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Quantifying and reducing uncertainty in down-hole shear wave velocities using signal stacking

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ABSTRACT: In down-hole seismic testing, shear wave velocities are interpreted from measured shear wave signals and the geometric positioning of the seismic source and receivers. The quality of the shear wave signals can be improved through signal stacking, however signal stacking decreases the production rate of down-hole seismic testing and has diminishing returns with each subsequent trace. In this paper we quantify the improvement in the interpreted shear wave propagation time that can be realized through signal stacking up to six traces. Results are presented for a seismic cone penetration test performed to a depth of over 30 m at a site in San Leandro, CA, America. We used an automatic seismic source to generate reproducible seismic waves. Each seismic test comprised of at least five left-side and right-side (reverse polarity) generated shear waves. In this paper we show how to quantify noise for each seismic test, how to calculate the signal to noise ratio for the shear wave, and how to calculate a probability distribution for the shear wave propagation time. We show that SNR increases with signal stacking; which results in a decrease in the shear wave interval time error. We quantified the interval time error by propagating the measurement uncertainty through the cross-correlation function and characterizing a probability distribution for the location of the peak cross-correlation. Quantifying down-hole seismic uncertainty does not require any special equipment or field procedures beyond recording at least two seismic traces. This work has implications in the engineering applications of shear wave velocity profiles. Sensitivity analyses can be performed considering the reported uncertainty in the shear wave velocity profile.

1 INTRODUCTION
Down-hole shear wave velocity profiles are interpreted by comparing adjacent recorded seismic traces. The quality of the traces can be improved through signal stacking, where the seismic test is repeated at a single depth and the recorded traces are added together. Signal stacking amplifies the systematic features in the trace such as the shear wave, compression wave, and any reflected waves. The improvement from signal stacking is diminished with each additional trace. Signal stacking also adversely affects the production rate of down-hole seismic testing.

In this paper we evaluate the benefit of signal stacking on down-hole seismic tests performed during a seismic cone penetration test (SCPTu). We quantify the noise and the signal to noise ratio (SNR) for each recorded trace. We calculate the SNR for the stacked signals. We then compared the error in the interval shear wave time to the calculated SNR.

This work was demonstrated using down-hole seismic tests performed during a SCPTu in San Leandro, California, United States. An automatic hammer was used to generate reproducible seismic waves at the ground surface. Seismic tests were repeated at least ten times, with half being inverse polarity, at each depth. The noise was quantified by subtracting signals to remove systematic features. The SNR was calculated by comparing the square of the amplitude of the seismic wave to the square of the amplitude of the noise. We assumed that the noise could be represented by a zero-mean Gaussian distribution. The noise-distribution was propagated through the cross-correlation function to estimate the error in the interval shear wave velocity time.

This paper contains three contributions to improve down-hole seismic profiles. First, we show how to calculate the noise in a set of down-hole seismic traces. Second, we show how to quantify the SNR for a trace and for stacked signals. Third, we show how to evaluate the error in the propagation time when comparing two seismic traces. Unsurprising-
ly, we observed that an increase in SNR reduces the error in the shear wave propagation time.

2 DOWN-HOLE SEISMIC TESTING

2.1 Cone Penetration Testing (CPTu)

Cone penetration testing (CPTu) is performed by advancing an instrumented probe into the soil at a rate of 2 cm/sec. The standard CPTu probe includes measures of the tip resistance (q_t), sleeve friction (f_s), dynamic fluid pressure (u_2), inclination, and temperature every 2.5 or 5 cm of depth. The cone tip resistance requires a systematic correction from q_c to a corrected value of q_t using q_t = q_c + u_2(1 - a). The net tip resistance (q_t - \sigma), depends primarily on the soil type, in-situ effective stress, and in-situ density. The fluid pressure depends on the ambient fluid pressure and induced fluid pressure due to cone insertion and soil shearing. The CPTu is a reproducible and efficient site investigation technique for profiling soil deposits and estimating soil properties.

