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Effect of inter-particle strength on K_0 correlation for granular materials

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ABSTRACT: The coefficient of lateral earth pressure at rest K_0 is an important state soil variable that is necessary to characterize the in-situ stress state of natural soil deposits. In this study, the values of K_0 measured for various granular materials were analyzed and the effect of inter-particle strength on the K_0-ϕ' correlation was presented. The inter-particle strength was referred to as the surficial frictional resistance mobilized on the contact areas of adjacent soil particles. The measured K_0 values indicated that denser and more angular materials tend to produce lower K_0 values, likely due to the interlocking effect as postulated by the Jaky's K_0 equation. It was also seen that neither the critical-state friction angle nor the peak friction angle for the Jaky's K_0 equation produced close match to the measured K_0 values. The use of peak friction angle in fact underestimated the values of K_0. A new K_0 correlation model based on the inter-particle strength analysis was presented. The effect of interlocking on the lateral effective stress was taken into account for the inter-particle strength K_0 correlation model. It was shown that the inter-particle strength model well described the dependency of K_0, a state variable, on the internal friction angle of soil, a strength parameter.

1 INTRODUCTION

The well-known K_0 equation by Jaky (1944, 1948) is commonly adopted in practice to estimate the values of K_0 that is given as a sole function of the internal friction angle (ϕ') of soils. It was established and analyzed based on the stress distribution and equilibrium condition assumed within the wedge-shaped sand pile (Mesri and Hayat 1993, Michalowski 2005). While the Jaky's K_0 equation has been widely used for various purposes in geotechnical engineering, the close correlation of K_0 to ϕ' is still interesting in that a pre-failure state parameter is well related to the failure-state parameter of the friction angle. It is indicated that the appropriate quantification of the friction angle is crucial for the K_0–strength correlation to be valid.

The peak friction angle (ϕ'_p) is composed of the critical-state friction angle ϕ'_c and the dilatancy angle ψ_p (Bolton 1986). While ψ_p is a state variable, ϕ'_c is an intrinsic variable that would give a unique K_0 value. According to the original Jaky's K_0 equation, the friction angle for the correlation should be ϕ'_c. However, it has been observed and reported that K_0 is not unique but varies with the relative density (Ishihara 1993, Wanatowski and Chu 2007, Lee et al. 2013). It was also reported that the calculated K_0 values using ϕ'_p are not in good agreement with measured values. All these implies that the K_0 correlation to ϕ' is still subject to some uncertainties in particular in regard to the value of friction angle that is to be adopted into the correlation.

In this study, the values of K_0 for granular materials are analyzed by focusing on the correlation of K_0 to the strength parameter of soils. Based on the work given in Lee et al. (2013, 2014), a new correlation of K_0 based on the inter-particle strength is explored and presented. The mobilized stress state between soil particles and inherent particle characteristics are considered in the K_0 correlation.

2 COEFFICIENT OF LATERAL EARTH PRESSURE AT REST

The coefficient of lateral earth pressure at rest (K_0) is an important state variable that is given by the ratio of lateral to vertical effective principal stresses. It specifies the geostatic stress state of soils and the following correlation proposed by Jaky is commonly used in practice (Jaky 1944, 1948):

$$K_0 = 1 - \sin\phi' \quad (1)$$

where ϕ' = internal friction angle of soils. While Eq. (1) has been re-visited and confirmed by several authors (Mesri and Hayat 1993, Michalowski 2005, Pipatpongsa et al. 2009), it is still interesting that the pre-failure state soil variable K_0 is well correlated to the strength parameter ϕ' that indicates failure.

In Eq. (1), the friction angle ϕ' corresponds to the repose angle of the sand wedge formed by pouring sand particles and thus is close to the critical-state friction angle ϕ'_c as discussed by Mesri and Hayat (1993). This then implies that K_0 is always unique for a given sand, which is however not the case of what is usually observed on actual sands. It is often observed that K_0 varies with the density condition showing increases with decreasing relative density (D_R).

3 EXPERIMENTAL TESTS

3.1 *Oedometer Tests*

To analyze the values of K_0 for various materials and stress conditions, thin-walled oedometer tests were conducted using three granular materials: Jumumjin sand, smooth glass beads and rough-surfaced etched glass beads (Lee et al. 2013). Triaxial (TX) tests were conducted on the test materials. The values of critical-state friction angle ϕ'_c were 37.8°, 27.1°, and 30.8° for Jumunjin sand, glass beads and etched glass beads, respectively. The peak friction angles ϕ'_p were also evaluated for different relative densities and confining stress levels.

