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Probabilistic assessment of laterally loaded pile performance in sand

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ABSTRACT: A probabilistic assessment of the lateral response of a pile in a uniformly graded dune sand is presented using CPT and load test data from a site in Perth, Western Australia. The assessment, which employs a new CPT-based design method, provides designers with an appreciation of the level of uncertainty associated with serviceability limit state design for laterally loaded piles in sand. It is shown that the lateral load range likely to induce a pile head rotation gradient of 1% (=0.57°) is significantly lower than the range anticipated from the CPT variability. Predictions of lateral response using the CPT method are not very sensitive to randomly generated q_c profiles with assigned mean and standard deviation values at each depth.

1 INTRODUCTION

The performance of laterally loaded piles is currently the subject of considerable attention due to the rapid growth in the numbers of wind turbines being constructed around the world. These turbines are usually founded on a single pile (or monopile) and experience large wind loads (as well as wave loads for offshore turbines). The lateral displacements of these piles generally govern their design as rotations of the mast in excess of about 1% can render the turbine un-serviceable. This paper presents a probabilistic assessment of lateral pile response using a new CPT based design method with the aim of providing designers with an appreciation of the level of uncertainty associated with serviceability limit state design for laterally loaded piles in sand.

The paper examines the effects of spatial variability of the ground parameters using an intensive series of CPTs performed at the site of lateral pile tests conducted in Perth, Western Australia. The primary calculation method for the sand's *p-y* curves is the direct CPT approach for sands proposed by Suryasentana & Lehane (2014, 2016) and this is shown to provide a reasonable prediction for the lateral response measured by the test piles using the average measured q_c profile and the *LAP* laterally loaded pile program (Doherty 2014). A Monte Carlo simulation is performed involving generation of random q_c profiles consistent with the assessed site variability and their combination with the *LAP* program to generate probability density functions for the load that would cause 1% rotation gradient (=0.57°) at the pile head. These functions are then used to examine the relationship between the uncertainty in the load required to cause a 1% rotation and the variability of the ground parameters. The paper also examines this relationship when employing the widely used American Petroleum Institute (API 2011) recommendations for lateral pile design in sand.

2 LATERALLY LOADED PILE ANALYSIS

The lateral load-displacement (*p-y*) curves recommended by the American Petroleum Institute (API 2011) are commonly used for the design of laterally loaded piles in sand. These recommendations are based on physical tests on relatively small diameter piles and simply require assessment of an operational friction angle for the sand. However, calculated lateral pile responses show a wide variability in practice, primarily because of the range of methods used by designers to determine ϕ' and the sensitivity of the *p-y* curves to the value of ϕ'. Such variability prompted the method developed by Suryasentana & Lehane (2014, 2016) which makes direct use of the CPT q_c value rather than inferred friction angles. This method, referred to here as the S&L method, was derived via a regression analysis on a large series of 3D Finite Element computations that predicted the lateral pile response in a variety of different sands and a cavity expansion approximation using Finite Elements to predict corresponding CPT q_c profiles in each sand deposit. The updated S&L formulation for a circular pile is provided in Equation (1) (Suryasentana & Lehane 2016), where D is the pile diameter and z, σ_v, σ'_v, q_c and G_{max} are the depth, vertical total stress, vertical effective stresses, CPT end resistance and small strain shear modulus at the level of the p-y spring. u_g is the water pressure at the ground surface (which is non-zero for offshore applications).

$$p = 4.5\, G_{max}\, y\,; \quad y/D \le 0.0001 \qquad (1a)$$

$$p = p_u\, f(y); \quad y/D \ge 0.01 \qquad (1b)$$

where

$$f(y) = 1 - exp\left(-8.9\left(\frac{y}{D}\right)\left(\frac{\sigma_v - u_g}{\sigma'_v}\right)^{0.5}\left(\frac{z}{D}\right)^{-1.25}\right) \qquad (1c)$$

$$p_u = 2.4\,\sigma'_v D\left(\frac{q_c}{\sigma'_v}\right)^{0.67}\left(\frac{z}{D}\right)^{0.75} \le q_c D \qquad (1d)$$

For simplicity, the very small strain component of the p-y curves (Equation 1a) is ignored in the following and Equation 1b is assumed to be applicable at all y/D values.