2.2 Seismic Cone Penetration Testing (SCPTu)

Rice (1984) described the combination of cone penetration testing with down-hole seismic testing to create the SCPTu. The seismic cone penetrometer includes a seismic sensor behind the friction sleeve. This push-in tool provides excellent coupling between the sensor and the in-situ soil; which can be a significant challenge in bore-hole seismic testing. In the work by Rice (1984), seismic shear waves were generated at the ground surface by striking the ends of a normally loaded wooden plank with a sledge hammer. Opposite ends of the plank were struck to invert the polarity of the shear wave. Seismic testing with a sledge hammer source was performed every metre of cone penetration, coinciding with the cone deployment rod lengths. Adjacent signals were compared to determine the interval propagation time. Rice (1984) found that the first cross-over point provided the most consistent characteristic point in the seismic trace. The first cross-over point is the point where the shear-wave and reverse-polarity shear wave traces cross. The first cross-over point for each seismic test depth was compared to determine the interval shear wave propagation time. The geometry of the seismic source and in-situ receiver are used to determine the interval propagation length. The shear wave velocity is the ratio of the interval propagation length to the interval propagation time.

Rice (1984) also compared a true-interval module to a pseudo-interval module. A true-interval cone uses two receivers at a fixed spacing along the cone probe. Signals from both receivers are recorded simultaneously when a seismic wave is generated at the ground surface. A pseudo-interval seismic cone uses a seismic receiver at a single location. The receiver is advanced through the soil to subsequent depths to collect a set of seismic traces. Rice (1984) observed a discrepancy between the true-interval and pseudo-interval results. He rebuilt the true-interval module using geophones that were paired based on similar resonant frequencies and oscillation characteristics. This revised true-interval probe provided results that were closer to the pseudo-interval technique. Styler et al. (2013) also observed a discrepancy using true-interval probes. True-interval modules may have a difficult to characterize systematic error due to slight variations in the sensors and mounting within the seismic tool. Furthermore, true-interval testing is still suspect to trigger-time errors when signal stacking is performed. Conversely, the errors in the pseudo-interval test are known and can be controlled and mitigated. For example, seismic testing at the end of the rod-stroke results in a very consistent and reproducible rod-stick up length and essentially eliminates the error due to probe insertion depth. Multiple tests and rejecting signals can resolve any significant trigger-time errors.

Robertson et al. (1986) presented the preliminary work on the development of the SCPTu technique. They demonstrated the viability of obtaining a down-hole seismic profile with the SCPTu. They stated that a typical survey included 40 hammer blows, but generally only 1 blow was required on each side of the beam.

Gillespie (1990) demonstrated the importance of generating reproducible seismic signals. He also interpreted the arrival of the shear wave using the first cross-over point. He showed that the generation of slightly different waveforms from the sledge hammer strike could shift the cross-over point. He showed that a metal beam provided a better seismic source than a wooden plank. To generate reproducible seismic waves, he created a pendulum with a fixed drop height to strike the metal beam.

Campanella and Stewart (1992) recommended cross-correlation over the first cross-over point. They said that the cross-over location can suffer from interference as it uses the time information at a single point. They show an example set of signals where the cross-over time had been delayed due to interference from a “step” distortion in the signal. Alternatively, the cross-correlation function compares the entire shear wave signal. For the same set of example signals, the cross-correlation interpretation was not as affected as the cross-over point.

Cross-correlation compares the entire seismic trace to determine the time shift which best aligns the two signals. The cross-correlation function can be solved with Equation 1. In this equation N is the number of discrete measurements in the signal, it is the length of the digital recorded data. The two signals are represented by x(i) and y(i). The first value of the cross-correlation function is at CC_{xy}(n=0), and
is simply the sum of the element-by-element multiplication of the two signals. When the two signals are aligned, the positive peaks and negative peaks are matched. The sum of the products for aligned signals is maximized as two negatives multiplied together results in a positive. Cross-correlation values are calculated for each time shift of \( n \), as the top signal is shifted in time. The maximum cross-correlation value corresponds to the time shift when the signals are most aligned. This time shift is taken as the propagation time.

\[
CC_{xy}(n) = \sum_{m=1}^{N-n} x(m)y(m-n)
\]

Howie and Amini (2005) further examined the interpretation methods for down-hole seismic testing. They performed numerical simulations of the down-hole seismic test and evaluated field data from a site in Richmond, British Columbia, Canada. They found generally good agreement between the first cross-over point and cross-correlation. They showed that soil-damping caused signal widening a dispersion with depth; but that this effect was somewhat countered by an increase in stiffness with depth.