The thin-walled oedoemeter test system was set-up and used to measure K_0 assuming that the K_0 condition is satisfied for the radial strain limit smaller than around 5×10^{-5} (Kolymbas and Bauer 1993). The thin-walled oedometer was made of aluminum with 0.13-mm thickness and 66-mm diameter. Strain gauges were installed on the surface of thin-walled cylinder. The height of samples prepared for thin-walled oedometer tests was 40 mm.

Test materials were placed in the thin-walled mold and compacted to achieve the target relative density. The test specimen was then subjected to loading-unloading-reloading process. Different relative densities were considered for the test specimens, D_R = 33% and 84% for Jumunjin sand and D_R = 56% and 80% and D_R = 55% and 81% for glass beads and etched glass beads, respectively.

3.2 *Measured K_0 Values*

Fig. 1 shows the values of K_0 with applied vertical effective stress (σ'_v) measured for different test materials at different relative densities. Note that the differences of maximum and minimum void ratios (e_{max} and e_{min}) was around 0.3 for Jumunjin sand while those for glass beads and etched glass beads were 0.1 and 0.2, respectively. The K_0 values of the dense sand were lower than of loose sand throughout the entire stress range for the tests. The results from dense sand showed the lower limit range of K_0 measured from all the test materials. The values of K_0 were higher duri-

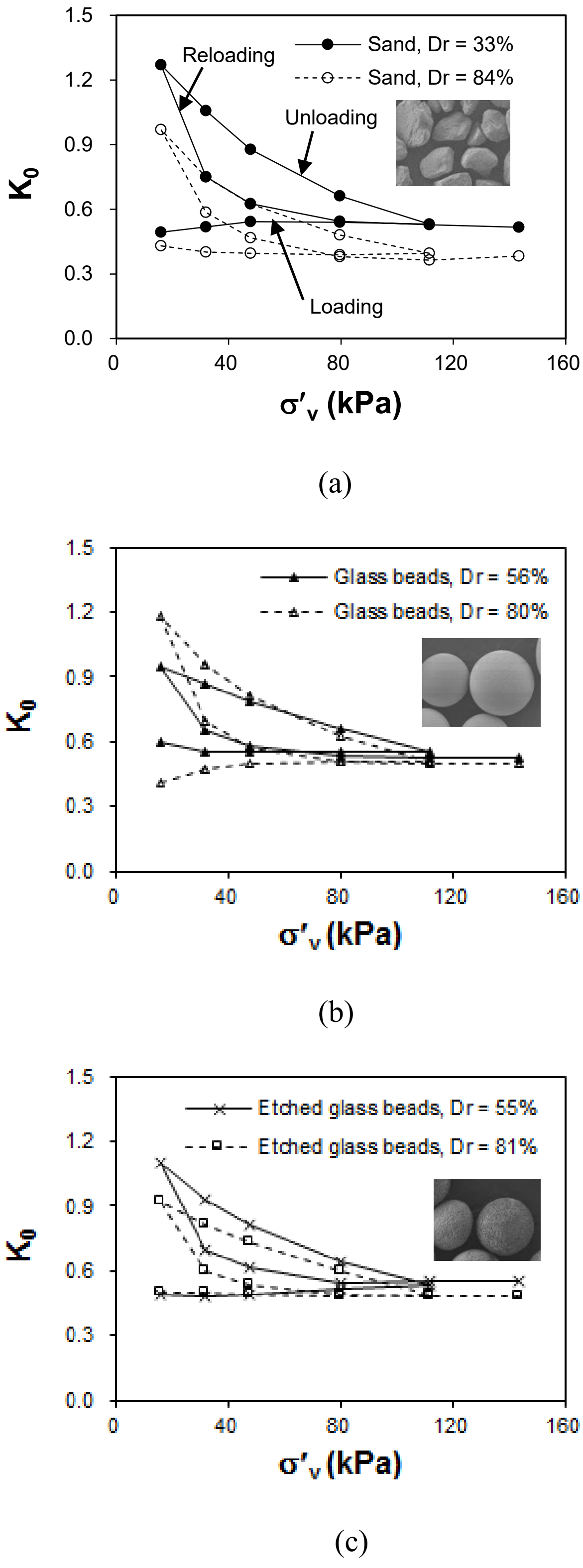

Figure 1. Values of K_0 for (a) Jumunjin sand; (b) glass beads and (c) etched glass beads.

ing unloading while those for reloading were between loading and unloading stages. Note that, when unloaded, the horizontal stress tends to be locked and less released while unloading process makes the vertical stress entirely released.

