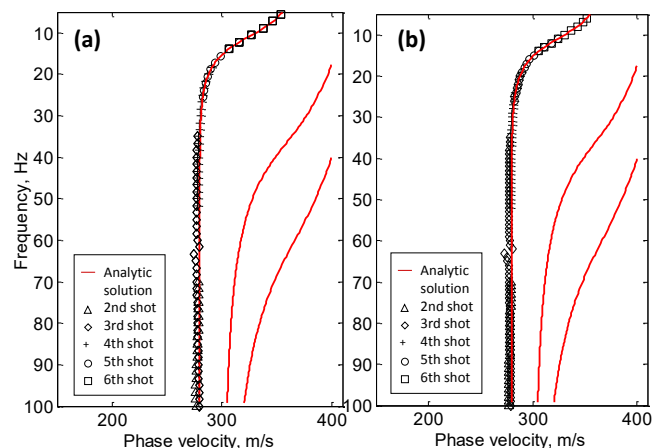


Figure 2. The dispersion curve obtained by (a) SASW method and (b) 2-station wavefield transformation for Model 1.

Limited to the two-station data and aliasing, the wavefield transformation is not able to separate different dominate modes. Instead of analyzing each pair of 2-station data, all SASW data can be combined by the source-to-receiver distance gather. Each pair of signals from different shots in a SASW testing is identified by the source-to-receiver distance. The SASW data is then rearranged in ascending order of source-to-receiver distance. Between two adjacent shots, signals having the same source-to-receiver distance can be simply stacked and averaged. The combined multi-offset data is a spatially non-uniform sampling of the wavefield. Equation (3) and (4) stem from of discrete-space Fourier trans-

form. But taking the form, they can be relaxed to accommodate data in that  $x_n$  is non-uniformly spaced.

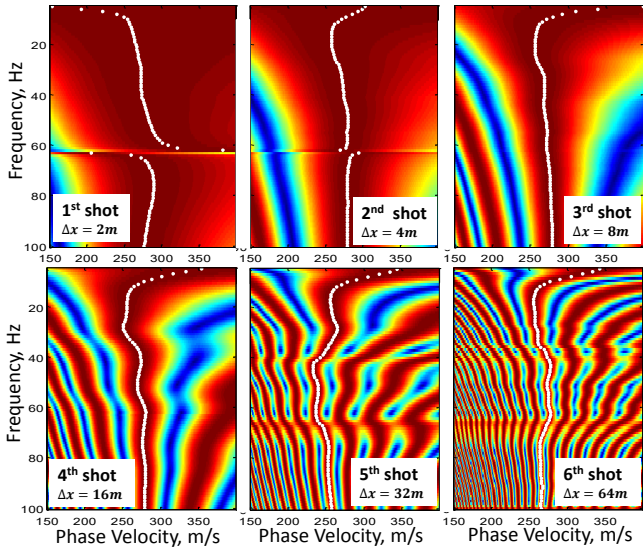


Figure 1. Results of 2-station wavefield transformation on each pair of SASW data for Model 2.

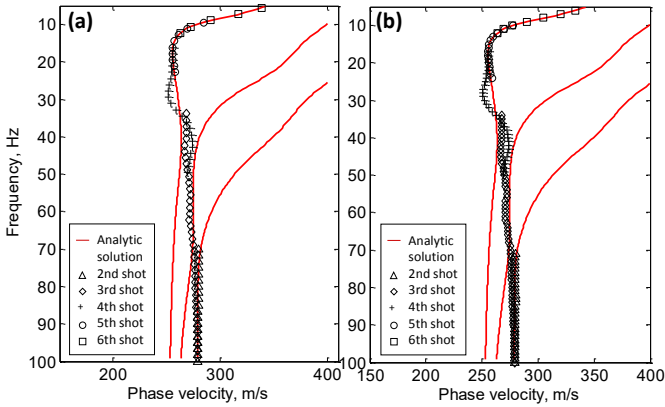


Figure 2. The dispersion curve obtained by (a) SASW method and (b) 2-station wavefield transformation for Model 2.