Improved automatic seismic sources have been developed (Styler et al. 2014, Mayne and McGillivray 2008, Casey and Mayne 2002) since the SCPTu was introduced. Casey and Mayne (2002) presents an automatic hammer source using internal sole-noids. Mayne and McGillivray (2008) present five iterations of a compact automatic spring loaded rotating hammer source. Styler et al. (2014) used a continuous vibrating source instead of a hammer-strike generated shear wave. The work presented in this paper used an automatic hammer-strike seismic wave source.

2.3 Signal stacking

Signal stacking is performed by repeating a test and summing up the results. In down-hole seismic testing, all of the left side hits would be summed up to form a left-side stack. For a series of identical tests with random error, the signal to noise ratio increases by a factor equal to \( \sqrt{n} \). For example, stacking 4 hits should double the signal to noise ratio.

Signal stacking is commonly used to improve bender element seismic signals measured in a laboratory. Bender elements are used to obtain the shear wave velocity across laboratory soil specimens. Lee and Santamarina (2006) emphasized the use of signal stacking over amplifiers and filters that can alter the signal spectrum. Brandenberg et al. (2008) developed a fast stacking procedure for bender element testing. This technique avoided the need to wait for residual seismic signals to dissipate before sending the next pulse. The time between pulses was randomized so that only the primary arrival of the shear wave was systematic and amplified during signal stacking. They note that this approach magnifies the SNR, but not as much as \( \sqrt{n} \).

Signal stacking is also used to improve in-situ seismic measurements. Hunter et al. (2002) recommended the use of signal stacking during down-hole seismic testing if interference from tube waves occurs due to poorly grouted borehole casing. They also suggested signal stacking during non-invasive shear wave refraction investigations as traffic and wind noise may coincide with the frequency range of interest. If the noise is at the same frequencies as the shear wave it cannot be easily filtered.

Meunier et al. (2001) used signal stacking to develop a permanent seismic monitoring technique using low energy stationary seismic sources. The objective was to monitor gas storage in a deep reservoir. Permanent seismic sensors were installed in-situ on the casing of three wells. To monitor small changes in the reservoir they needed permanent stationary seismic sources. This constraint prevented the use of a mobile 30000 lb vibrator. Their solution was a low energy piezoelectric seismic source. After stacking 1280 signals the results were comparable to the large vibrator results.

Signal stacking is a powerful technique to increase the signal to noise ratio. It is used for seismic testing in the laboratory and in-situ.

3 SITE INVESTIGATION AND INTERPRETATION METHODS

Our objective for this paper was to evaluate how much signal stacking improves the interpretation of the down-hole Vs profile from a SCPTu. We achieved this by quantifying the SNR from the recorded traces and the stacked traces. We compared the calculated SNR to the 95% coverage window for the uncertainty in the shear wave propagation time. Calculating the propagation time error compares two signals, so we compared the error to the smaller SNR of the two signals.

This evaluation was performed on a SCPTu in San Leandro, California, United States. This site geological surface map indicates that this area comprises Young Quaternary Alluvium overlying Older Quaternary Alluvium with possible channels of Quaternary Intertidal Deposits. Figure 1 shows the SCPTu profile for this site. There are three fine grained layers from 0.5 m to 9.0 m, 12.0 m to 17.5 m, and 22.0 to 28.5 m. These layers are separated by free-draining sands. The interpretation of the velocities in the fourth column of SCPTu profile are described within this paper.

The seismic waves were generated using an automatic hammer source and received down-hole using a pseudo-interval seismic cone. The seismic
source was located 57 cm from the test-hole. The data acquisition system included amplification, analog-to-digital conversion, and data recording. The amplification was manually adjusted by the CPT operator during the SCPTu. The analog signal from the geophone is amplified prior to digital conversion. The operator had the option of accepting or rejecting each recorded seismic trace. Reasons for rejection included over or under amplification, oversaturated geophone signals, or a subjective evaluation of the quality of the seismic trace. Data was recorded at a sampling rate of 20 kHz. The seismic tests were performed at 1 m intervals corresponding to the end-of-stroke rod breaks. Seismic testing consisted of automatic striking of left and right side hits. Each test was repeated at least five times per side.

