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Calibration of Vs Prediction Model based on SPT-N using Conditional Probability Theory

T. Kishida
Pacific Earthquake Engineering Research Center, University of California, Berkeley, USA

C-C. Tsai
National Chung Hsing University, Taiwan

ABSTRACT: Prediction models of shear wave velocity (V_s) based on the standard penetration test (SPT) blow counts (N) are widely used in design practice. However, application of these models is limited because these models are typically ranged between regions. Moreover, it is difficult to calibrate the regression parameters for a site specific condition if multicollinearity exists in the model. This paper proposes a calibration procedure for developing a site specific V_s prediction model. The framework is based on conditional probability theory by developing correlations of model parameters from a global database. An application example is presented to develop the site specific V_s prediction model based on the available local N measurements. The framework of the conditional probability theory provides the rational approach to calibrate the site specific V_s prediction model.

1 INTRODUCTION

Standard penetration test (*SPT*) blow count (N) is available at many sites throughout the world. On the other hand, shear wave velocities (V_s) are less available compared to N. Since V_s profiles are required for dynamic analysis and site characterization, the correlation of V_s and N is widely used for many design practice. Brandenberg et al. (2010) and Wair et al. (2012) summarized the previous studies and showed that most prediction models use the functional form of $V_s = AN^B$. Brandenberg et al. (2010) also presented multiple linear regression models for V_s by using N and effective overburden stress (σ'_{vo}) as predictor variables, showing that it improves the prediction capabilities. However, it is recognized that V_s prediction models range widely between sites that is an importance issue to improve the prediction capability (Brandenberg et al. 2010 and Wair et al. 2012). Based on this reason, this paper proposed an approach to calibrate V_s prediction model for the site specific condition based on the conditional probability framework.

2 V_S PREDICTION MODELS CONDITIONED ON N MEASUREMENTS

2.1 *Model Development*

It is assumed that N is modeled with σ'_{vo} by a simple regression equation,

$$\ln N = b_0 + b_1 \ln \sigma'_{vo} + \varepsilon_N \tag{1}$$

where ε_N represents the residuals that follows normal distribution with standard deviation of σ_N. Similarly, it is assumed that V_s is modeled with σ'_{vo} by a simple regression equation,

$$\ln V_s = c_o + c_1 \ln \sigma'_{vo} + \varepsilon_{Vs} \tag{2}$$

where ε_{Vs} are residuals following normal distribution with standard deviation of σ_{Vs}. Correlation between ε_N and ε_{Vs} is also defined as ρ_{NVs} from Equations (1) and (2). Based on the conditional prediction of V_s given N measurement, the following formula is obtained.

$$E[\ln Vs \mid \ln N] = \beta_0 + \beta_1 \ln N + \beta_2 \ln \sigma'_{vo} \tag{3}$$

where

$$\beta_0 = c_0 - b_0 \frac{\sigma_{Vs}}{\sigma_N} \rho_{NVs} \tag{4}$$

$$\beta_1 = \frac{\sigma_{Vs}}{\sigma_N}\rho_{NVs} \tag{5}$$

$$\beta_2 = c_1 - b_1\frac{\sigma_{Vs}}{\sigma_N}\rho_{NVs} \tag{6}$$

Standard deviation is calculated as;

$$\sigma^2_{Vs|N} = \sigma^2_{Vs}\left(1-\rho^2_{NVs}\right) \tag{7}$$

Equations (3) to (7) become the basis of conditional probability approach.

2.2 *Variation of Simple Regression Parameters*

Database of V_s and N was developed from three data resources of Taiwan Strong Motion Instrument Program (TSMIP), Kyoshin Network (K-NET) by National Research Institute for Earth Science and Disaster Prevention and Oakland International Airport (OAK). Data from TSMIP and K-NET are available for public. The data for OAK was obtained from the previous study (Arulnathan et al. 2009). The database includes approximately 1,400 soil profiles. Effective stresses were computed for each boring from the available unit weight and depth of water table. If the depth of water table is not available, it was determined when the P-wave velocity becomes 1,500 m/s. The N is corrected to N_{60} with the associated correction factors from past studies.