The result of non-uniform MASW analysis using SASW data for the normal case is shown in Fig. 5(a). For comparison, the dispersion analysis using uniformly spaced MASW data of the same offset range is shown in Fig. 5(b). The non-uniform wavefield transformation from the SASW testing shows some aliasing patterns in the 2-D spectrum due to sparse sampling in the space domain. Yet the correct dispersion curve can be clearly distinguished from the aliasing pattern in the  $f$ - $v$  domain. The results for the case of reversal profile are shown in Fig. 6. Although there are some aliasing effects due to non-uniform spatial sampling, particularly in the left part of the spectral image (Fig. 6a), dominated modes can still be recognized successfully. On the contrary, the apparent experimental dispersion curves extracted from the conventional SASW analysis (Fig. 4) gradually sway from one mode to another. It is difficult to tell whether higher modes dominate at certain frequency range. The SASW method samples phase

angles at only two locations, and yields location-dependent apparent velocities from the linearized slope of the phase angle versus distance. This inconsistency in the experimental dispersion data between difference receiver spacings (locations) is often misinterpreted as a result of laterally variable profile. The MASW method, on the other hand, provides visualization of dispersion relation through 2-D wavefield transformation, which yields consistent experimental dispersion curve and allows identification or separation of multiple modes.

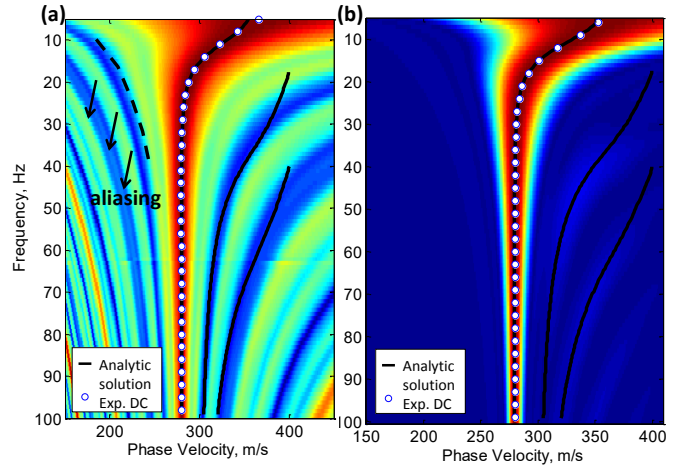


Figure 5. (a) Non-uniform MASW analysis using 6-shot SASW data vs. (b) Uniform MASW analysis using equally-spaced multi-station data of the same offset range for Model 1.

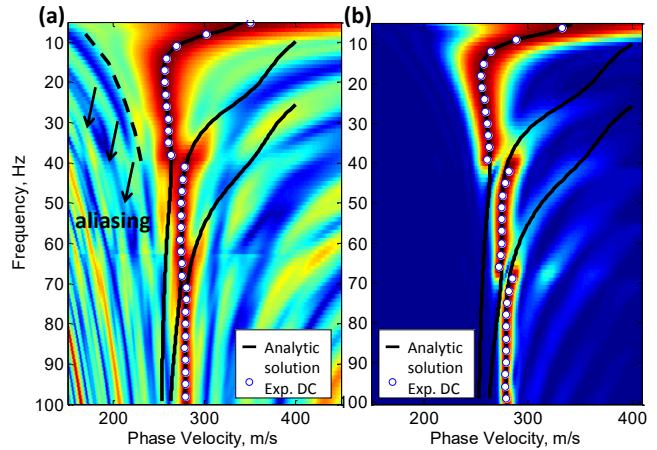


Figure 6. (a) Non-uniform MASW analysis using 6-shot SASW data vs. (b) Uniform MASW analysis using equally-spaced multi-station data of the same offset range for Model 2.