3.1 Reproducible seismic signals

Figure 2 presents a cascade plot of the seismic traces to the end of hole. In this figure the raw data from each seismic trace was stacked, divided by 10 to avoid overlapping for figure clarity, and then plotted at the seismic sensor depth. In a cascade plot, the shear wave velocity is approximately the slope of the characteristic points in the signals. This figure indicates that the shear wave velocity increases at 8 m and 18 m depth. The exact shear wave velocity is different than this approximation as the shear wave propagation length is longer than the seismic sensor depth. A cascade plot provides a quick visual review of the collected seismic dataset.

Figure 3 shows two sets of recorded seismic tests in detail. This figure shows ten traces at 18.6 m and eleven traces at 19.6 m that have been scaled to plot in clarity on the depth-axis scale. Sets of adjacent seismic traces are compared to interpret shear wave interval propagation time. Small errors in the interpretation for the propagation time can result in larger errors in the shear wave velocity.

Figure 4 shows a close-up view of the raw recorded seismic signals at a geophone depth of 19.6 m. These signals have not been filtered. They have not been scaled to a common amplitude. This figure includes six right side (red) and five left side (blue) seismic traces. It is difficult to discern the individual traces at this scale due to the reproducibly of the seismic hits and received signals.

3.2 Quantifying noise

A digital signal is a finite time-series of measurements. Each measurement is the sum of the underlying value and noise: Trace = Signal + Noise. We have assumed that the noise can be represented as a
distribution is required for a subsequent analysis of the uncertainty of the location for the cross-correlation peak. It represents the measurement uncertainty in our digital signals.

In a down-hole seismic test a long trace is recorded to capture a seismic wave that may only occur over a time window less than 100 ms. Based on this, we incorrectly assumed that the noise could be characterized over the first 50 ms or last 50 ms of each recorded trace. These windows can be seen in Figure 2. We unexpectedly observed systematic features over these time-windows. The first 50 ms of the signals collected at a geophone depth of 19.6 m are shown in Figure 5. This figure shows eleven raw traces. What is immediately apparent is that every trace is aligned. They are in-phase — this shows that the variation is not random. Figure 6 shows the last 50 ms. Again, repeated tests show systematic features in the signal; but the left and right are not in-phase. This may be due to continued resonance of the geophones or from reflected waves. It is clear from these reproducible seismic tests that the noise cannot be characterized over small time-windows before or after the main shear wave signal.

We separate the signal from the noise, the systematic from the random, by comparing repeated seismic traces. We calculated the mean trace \( T_m \) and the average deviation from the mean trace. The mean trace is simply the stacked trace divided by \( N \), the number of stacked signals. The standard deviation for the noise is the average deviation from this mean trace. It is calculated over all \( N \) signals with Equation 2. In this equation \( n \) is the total number of sample points. For 5 recorded traces with 5,000 points each, \( n \) is 25,000.

\[
\sigma_n = \sqrt{\frac{\sum_{i=1}^{N} (T_i - T_m)^2}{n-1}}
\]  

(2)

Figure 7 shows a scatter plot of the dispersion of the data for the right-side hits at a depth of 19.6 m. The noise was calculated with Equation 2 to a value of 0.0029 V. This figure includes two lines representing ±1.

3.3 Calculating the signal to noise ratio (SNR)

The SNR quantifies the quality of the recorded seismic trace. It represents how strongly the underlying seismic signal \( T_i \) is separated from the background noise \( \sigma_n \). We calculated the SNR with Equation 3 as the ratio of the maximum absolute value of the signal divided by three times the standard deviation of the noise. We used extreme values for the signal and noise in order to avoid penalizing traces that were recorded over longer durations. The extreme value for the signal typical occurs as a peak
the standard deviation of the noise (Equation 2) as the maximum value for the noise.

\[
SNR = \frac{\text{Max}(|\text{abs}(T_i)|)}{3\sigma_n}
\]  