The values of K_0 for dense sand were lower than for glass beads indicating the effect of angularity and interlocking. The difference of glass beads and etched glass beads, on the other hand, indicated the degree of surface roughness. As compared from Figs. 1(b) and (c), the values of K_0 between untreated and etched glass beads were not significantly different.

Fig. 2 shows the measured versus calculated values of K_0 using the peak (ϕ'_p), critical-state (ϕ'_c) and inter-particle (ϕ'_s) friction angles obtained for the test materials. The values of ϕ'_s were estimated as equal to 26°, 17° and 21° for Jumunjin sand (JS), glass beads (GB) and etched glass beads (EGB), respectively, from the previously reported results in Procter and Barton (1974) and Andrawes and El-Sohby (1973). The values of K_0 obtained from ϕ'_c and ϕ'_s were constant as these friction angles were unique and constant. The calculated K_0 values with ϕ'_p showed the lowest range, while those using ϕ'_s were in the upper bound range. From Fig. 2, it is seen that the correlation of K_0 to ϕ'_c is approximately valid for uniformly round materials, whereas the particle interlocking affects the correlation for irregularly-shaped natural sands. It was also indicated that the correlation to ϕ'_p tends to underestimate K_0 values.

4 INTERPARTICLE STRENGTH MODEL

4.1 *Stress State*

Fig. 3 shows the mobilized stress state for the K_0 condition compared with those of active (σ'_a) and passive (σ'_p) stress states. For the mobilized friction angle ϕ'_{mob} in Fig. 3, K_0 can be given as the following relationship:

$$K_0 = \frac{\sigma'_{h0}}{\sigma'_{v0}} = \frac{1-\sin\phi'_{mob}}{1+\sin\phi'_{mob}} \qquad (2)$$

Eqs. (1) and (2) both represent the correlation between K_0 and strength characteristics. Eq. (2) may be however more straightforward as ϕ'_{mob} indicates the mobilized strength, which is difficult to quantify (Mesri and Hayat 1993).

Although the in-situ K_0 vertical and horizontal stresses of σ'_{v0} and σ'_{h0} given by ϕ'_{mob} exist within the zone below failure envelop, the inter-particle stresses acting on particle contacts would be at the limit stress states that is just about to initiate particle slips. The limit stresses are then related to the frictional resistance of particle surfaces, which can be characterized using the inter-particle friction angle ϕ'_s.

Fig. 4 shows the K_0 stress state [Fig. 4(a)] and the inter-particle stress states at particle contact [Fig. 4(b)]. The inter-particle stresses σ'_{np} and τ_p in Fig. 4(b) represent the limit stress state given by the inter-particle friction angle ϕ'_s. The inter-particle stresses will continuously develop until particle slip occurs until the global geostatic condition is reached. For the