The *LAP*, laterally loaded pile program (Doherty 2014), was used to calculate lateral pile response as it incorporates Equation (1) in one of its available p-y options. The program uses beam elements to represent pile sections and requires specification of the diameter and flexural rigidity of these sections and their corresponding yield moments. The soil unit weight and position of the water table are specified in addition to the CPT q_c profile (which can be copied directly from the CPT Contractor's data files). The program generates about ten p-y springs per metre depth.

3 SHENTON PARK LATERAL LOAD TEST

This paper uses Equation (1) to examine the sensitivity of variations in CPT q_c to the performance of a laterally loaded test pile conducted at Shenton Park, Perth, Western Australia. The stratigraphy at the Shenton Park site comprises a 5m to 7m thick deposit of siliceous dune sand overlying weakly cemented limestone. The sand is sub-angular to sub-rounded with D_{50}, D_{60} and D_{10} values of 0.42mm, 0.47mm and 0.21mm respectively. The water table is typically just above the top of the limestone and the sand has a low level of saturation (< 15%), which varies seasonally by up to about 5%. A full description of the properties of the Shenton Park sand is provided in Lehane et al. (2004).

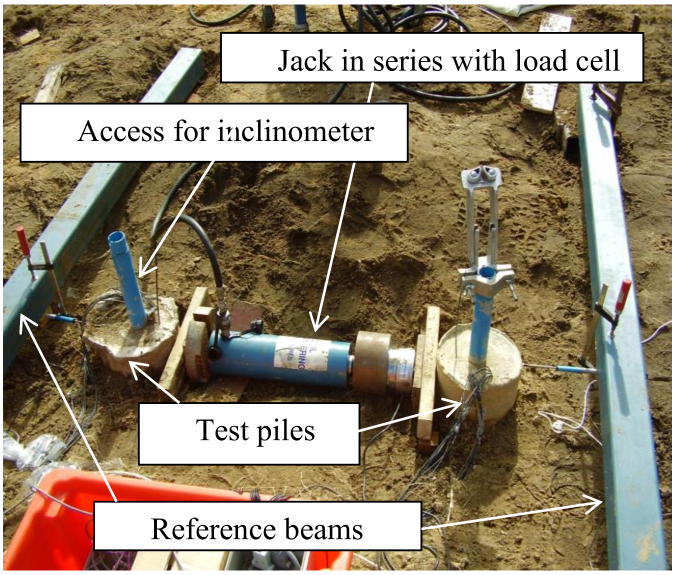

Figure 1. Lateral load test set up at Shenton Park

Two, 225mm diameter, 3.5m long grout piles were constructed using the continuous flight auger (CFA) technique. Lateral tests were performed as shown on Figure 1 by pushing the piles apart using a jack aligned approximately 0.15m above ground level. The q_c values recorded in 12 No. CPTs performed within 10m of the test piles are plotted on Figure 2. The coefficient of variations (CoVs) of the q_c values in the lateral direction are also shown on this figure and indicate a higher level of variability near the surface (which is typical of many sites).

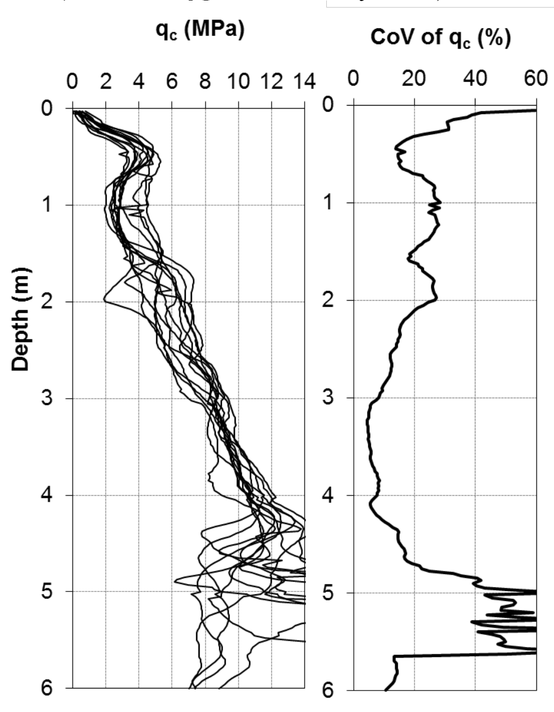

Figure 2. CPT end resistances and their variation at Shenton Park

The lateral load – displacement variation calculated with *LAP* using an initial uncracked EI value of 2420 kNm^2 and yield moment (M_y) of 9 kNm is compared with the average measured response (of the two test piles) on Figure 3. It is noteworthy that the lateral displacements of each of the two test piles at any given load differed by less than 5 to 10%.