By running the simple regression analyses with Equations (1) and (2) for the developed database, Figure 1 was obtained. Mean, standard deviations, and correlations of these parameters are also presented in Equations (8) to (10).

2.3 *Variation of β_0, β_1, β_2 and σ_{VsN} for prediction model of Vs with N*

Figure 2 shows the scatter plots of β_0, β_1, and β_2 for clay, silt and sand obtained from the aforementioned data sources. The figure shows that there is no clear difference in the distribution of the parameters among clay, silt and sand. This observation implies that regional factors such as geologic conditions contribute more to the variation of β_0, β_1, and β_2 more than soil type.

$$\begin{bmatrix}\hat{b}_0 & \hat{b}_1 & \hat{c}_0 & \hat{c}_1 & \hat{\sigma}_N & \hat{\sigma}_{Vs} & \hat{\rho}_{VsN}\end{bmatrix} = \begin{bmatrix}0.484 & 0.489 & 4.21 & 0.251 & 0.840 & 0.322 & 0.400\end{bmatrix} \tag{8}$$

$$\begin{bmatrix}\sigma_{b0} & \sigma_{b1} & \sigma_{c0} & \sigma_{c1} & \sigma_{\sigma N} & \sigma_{\sigma Vs} & \sigma_{\rho VsN}\end{bmatrix} = \begin{bmatrix}1.97 & 0.436 & 0.763 & 0.163 & 0.212 & 0.115 & 0.276\end{bmatrix} \tag{9}$$

$$\begin{bmatrix}
1 & \rho_{b0b1} & \rho_{b0c0} & \rho_{b0c1} & \rho_{b0\sigma N} & \rho_{b0\sigma Vs} & \rho_{b0\rho VsN}\\
\rho_{b0b1} & 1 & \rho_{b1c0} & \rho_{b1c1} & \rho_{b1\sigma N} & \rho_{b1\sigma Vs} & \rho_{b1\rho VsN}\\
\rho_{b0c0} & \rho_{b1c0} & 1 & \rho_{c0c1} & \rho_{c0\sigma N} & \rho_{c0\sigma Vs} & \rho_{c0\rho VsN}\\
\rho_{b0c1} & \rho_{b1c1} & \rho_{c0c1} & 1 & \rho_{c1\sigma N} & \rho_{c1\sigma Vs} & \rho_{c1\rho VsN}\\
\rho_{b0\sigma N} & \rho_{b1\sigma N} & \rho_{c0\sigma N} & \rho_{c1\sigma N} & 1 & \rho_{\sigma N\sigma Vs} & \rho_{\sigma Vs\rho VsN}\\
\rho_{b0\sigma Vs} & \rho_{b1\sigma Vs} & \rho_{c0\sigma Vs} & \rho_{c1\sigma Vs} & \rho_{\sigma N\sigma Vs} & 1 & \rho_{\sigma N\rho VsN}\\
\rho_{b0\rho VsN} & \rho_{b1\rho VsN} & \rho_{c0\rho VsN} & \rho_{c1\rho VsN} & \rho_{\sigma Vs\rho VsN} & \rho_{\sigma N\rho VsN} & 1
\end{bmatrix} = \begin{bmatrix}
1 & -0.955 & 0.414 & -0.394 & 0 & 0 & 0\\
-0.955 & 1 & -0.402 & 0.455 & 0 & 0 & 0\\
0.414 & -0.402 & 1 & -0.938 & 0 & 0 & 0\\
-0.394 & 0.455 & -0.938 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0.373 & 0\\
0 & 0 & 0 & 0 & 0.373 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \tag{10}$$