### 3 DILEMMAS IN FIELD TEST CONFIGURATION

The 2-D wavefield transformation can be generalized to accommodate SASW data. It has been demonstrated that the multi-station wavefield transformation is superior in dispersion analysis. It avoids unwrapping of phase angles, a major source of human error in conventional SASW analysis, enable

identification and separation of multiple modes, and provides visual appraisal of complicated dispersion phenomenon. With modern seismograph, the multi-station data is normally collected by a number of receivers with constant receiver spacing. In a MASW survey, three spatial parameters need to be decided, including near offset ( $x_0$ ), receiver spacing ( $\Delta x$ ), and receiver spread length ( $L=N\Delta x$ ). To avoid spectral aliasing in the space domain, receiver spacing should be sufficiently short. This will limit the spread length with finite channels of the seismograph. On the other hand, spread length should be as long as possible to have good depth coverage and identify different dominant modes. However, too long of a receiver spread is more susceptible to lateral variation and has poor lateral resolution. As to the near offset, short  $x_0$  is desired to avoid far field effect while long  $x_0$  is preferred to avoid near offset. These tradeoffs in selecting proper  $x_0$ ,  $\Delta x$ , and  $L$  make the field testing ambiguous and subjective.

#### 4 STANDARDIZED MASW SURVEY

Tradeoffs are involved when selecting the testing configuration. To resolve the aforementioned dilemmas, a multi-shot common receiver configuration survey is proposed as a standard approach to acquire multi-station data for 2-D wavefield transformation. The multi-shot common receiver configuration is illustrated in Fig. 7, in which the geophone spread is fixed and data of different source-to-receiver distance are collected by gradually increasing the near offset in multiples of receiver spacing. More equally-spaced data can be recorded by these walk-away shots, which allows assembling MASW-imitating data by the source-to-receiver distance gather. The offset range is initially from  $x_0$  to  $(x_0+L)$  when using the conventional multi-station configuration. After  $K$  consecutive walk-away shots, all collected field data can be synthesized into one seismic record according to the source-to-receiver offset. The synthesized seismogram has an extended offset range from  $x_0$  to  $(x_0+KL)$ .

The receiver spacing  $\Delta x$  ideally is set to be half of the minimum depth of exploration. This can be relaxed since the aliasing can often be differentiated from the surface wave dispersion. The nearest offset  $x_0$  is about the same distance as the minimum depth of exploration. The receiver spread  $L$  is set to be in the target range of interest. The number of walk-away shots times the spread length  $L$  should be greater than if not doubled the desired depth of exploration. This way, requirements of the lateral resolution and depth of investigation can be met simultaneously. Even with a small number of receiving channels, the receiver interval can be kept small enough without sacrificing the total offset range. The near field and far field effects can also be mitigated

since it establishes an expansive offset range in the same spatial range by synthesizing seismic records with different nearest source-to-receiver offsets.

The dispersion analysis of the SASW test can be replaced by the wavefield transformation technique. However, there is a nice feature about the SASW field testing. The near offset is proportional to the receiver spacing and different types of source are often adopted from small hammer in short spacing to large weight drop in long spacing, while single shot is normally used to conveniently collect the MASW data. The useful frequency range of dispersion curve is affected not only by receiver spread length, but also by the type of source. For example, when a large hammer is used, it is difficult to obtain high frequency component regardless of how small the offset is. Likewise, when a small hammer is used to generate high-frequency component, it is difficult to obtain low frequency component regardless of how long the receiver spread is. Using the multi-shot common receiver MASW survey, different types of sources can be used for different walk-away shots. As a general rule of thumb, handheld hammer, sledge hammer, and large weight drop can be used for short, intermediate, and long near offset, respectively. To maximize the bandwidth of dispersion curve, multiple sources can be used for each walk-away shot. For each frequency component, the optimal combination of source and offset can be selected to mitigate the near and far field effects as well as maximize the obtainable bandwidth for the dispersion curve.