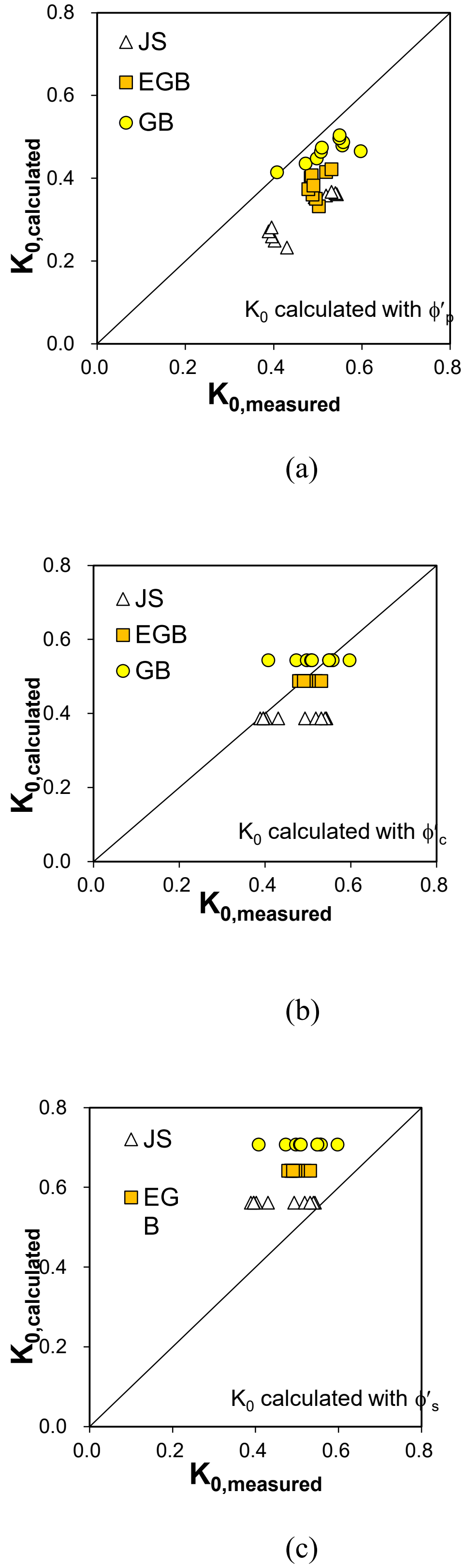

Figure 2. Measured versus estimated K_0 values of all materials for (a) ϕ'_p; (b) ϕ'_c and (c) ϕ'_s.

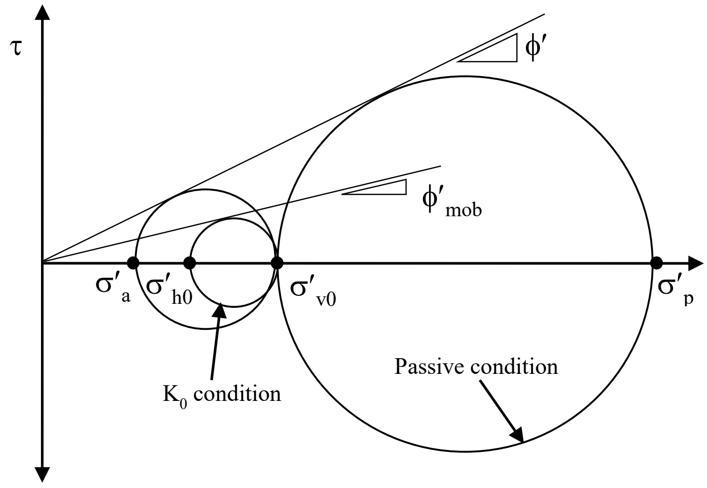

Figure 3. Stress states for K_0 condition.

inter-particle stresses acting on particle surfaces shown in Fig. 4(b), the local stress ratio K_{0p} between the local principal stresses of σ'_{vp} and σ'_{hp} can be given as follows:

$$K_{0p} = \frac{\sigma'_{hp}}{\sigma'_{vp}} = \frac{1-\sin\phi'_s}{1+\sin\phi'_s} \tag{3}$$

where K_{0p} = inter-particle stress ratio; σ'_{vp} and σ'_{hp} = vertical and horizontal principal stresses at particle contacts, respectively; and ϕ'_s = inter-particle friction angle. Due the stress concentration at contact areas, σ'_{hp} and σ'_{vp} are larger than σ'_{h0} and σ'_{v0}. However, the ratios of σ'_{hp} to σ'_{vp} and σ'_{h0} to σ'_{v0} would be similar as equilibrium conditions hold for both local and global stress states, implying that K_{0p} would also be similar to K_0.

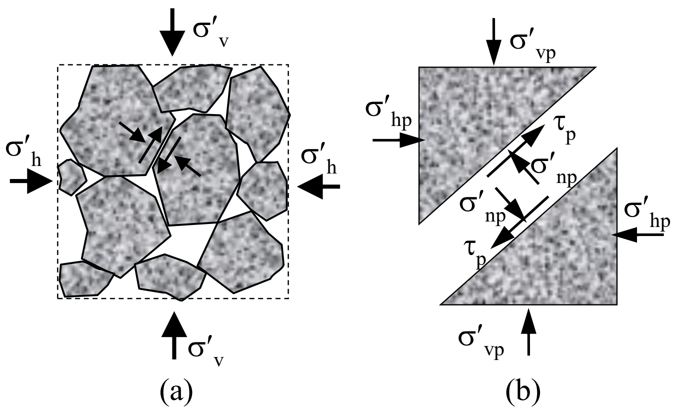

Figure 4. Stress states for (a) K_0 condition and (b) inter-particle state.