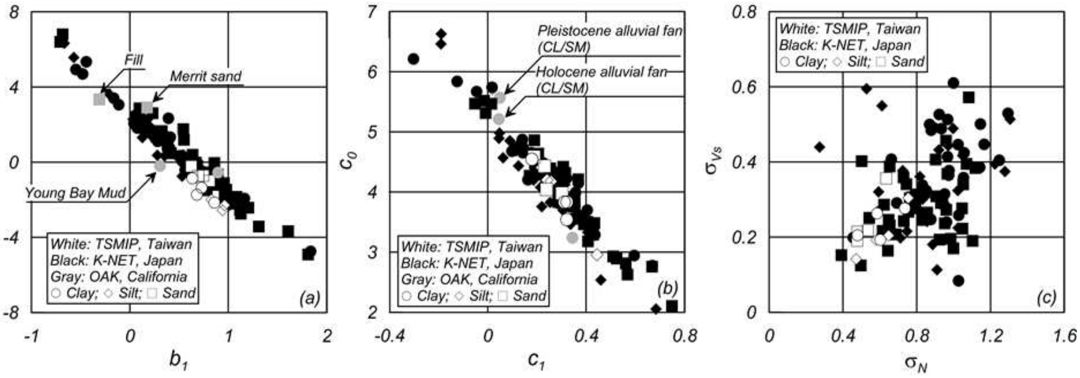

Figure 1. Variation of b_0, b_1, c_0, c_1, σ_{Vs}, and σ_N for clay, silt, and sand

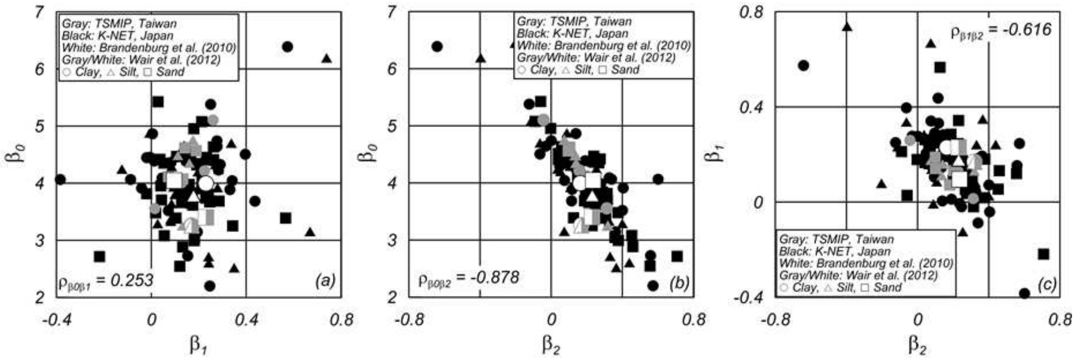

Figure 2. Variations of (a) β_0 and β_1, (b) β_0 and β_2, and (c) β_1 and β_2 for clay, silt and sand. Figures also show the parameters from the previous studies by Brandenberg et al. (2010) and Wair et al. (2012).

Figure 2 also shows the regression parameters by Brandenberg et al. (2010) and Wair et al. (2012) for comparison. The results by Brandenberg et al. (2010) locate nearly the center of distributions obtained from the database, where the results by Wair et al. (2012) are slightly shifted from the center of distributions, but still within the range of the parameter distributions.

3 CALIBRATION OF SITE SPECIFIC VS PREDICTION MODEL WITH AVAILABLE N

This section describes the development of a site specific V_s prediction model where only N measurements are available from several borings, but no V_s measurements. The c_0 and c_1 in Equation (2) are estimated conditioned on the b_0 and b_1 measurements as follows:

$$\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} \hat{c}_0 \\ \hat{c}_1 \end{bmatrix} + \Sigma_{12}\Sigma_{22}^{-1} \begin{bmatrix} b_0 - \hat{b}_0 \\ b_1 - \hat{b}_1 \end{bmatrix} \quad (11)$$