The LAP calculation on Figure 3 adopted the mean q_c profile derived using the CPTs plotted on Figure 2. The EI and M_y values employed were measured directly in a separate on-site calibration exercise, which involved lateral loading at the heads of identical piles after excavation of a large 1m deep pit in the vicinity of the piles (Luff 2007).

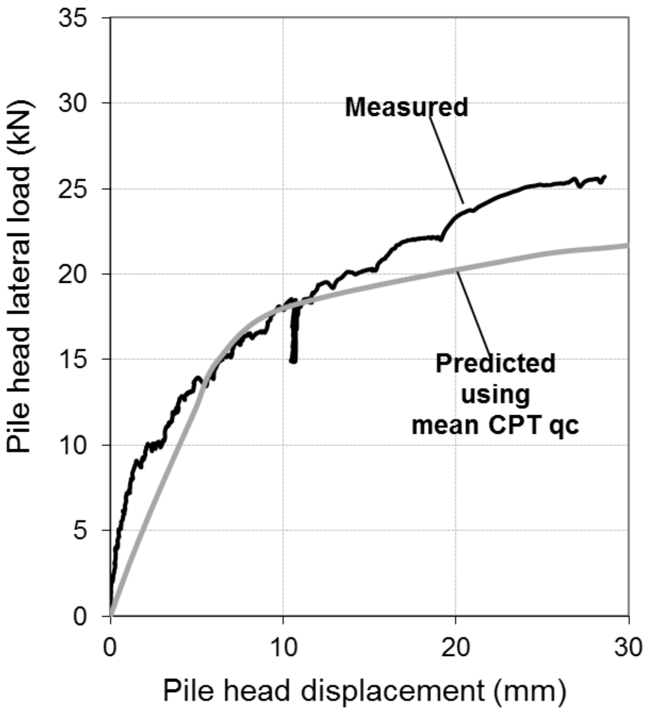

Figure 3. Measured and calculated pile head lateral load-displacement variations

Figure 3 shows that the S&L method provides a reasonable prediction of the lateral pile response, although the initial lateral stiffness is under-estimated. The lateral load required to cause a rotation of 1/100 at the location of the applied load is referred to here as $H_{0.01}$, and was approximately 18 kN for the case on Figure 3 (at which stage the pile head displacement was about 11.5 mm).

4 PROBALISTIC ANALYSIS

4.1 Predictions for Shenton Park using Equation 1

Fifty random CPT profiles for the Shenton Park site were generated using Microsoft Excel © for a Monte Carlo analysis. The random generator employed to generate these profiles assumed a normal distribution and hence required specification of a mean CPT q_c value and corresponding standard deviation. As described by Phoon & Kulhawy (1999) and illustrated in Figure 4, the spatial variation of q_c at any depth z can be decomposed into a smoothly varying trend function (q_{ct}) and a fluctuating component q_{cf}, i.e.

$$q_c(z) = q_{ct}(z) + q_{cf}(z) \qquad (2)$$

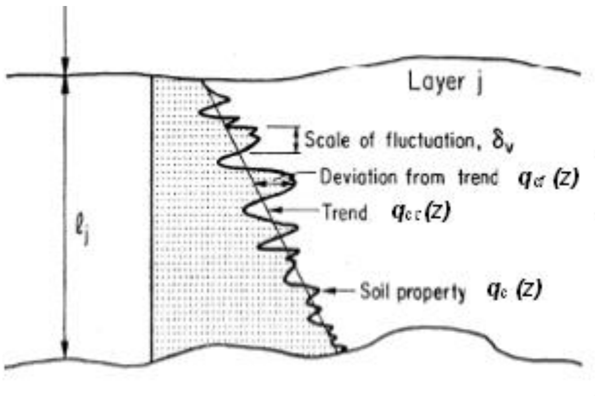

Figure 4. Statistical representation of q_c profile (Phoon & Kulhawy 1999)

The fluctuating component, $q_{cf}(z)$, represents the inherent variability of q_c and its mean and variance should not be depth dependant. The vertical scale of fluctuation (δ_v) at the Shenton Park site was assessed to be 0.5m (but 0.25m in the upper 0.5m) assuming a general trend for q_c increasing from zero at the surface to about 3.8 MPa at 0.5m, reducing to about 2.5 MPa at 1m depth and then increasing linearly with depth to 7.3 MPa at 3m depth. The coefficient of variation assumed for lateral variability of q_c is shown on Figure 2.