The particle interlocking is another factor that should be considered for K_{0p} of Eq. (3) and K_0 as ϕ'_s only represents the frictional resistance of particle surfaces. The particle interlocking affects the strength mobilization, stress propagation and thus the stress ratio. The mobilized friction angle ϕ'_{mob}, which defines the global stress state, in fact includes the effect of particle interlocking. This can be evaluated by combing the effect of particle interlocking and the inherent frictional resistance of the inter-particle friction angle ϕ'_s or critical-state friction angle ϕ'_c.

While both ϕ'_s and ϕ'_c can be adopted, ϕ'_c would be better option as it is more commonly used for soil characterization in practice. The K_0 correlation of Eq. (3) can then be modified in terms of ϕ'_c introducing a certain correlation parameter to consider the effect of particle interlocking given as follows:

$$K_0 = \frac{1-\sin(\beta\cdot\phi'_c)}{1+\sin(\beta\cdot\phi'_c)} \tag{4}$$

where β = correlation parameter that represents the effect of particle interlocking on K_0. As the strength characteristics of soil was logically included in Eq. (4), it justifies and explains well the sequence and validity of the K_0 correlation to strength.

In order to evaluate the values of β, back-analysis was performed using the measured K_0 values and Eq. (4). The back-calculated values of β are shown in Fig. 4 as a function of relative density (D_R). It is seen that β increases with increasing D_R showing a reasonably tight correlation. These results are reasonable as β indicates the effect of particle interlocking on the values of K_0. The correlation of β to D_R obtained from the results given in Fig. 14(b) is given as follows:

$$\beta = a\cdot\left[D_R(\%)\right]^b \tag{5}$$

where a and b = correlation parameters. The values of *a* and *b* were 0.1 and 0.44, respectively, for the test results adopted in this study.

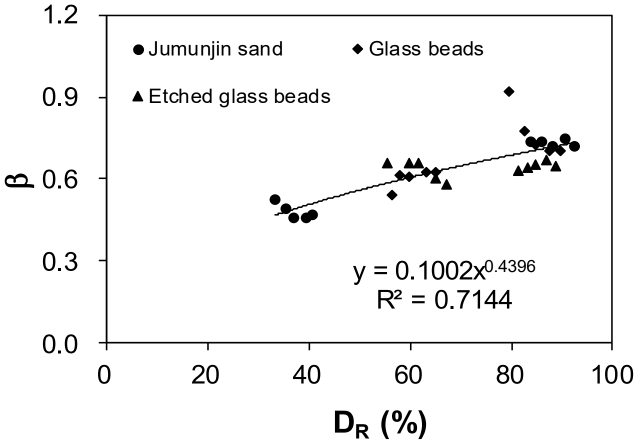

Figure 5. Values of β for test materials.

5 SUMMARY AND CONCLUSIONS

In this study, the correlation of K_0 to strength was explored focusing on the effect of inter-particle strength. Test results from thin-walled oedometer tests were adopted to analyze the values of K_0. Different granular materials of clean sand, glass beads and etched glass beads were used in the tests.

The values of K_0 were lower for higher relative densities for all test materials. Jumunjin sand showed higher K_0 values than glass beads, which was attributed to higher angularity and interlocking effects. The measured K_0 values indicated that denser and more angular materials tend to produce lower K_0 values. It was found that the Jaky's K_0 equation using the critical-state friction angle ϕ'_c produces satisfactory results for granular soil with uniform particles while did not match well for irregularly angular particles. The application of ϕ'_p underestimated K_0 values.

The inter-particle strength model was presented to describe the correlation of K_0 to strength characteristics of sand. It was explained that the mobilized strength at particle contacts plays key role for the mobilized K_0 stress state. The inherent particle strength was introduced to establish a modified K_0 correlation with consideration of interlocking effect. The correlation parameter was evaluated, which was given as a function of relative density.

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