By substituting Equations (8), (9) and (10) into (11), the following expressions are obtained:

$$c_0 = 4.21 + 0.132 b_0 - 0.132 b_1 \quad (12)$$

$$c_1 = 0.0689 + 0.0381 b_0 + 0.335 b_1 \quad (13)$$

Similarly, the σ_{Vs} is obtained conditioned on σ_N as follows:

$$\sigma_{Vs} = 0.152 + 0.202 \sigma_N \quad (14)$$

Therefore, substituting Equations (12), (13) and (14) into (4), (5) and (6), β_0, β_1 and β_2 in Equation (3) are obtained conditioned on the b_0, b_1 and σ_N as follows:

$$\beta_0 = 4.21 + \left(0.0512 - \frac{0.0608}{\sigma_N}\right) b_0 - 0.132 b_1 \quad (15)$$

$$\beta_1 = 0.080 + \frac{0.0608}{\sigma_N} \quad (16)$$

$$\beta_2 = 0.0689 + 0.0381 b_0 + \left(0.254 - \frac{0.0608}{\sigma_N}\right) b_1 \quad (17)$$

Urayasu, Japan where the lateral spread was observed after 2011 Tohoku earthquake (Ashford et al. 2011) is selected as an example site. Figure 3 shows the variation of N_{60} against σ'_{vo} for sand deposit beneath the fill. The figure shows that b_0 and b_1 in Equation (1) are 1.77 and 0.182, respectively. The standard deviation of σ_N is obtained as 0.663. Based on these variables, β_0, β_1 and β_2 are obtained from Equations (15) to (17); hence the V_s prediction model conditioned on these measurements is obtained as follows:

$$\ln V_s = 4.11 + 0.172 \ln N + 0.166 \ln \sigma'_{vo} \tag{18}$$

Standard deviation is calculated based on Equations (14) as follows:

$$\sigma_{Vs|N} = 0.262 \tag{19}$$

By following this approach, the site specific V_s prediction models can be calibrated by using the global database and the available N measurements at application sites.

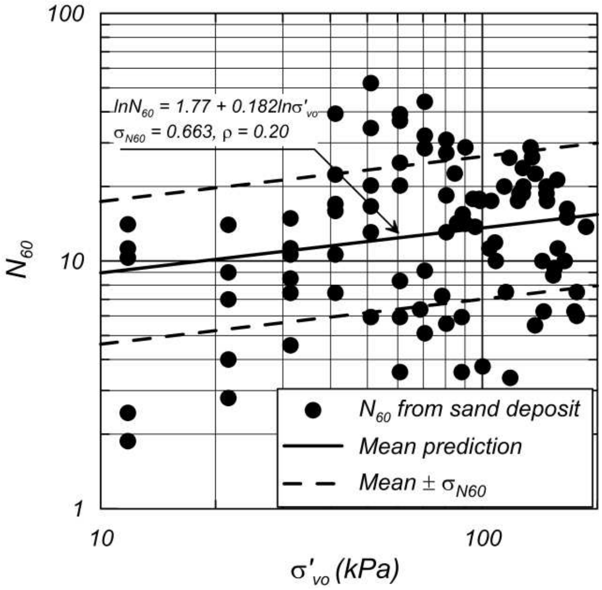

Figure 3. Variation of N_{60} for sand deposit beneath artificial fill against σ'_{vo}.

4 CONCLUSIONS

The V_s prediction model conditioned on SPT N is discussed in this paper. This study focuses on the calibration of V_s prediction model with the site specific database, where previous studies mainly focuses on the improvement of the model based on soil type such as clay, silt and sand.

Simple regression parameters of $\ln V_s$ and $\ln N$ against $\ln\sigma'_{vo}$ were computed from global database. Based on these analyses, means, standard errors and correlations of these regression parameters are obtained as a basis to develop the site specific V_s prediction models.

Methodology to obtain the site specific V_s prediction model is described when only N are available from several borings, but no V_s measurement. The proposed approach can reflect the trend observed in $\ln N$ vs. $\ln\sigma'_{vo}$ at application sites whereas previous studies have to assume the same site condition between where V_s prediction model was developed and the application site. This is a significant improvement to adjust the V_s prediction models to the local site conditions. The proposed approach can be applicable to any sites since no specific assumption exists behind the development of conditional V_s prediction model with N.

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