Signal-stacking traces increases the amplitude of the trace and the variance noise. The amplitude of the trace is increased by approximately a factor equal to the number of stacked signals (N). The increase in noise is calculated as the square root of the sum of the variances of the individual trace. The effect of signal stacking on SNR is shown in Equation 4. If both the trace and the variance (\(\sigma_n^2\)) increase by N, then the SNR increases by N^{0.5}. Stacking 4 traces should increase the SNR by a factor of 2.

\[
SNR = \frac{\text{Max}(|\text{abs}(\sum_{i}^{N}T_i)|)}{3\sqrt{N\sigma_n^2}}
\]  

At a depth of 19.6 m there were six right-side seismic traces recorded. Each of these six traces is shown on the left side of Figure 8. These are the raw data plots and have not been normalized or scaled. Each plot includes an annotation for the signal to noise ratio. The right side of Figure 8 shows the effect of stacking each subsequent seismic trace. The signal to noise ratio increases with each hit. It increases by the expected rate – a factor equal to the square root of the number of stacked traces.

3.4 Quantifying Vs uncertainty

We developed a technique to quantify the uncertainty in the estimated propagation time using the cross-correlation function. This was accomplished by characterizing the measurement uncertainty in our recorded seismic signals and propagating it through the cross-correlation function. We then created a probability distribution for the location of the cross-correlation peak; which corresponds to the propagation time.

The characterized measurement uncertainty is propagated through the cross-correlation function, Equation 1. Each point in the cross-correlation function is the result of a large summation. Following the central limit theorem, a large sum of independent random variables will be approximately normally distributed. To calculate this distribution, we only need to determine the mean and variance of the inner product. We do not need a complete characterization of the product of two normal random variables. The first step is to normalize both signals using the characterized standard deviation of the noise. In these normalized signals, every single point has a standard deviation of 1. The variance of
the product of two independent normal distributions, each with a standard deviation of 1, is given in Equation 4 – as shown by Donahue (1964).

\[ \sigma^2 = \mu_x^2 + \mu_y^2 + 1 \]  

(4)

The variance of a sum is simply the sum of the variances. We have shown (Eq. 4) that the variance of a product of independent normal random variables, each with a standard deviation of 1, is also a sum. Therefore, simply using the associative property of addition, we can calculate the variance of the cross-correlation function using Equation 5. The variance is simply the sum of the square of each noise-normalized signal plus the length of the recorded signals.

\[ \sigma^2 = \sum_{m=1}^{N} \left( \frac{x(m)}{\sigma_x} \right)^2 + \left( \frac{y(m)}{\sigma_y} \right)^2 + 1 \]  

(5)

The mean of the cross-correlation function is solved using Equation 1 with the noise-normalized signals.

Evaluating the uncertainty in the cross-correlation function does not yet provide a probability distribution for the shear wave propagation time. Figure 9 shows the cross-correlation function of the first right-side hits of the signals shown in Figure 3. Figure 9 includes the range of the standard deviation calculated with Equation 5. The axis limits in Figure 9 were selected to show the detail in the cross-correlation peak. When uncertainty in the cross-correlation value is considered, it can be seen that the exact location of the peak of the cross-correlation function becomes uncertain.

We have been unable to determine a closed-form solution for the probability that each point in the cross-correlation function is larger than every other point. The two options are to perform numerical integration or a Monte-Carlo simulation. We selected a Monte Carlo simulation for the simplicity in execution. We generated 100,000 realizations of the cross-correlation function and calculated a histogram for the location of the maximum value. This histogram is converted into a probability density function, as shown in Figure 10, by dividing the histogram by the number of realizations.

A cumulative distribution function is an alternate representation of the results shown in Figure 11. It is an integration of the probability distribution function. It is a cumulative sum, or running sum, of the probability distribution function. The y-axis is the probability that the propagation time is less than the value on the x-axis. At 50% the propagation time may equally be higher or lower. This 50% value is an average, but not necessarily the arithmetic

Figure 8. Effect of signal stacking individual traces from right-side seismic testing performed at a geophone depth of 19.6 m with the left side figures representing each trace and the right side figures representing the stacked trace; each plot is annotated with the calculated SNR showing the increase with signal stacking.

Figure 9. Maximum peak from cross-correlation (Eq. 1) of the first right side hit at 18.6 m and 19.6 m with a calculated (Eq. 5) cross correlation standard deviation of 3144.
average. In Figure 11 we have annotated the values at 2.5%, 50%, and 97.5%. These three numbers provide an average propagation time and the width of a 95% coverage window. The results in Figure 11 have an average velocity of 361 m/s with a 95% coverage window of 22 m/s.