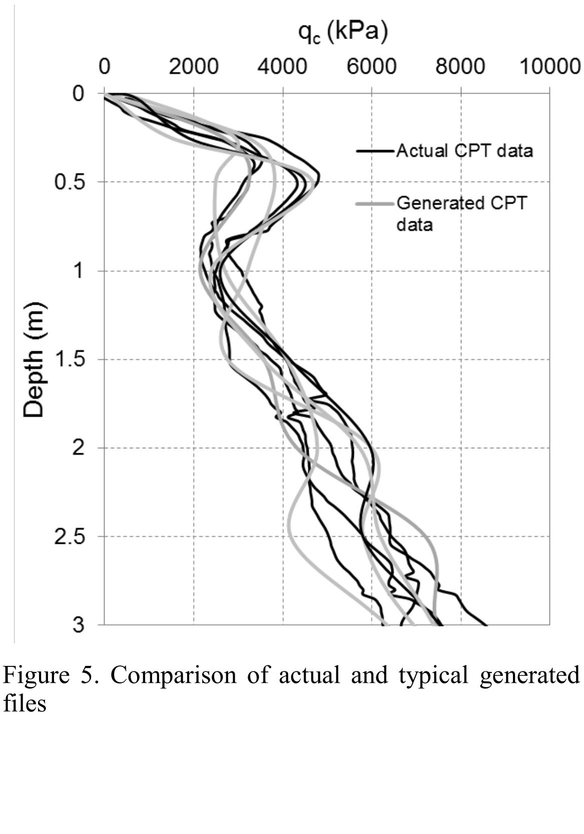

Figure 5. Comparison of actual and typical generated q_c profiles

Examples of randomly generated q_c traces are compared in Figure 5 with CPT data recorded at the site. The randomly generated data reflect the measurements well, although some minor localised peaks are not captured. Trials using a smaller δ_v value led to far greater 'peakiness' in the generated profiles which did not reflect the measurements.

The *LAP* program was then used to derive $H_{0.01}$ values with a node spacing of 0.025m for each of these fifty CPT profiles, assuming the *p-y* response of the sand is given by the S&L method. Calculations were performed assuming (i) the pile yield moment of 9 kNm and (ii) the pile remained elastic during lateral loading. Mean values of $H_{0.01}$ of 18.7 kN and 26 kN were calculated for analysis set (i) and (ii) respectively. The larger $H_{0.01}$ values determined for the elastic pile may be anticipated from extrapolation of the initial stages of the *LAP* calculated load-displacement curve on Figure 3.

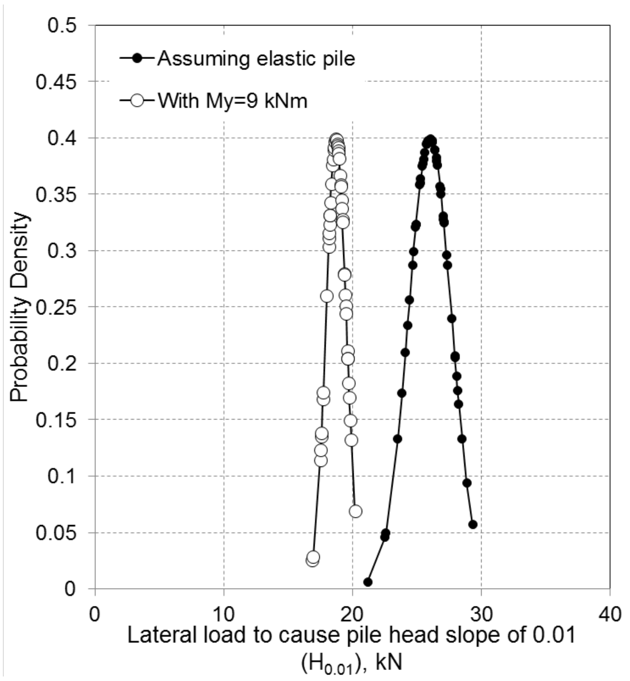

Figure 6. Probability densities for $H_{0.01}$ at Shenton Park, with and without pile yield

The probability density functions determined using the 50 computations of $H_{0.01}$ for each analysis set are plotted on Figure 6. It is clear that there is a very narrow spread in calculated $H_{0.01}$ values, despite the relatively wide range of q_c resistances; this trend is consistent with the close similarity of the measured responses of the two test piles.