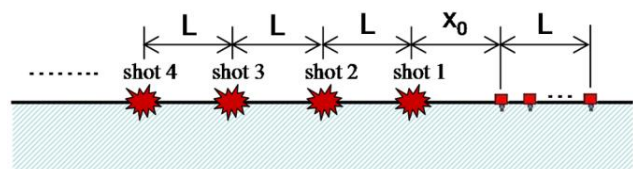


Figure 7. The multi-shot common receiver configuration.

A field example was conducted to demonstrate the advantages of the multi-shot common receiver survey. The test was carried out on a grass field in the campus of National Chiao Tung University, Taiwan. An area about 50 m long was to be tested with a desired investigation depth up to 30 m using a 24-channel seismograph. The receiver spread consisted of 24 geophones with 2 m spacing used. A small handheld hammer and large 12-lb sledge hammer were used as the sources at near offset 4 m and 27 m, respectively. The results of dispersion analysis using each single shot and the multi-shot source-to-receiver distance gather are shown in Fig. 8. The small handheld hammer at short near offset yielded good high frequency components from 30 Hz to 120 Hz, while the big sledge hammer at long near offset produced low frequency energy from 15 Hz to 55 Hz. The bandwidth of experimental dispersion curve is rather limited by either case. A higher

resolution dispersion image and larger bandwidth of experimental dispersion curve were obtained by multi-shot source-to-receiver distance gather, as shown in Fig. 8c. The improvement from the multi-shot common receiver configuration is quite pronounced.

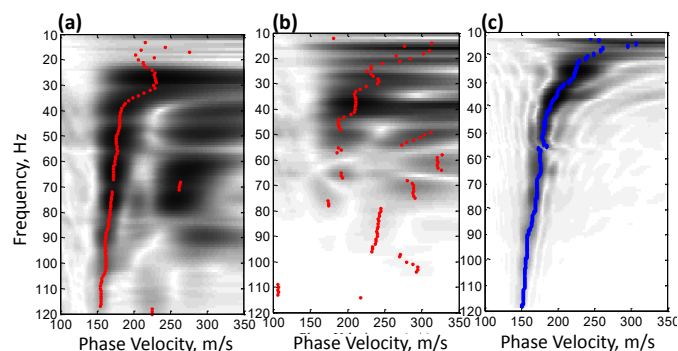


Figure 8. The dispersion analysis for (a) single shot at  $x_0 = 4\text{m}$  with a small handheld hammer, (b) single shot at  $x_0 = 27\text{m}$  with a small handheld hammer, and (c) two shots combined by source-to-receiver distance gather.

## 5 CONCLUSIONS

The data reduction method for dispersion relation in a surface wave testing is conventionally associated with certain method of data acquisition. Typical examples are phase angle analysis in the two-station SASW method and 2-D multi-station wavefield transformation of surface wave in the MASW method. Limited to the two channel data, conventional SASW dispersion analysis suffers from possible phase un-wrapping errors, inefficient data filtering and synthesis, and inability to distinguish multiple modes. The MASW method is generalized to accommodate SASW data. Numerical simulations were performed to demonstrate its feasibility and advantages for analyzing SASW data. It avoids un-wrapping of phase angles, a major source of human error in conventional SASW analysis, enable identification and separation of multiple modes, and provides visual appraisal of complicated dispersion phenomenon.

Tradeoffs are involved in the MASW testing configuration when spatial resolution, effect of lateral heterogeneity, spectral resolution, investigation depth, and near and far field effects are considered. To resolve the dilemmas in the field configuration, a multi-shot common receiver configuration survey is proposed to acquire multi-station wavefield for 2-D wavefield transformation analysis. A field example demonstrated that this new approach provides a more definitive guideline for MASW testing and maximize the obtainable bandwidth for the dispersion curve.

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