This approach quantifies the uncertainty. It permits engineering applications to use the reported data appropriately and to perform sensitivity analyses. In this paper we evaluated how signal stacking can be used to reduce the uncertainty in the shear wave propagation time.

4 RESULTS

Figure 12 shows depth profiles for the SNR for both the left and right side results. Signal stacking always increased the SNR. The left-most point represents the SNR from just the first trace. As each subsequent trace was stacked the SNR increases. In this SCPTu, the SNR generally increased to 20 m depth, then decreases below 20 m. In this SCPTu, the changes in SNR did not appear to coincide with any particular features in the profile. We did not see a loss of signal strength immediately below interfaces with high tip resistance that could reflect a portion of the seismic signal.

Figure 13 includes three depth profiles. The first one shows both the left-side and right-side interpreted shear wave velocities using the completely stacked signals. Each depth interval is represented by a block that indicates the range of the 95% coverage window. The error in this profile was so low that these windows are difficult to discern on this scale. This profile also showed remarkable consistency between the left-side and right-side interpreted shear wave velocities. Unlike cross-over points, the use of cross-correlation allows both the left-side and right-side set of data to be interpreted independently. The reproducibility of the left-side and right-side confirms the interpreted results. The middle profile shows the interval propagation time. This is the 50% value from the CDF calculated for
Figure 14 compares the width of the 95% coverage window to the calculated SNR. These results were calculated for each additional signal from signal stacking. Calculating the uncertainty of the shear wave interval propagation time requires comparing two signals, each with their own SNR. The results presented in Figure 14 show the minimum SNR. It was assumed that the smaller SNR would govern the error in the propagation time. This figure shows that increase the SNR decreases the error in the propagation time.

5 DISCUSSION

Signal stacking raw seismic traces amplifies the systematic features in the signal. This is shown clearly in Figure 8. A stronger signal, with a higher signal to noise ratio, corresponds with a lower error in the estimated propagation time. This is shown in Figure 14. Therefore, signal stacking can be used to improve the estimate of the shear wave propagation time and shear wave velocity.

At depth the signal strength decreases. This is due to geometric attenuation and soil damping. A decrease in SNR with depth below 20 m was observed and is shown in Figure 12. This decrease can be countered by signal stacking. Figure 8 and Figure 12 both show that stacking signals increases the SNR.

The required number of signals to stack to achieve an acceptable error in $V_s$ may exceed what can reasonably be accomplished while maintaining acceptable SCPTu production rates. Signal stacking has a positive impact on SNR, but it may not be enough to achieve $V_s$ uncertainty objectives. An alternative to signal stacking is to increase the propagation interval. Increasing the propagation interval reduces the significance of the error in the propagation time.

In this paper we presented the effect of signal stacking at one SCPTu using a reproducible automatic seismic source. We have not examined or isolated equipment-specific or site-specific effects on these results. We expect that the observed SNR is equipment and site dependent. Consequently, the results shown in Figure 14 cannot yet be used to specify SNR targets for other field programs. Further work is needed to examine the effect of different sites and different equipment on the SNR and propagation time error.

6 CONCLUSIONS

From this work analyzing a single profile we have supported the following conclusions:

- SNR increases with signal stacking,
- The error in the propagation time decreases with higher SNR, and
- The decrease in SNR with depth can be countered by signal stacking.

These general conclusions are completely expected, however the specific observed trends may be site-dependent and equipment dependent. The equipment used at this site produced remarkably consistent shear wave signals which provided a small quantified noise and high SNR.

The main contributions contained within this paper are the techniques we used to demonstrate the effect of SNR and signal stacking on the uncertainty in the shear wave velocity. We presented interpretation methods that can be used to:

- Quantify signal noise through signal-subtraction,
- Quantify SNR in down-hole seismic signals, and
- Calculate the error in the shear wave propagation time using the cross-correlation function.

These techniques are applicable beyond the SCPTu presented within this paper. They can be applied to any SCPTu that recorded more than one trace per seismic test.

7 REFERENCES


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