The calculated coefficients of variation (CoV) of $H_{0.01}$ allowing for pile yield and assuming elastic piles are 0.042 and 0.064 respectively. These coefficients are more than five times smaller than the mean CoV for the q_c data at Shenton Park. It is therefore clear that high, normally distributed, variability in q_c values at a site does not lead to a correspondingly high variability in $H_{0.01}$ values.

The significantly lower CoV for $H_{0.01}$ compared to that of the CPT q_c arises because of the assumption of randomly distributed q_c values. Separate analyses to confirm this observation were performed using the following two q_c profiles:

$$q_c\ (\text{lower}) = q_{c,mean}\,(1 - \text{CoV}) \qquad (3a)$$

$$q_c\ (\text{upper}) = q_{c,mean}\,(1 + \text{CoV}) \qquad (3b)$$

Output from the analyses indicated that, in line with Equation (1d), the ratios of the ultimate pressures developed on the piles in the q_c (lower) profile to those developed in the q_c (upper) profile were equal to $[(1\text{-CoV})/(1\text{+CoV})]^{0.67}$, which amounts to 0.76 at CoV=0.2 and 0.23 at CoV=0.8. The corresponding ratios of $H_{0.01}$ were a little higher than these values because of the dependence on y/D (Equation 1c) and varied from 0.84 for a CoV=0.2 to 0.37 for CoV=0.8.

4.2 Additional calculations using Equation 1 and the stratigraphy at Shenton Park

Further analyses were performed adopting the same pile configuration as the test piles at Shenton Park, assuming the mean q_c profile, with the pile *EI*=2420 kNm^2 and yield moment, M_y= 9 kNm. However, for these cases, the random generation of 50 CPT profiles adopted a constant CoV for q_c over the depth of the piles. The probability density functions determined in these analyses using *LAP* (with *p-y* curves given by Equation 1) for q_c CoVs of 0.2, 0.4, 0.6 and 0.8 are presented on Figure 7. These again confirm the low sensitivity of the computed $H_{0.01}$ values to the variability in q_c values. The mean $H_{0.01}$ value was about 18 kN for all cases. More significantly, the calculated CoV for $H_{0.01}$ was only 10% of the CoVs of the q_c profiles for CoV=0.2 increasing to 20% of the of the CoVs of the q_c profiles at CoV=0.8.

An appreciation of the relative insensitivity of $H_{0.01}$ values at Shenton Park to variability of the q_c values may be better understood in terms of probabilities. If, for example, the CoV for q_c at the sand site is 0.4 (this CoV is considered relatively high) and the expected $H_{0.01}$ value is the mean given by equation 1 i.e. 18.6 kN, the foregoing analyses suggest that there is a probability of less than 0.001 that $H_{0.01}$ exceeds 22 kN.

Separate analyses were also performed assuming that CPT q_c values at Shenton Park. These showed that:

- closely comparable trends are observed if the q_c values are assumed to be log normally distributed (for CoV<0.5);
- as for the variable CoV analyses, the CoVs of $H_{0.01}$ assuming an elastic pile were about 50% higher than those adopting M_y=9kNm

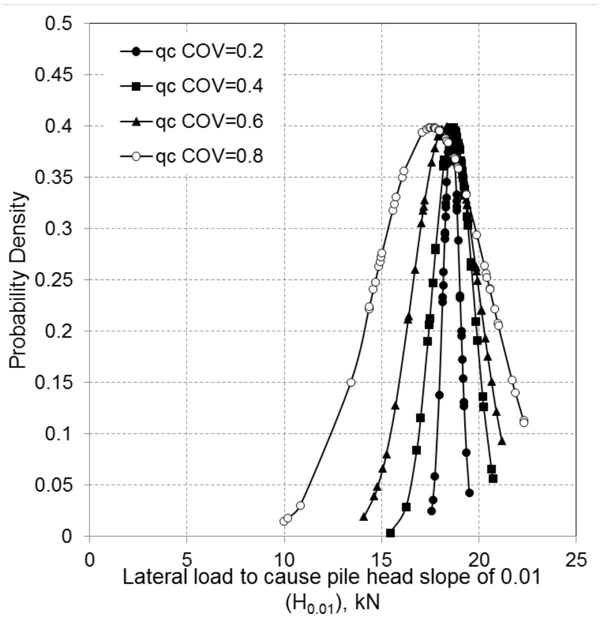

Figure 7 Probability densities for various (constant) COVs for q_c at Shenton Park (M_y=9 kNm)

4.3 *Predictions for Shenton Park using the API (2011) method*

One of the most popular means of determining the lateral response of piles in sand is the method recommended in API (2011). The *p-y* curves derived using this method are a function of the relative depth (*z*/D) and the sand's friction angle (ϕ').

For the purposes of this study, use is made of the site specific correlation given in Lehane et al. (2004) between the relative density (D_r) of the Shenton Park sand and the CPT q_c value. The sand's friction angle was then determined using the correlations with D_r presented by Bolton (1986). The mean of Bolton's expressions for triaxial compression and plane strain ϕ' values was used in calculations. The CoVs for q_c from which D_r values were derived were the same as the site specific values shown on Figure 2; the associated randomly generated profiles of ϕ' at Shenton Park are shown on Figure 8. The mean value of ϕ' determined was 39.6°, which is about 3 degrees less than the operational angle backfigured by Li & Lehane (2010) for a cantilever retaining wall at Shenton Park. Interestingly, the average generated CoV for ϕ' over the whole profile (to 3.5m depth) is only 0.068, which is considerably less than the CoV for q_c.

The calculations made using LAP and the API (2011) sand model with the randomly generated friction angle profiles plotted on Figure 8 show an even smaller spread of $H_{0.01}$ compared to that calculated using the direct CPT method (i.e. Equation 1). The CoV calculated for $H_{0.01}$ for an elastic pile using API is only 0.047 compared with 0.064 obtained using Equation 1. This trend arises because of the low CoV for ϕ', which effectively leads to very similar average ϕ' values and hence *p-y* curves within the lateral zone of influence of the pile.

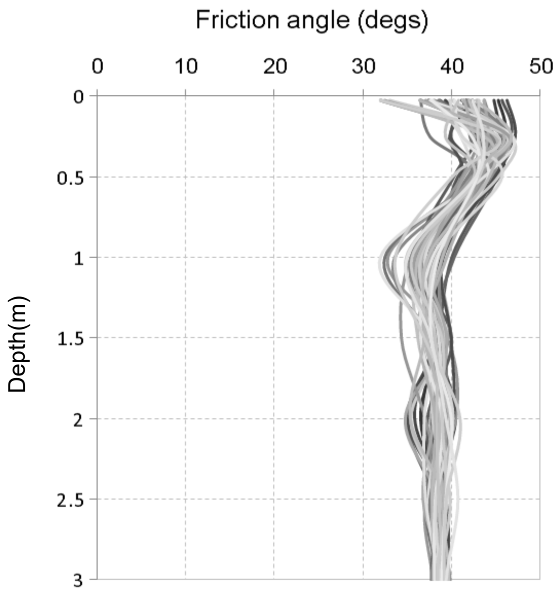

Figure 8. Randomly generated ϕ' values at Shenton Park

The mean $H_{0.01}$ value of 27.6 kN determined using API and the ϕ' values on Figure 7 was very close to that calculated an elastic pile using Equation 1 (=26 kN; see Figure 3). Given the very low variability in calculated $H_{0.01}$ values, it follows that, when using API, it is more important to obtain confidence in the method employed to determine ϕ'. Unfortunately there are a wide range of methods in common use to assess ϕ', with the consequence that most assessments are highly subjective. Back-analysis of the Shenton Park tests using the API method suggests that the mean operational friction angle was about 40° whereas, as part of a settlement prediction exercise at Shenton Park (Lehane et al. 2009), practitioners estimated ϕ' values at Shenton Park of between 30° and 36°. In this respect, direct methods, such as Equation (1), provide a clear advantage over indirect methods such as API (2011).

5 EXTRAPOLATION TO LARGE DIAMETER PILES AND LARGER ROTATIONS

4.1 *Extrapolation to larger diameter*

A number of analyses were performed involving application of a lateral load at pile head level to 2m diameter, 20m long, 50mm wall thickness (t), steel tubular piles in saturated sand; the sand surface was 1.3m below the point of lateral load application. The same statistical approach as outlined above for the (relatively small scale) Shenton Park tests was employed. The analyses adopted Equation (1) to derive *p-y* springs and assumed that the sand had a constant relative density (D_r). The following relationship proposed by Jamiolkowski *et al.* (1985) for a normally consolidated sand of medium compressibility was used to relate q_c with D_r:

$$q_c \text{ (kPa)} = 97 (\sigma'_v)^{0.5} 10^{1.52D_r} \qquad (3)$$

where σ'_v is given in kPa and D_r is expressed as a fraction. The pile was assumed elastic and the CoV for q_c at any given depth was taken as 0.4 i.e. q_c had a relatively high variability.

These analyses showed that the value of $H_{0.01}$ had approximately the same level of sensitivity to q_c variability as that shown by the smaller pile at Shenton Park i.e. for a q_c CoV of 40%, the CoV for $H_{0.01}$ was only 5.5%.

4.2 *Extrapolation to larger pile head rotation*

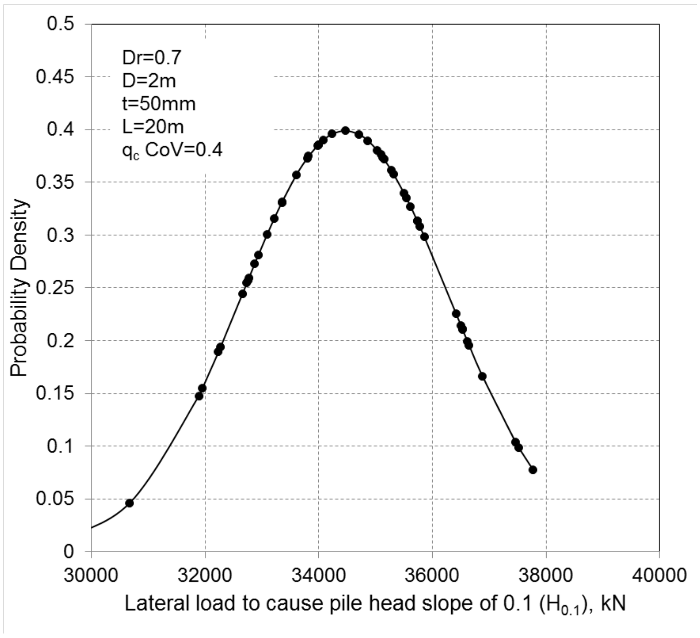

Figure 9 $H_{0.1}$ values for steel pipe pile in saturated sand with D_r=0.7

To further examine the sensitivity to random variations of q_c profiles with a given CoV, calculations were carried out using Equation (1) to determine the likely range of loads causing a pile head rotation of 10%; the same pile geometry and sand conditions as employed above were employed i.e. a 2m diameter pipe pile in a uniform saturated sand with D_r=0.7. The CoV for q_c at any given depth was also taken as 0.4 and the pile was assumed elastic.

The results from the computations are presented on Figure 8. The results equate to a CoV of about 5.5% for loads causing a rotation of 10% of the pile head, $H_{0.1}$ i.e. the CoV for both $H_{0.1}$ and $H_{0.01}$ was about 14% of the CoV for CPT q_c variability. As for the Shenton Park case, a much wider range of $H_{0.1}$ values are obtained if the lower-bound and upper-bound q_c values are employed in the calculations (rather than randomly generating q_c profiles with a given mean and CoV).

6 CONCLUSIONS

The calculations summarised in this paper have shown that the lateral load range likely to induce a given level of pile head rotation for a pile in sand is significantly lower than the range anticipated from the CPT q_c variability.

The CPT-based method for laterally loaded piles proposed by Suryasentana & Lehane (2014,2016) is seen to provide good predictions for the lateral response of a test pile at a medium dense sand site. Predictions using this method and using the API sand method have a low sensitivity to randomly generated q_c or ϕ' profiles that are normally distributed at any given depth.

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